

ERROR SENSITIVITY STUDY FOR SIDE COUPLED MUFFIN TIN STRUCTURES USING A FINITE DIFFERENCE PROGRAM

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Abstract

The sensitivity of three previously proposed side coupled standing wave muffin-tin structures are estimated using a finite difference program. The muffin tins consist of rectangular cavities only. Such geometries can be discretized by a program like MAFIA without errors. But as soon as small errors in the cavity dimensions are present, MAFIA is not able to discretize them with sufficient accuracy. Therefore a finite difference program adjusted to the problem has been written that discretizes the cavity volume exactly for all cavities where the position of two of the six cavitywalls is perturbed. This program was used to calculate the shunt impedance of a big number of structures with random errors in the cavity dimensions.

I. INTRODUCTION

A class of linear accelerators has been previously proposed, which should operate at 120 GHz and could be manufactured cheaply by silicon etching or LIGA due to their rectangular geometries [2],[3]. The typical structure dimensions are 1 mm. Because the structures are so tiny, it seems almost impossible to tune them after manufacturing. Therefore it is necessary to evaluate the effects of errors in the geometry.

In a previously published paper [4], the error sensitivity of the geometries was already estimated using a lumped circuit approach. As will be seen from the present paper, the lumped circuit approach was valid. The present paper deals with calculating directly the fields in the perturbed structures and reports the averages of the calculated shunt impedances.

II. THE PROBLEM OF CALCULATING THE FIELDS IN A PERTURBED STRUCTURE

The standard linac structure operates in $2\pi/3$ -mode. The left part of Fig. 1 shows the accelerating field in a quarter of a $2\pi/3$ muffin tin. Perturbing the position of the lower wall of the 2.nd and 3.rd cavity by a fractional amount of ± 1 percent respectively changes the field pattern to that of the right of Fig. 1. Calculating the field in this geometry with MAFIA [1] is possible, but the closeness of three meshlines at the lower boundary gives rise to numerical problems.

As will be seen later, a perturbation of 1% is not tolerable. As the perturbations become smaller, the distance of the mesh-lines decreases also and the structures become untractable with MAFIA.

III. HOW TO SOLVE THE PROBLEM

In this special problem, where only the lower planes $y = y_n$ of the cavities are perturbed, calculation of the fields is possible with finite differences. The trick is, not to extend the mesh

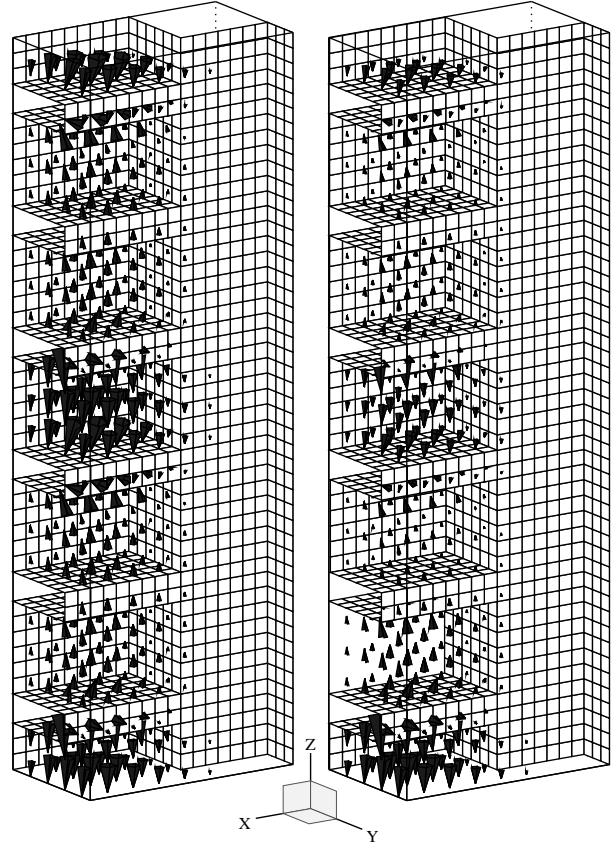


Figure 1. Left: Accelerating field in the ideal structure, Right: Field in the perturbed structure

planes $y = y_n$ over the whole volume, but to define local mesh-planes in the parts of the volume that are only partly coupled.

In figure 2 two cells of a perturbed muffin tin are shown. It can be seen, that local within every cavity the grid can be made as in a single cavity, since the fields inside the tins are decoupled from each other.

