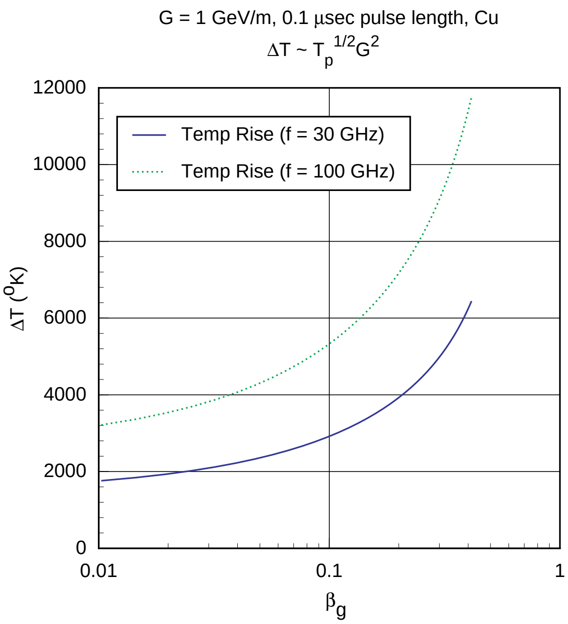
Pulsed Temperature Rise

The pulsed temperature rise for a short wavelength accelerator with a 100 nS long pulse and a gradient of 1 GeV/m is plotted below.



These curves were arrived at using relationships for cylindrical waveguides. Use the following notation

a	iris radius	G	gradient	λ	wavelength
b	outer radius	β