This idea has been implemented as a computer code called GdfidL [5]. It handles the non regular grid as a 6 times linked list, where every cell has indices to its six neighbours. With this grid definition, it is not necessary to discretize inside the metallic parts of a structure. This has the effect, that for the structures considered here, the number of unknowns to be handled shrinks to about 60% of the number necessary for a regular grid.

The figure 3 show the positions of grid planes $z = z_n$ and $y = y_n$ in the cases of a regular and a non regular grid. For clarity, these grids discretize cavities with errors of 5 percent, because smaller errors are almost invisible.

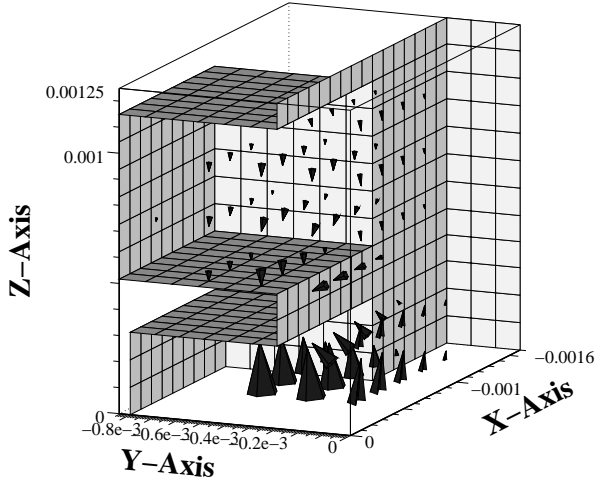


Figure 2. $2\pi/3$ mode in a perturbed muffin tin. The y-extension of the second cavity is perturbed by 5%.

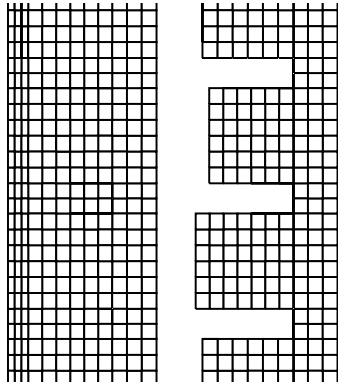


Figure 3. Left: Regular grid, Right: irregular grid, represented as a linked list

IV. ESTIMATION OF THE SENSITIVITIES

To estimate the sensitivity of the muffin tins, the shunt impedance r/Q was calculated for a number of structures with random errors in the cavities. The average of these r/Q 's is a measure of the sensitivity.

GdfidL can discretize arbitrarily small perturbations of the positions of the lower or higher walls of the cavities, but cannot discretize small perturbations in the x-extension of the cavities. Therefore only the positions of the lower cavity walls are perturbed.

GdfidL calculates fast and in small memory, but cannot calculate a side coupled muffin tin in full length on the computer I used. Therefore only 20 mm long sections of the muffin tins were calculated. These sections contain 24 cavities in the case of the single periodic $2\pi/3$ structure and 48 cavities in the case of the side coupled structures.

The figures 4 and 5 present the averages and standard deviations of the shunt impedances, scaled with shunt impedances of the ideal structures. The ordinates are the standard deviations of the relative errors in the y-extension of the cavities. Every marker in the curves is the average of 20 shunt impedances of structures with the same standard deviation of errors. The wall of every single cavity was randomly perturbed, but the perturbations were limited to 2 times the standard deviation.

The exact geometries that are analyzed here are described in [2] and [3].

V. CONCLUSION

It seems that in any case 0.5% errors in the positions of the lower wall are tolerable for 90% of the optimum. The lumped circuit approach [4] predicted 0.2% frequency errors would be ok for the $2\pi/3$ structure, 0.1% for geometry 2 and geometry 3.

The lumped circuit dealt with frequency errors, this paper deals with geometric errors. But since all the muffin tins have approximately the same extension in y- as in x-direction, the frequency of a single cell depends in the same way on the y-extension as on the x-extension. The frequency of a single cell of a muffin tin is approximately given by $(\frac{\omega}{c})^2 \approx (\frac{\pi}{w})^2 + (\frac{\pi}{w+\Delta y})^2$. Therefore a relative error $\Delta y/w$ in the y-extension only, as it was analyzed here, has the effect of a relative frequency error of $\delta f/f_0 \approx -\frac{\delta y/w}{2}$.

This means: when the lumped circuit allows a frequency error of 0.2%, this is equivalent to a geometric error in the y-extension of $2 \times 0.2\%$. The lumped circuit approach was valid.

VI. ACKNOWLEDGEMENT

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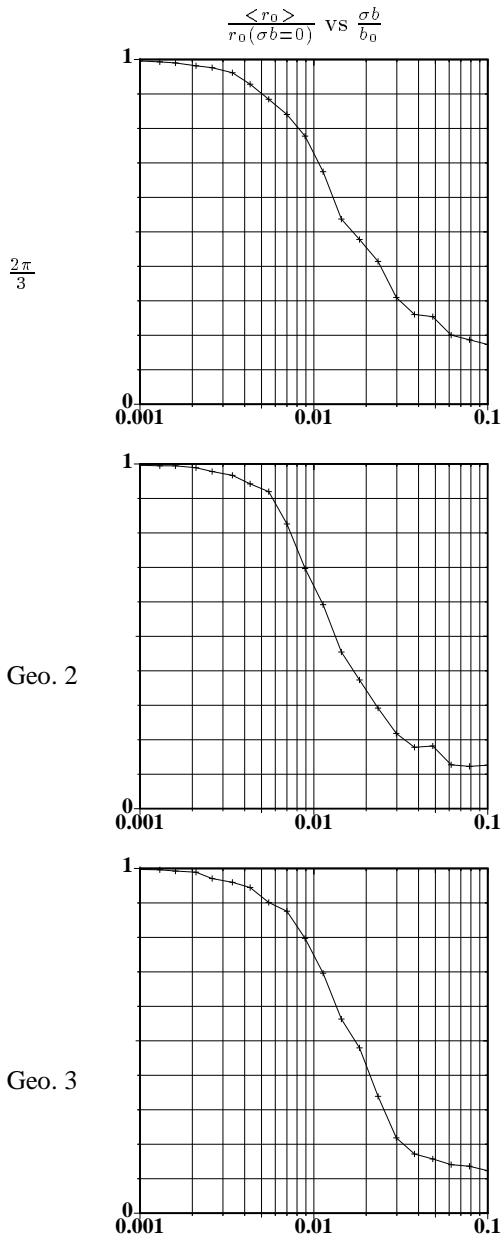


Figure 4. Mean ($\langle r_0 \rangle$) of the shunt impedances as a function of the standard deviation of the errors ($\frac{\sigma b}{b_0}$).

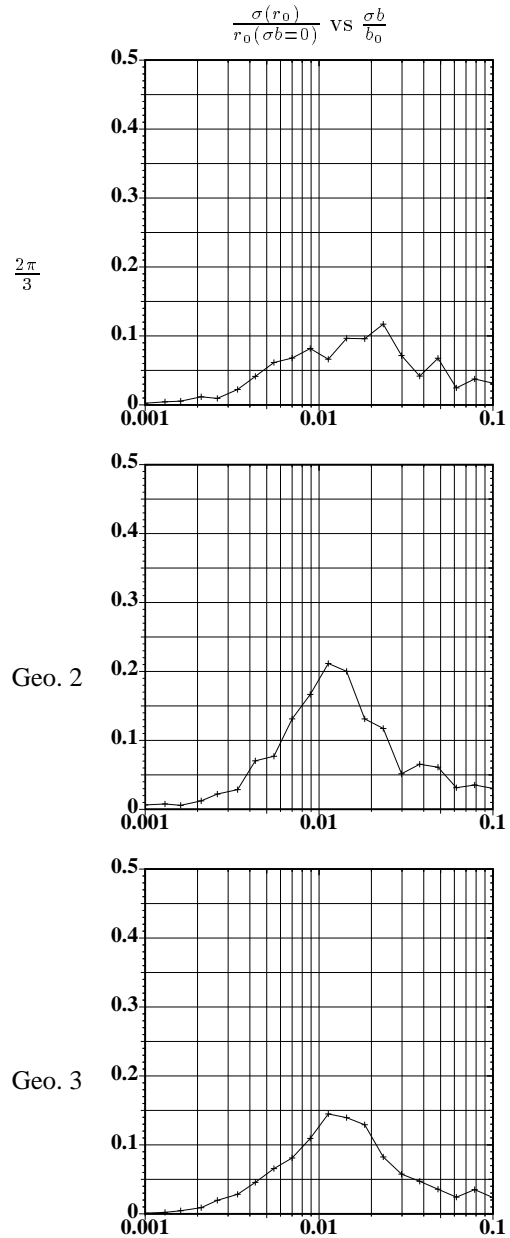


Figure 5. Standard deviation ($\sigma(r_0)$) of the shunt impedances as a function of the standard deviation of the errors ($\frac{\sigma b}{b_0}$).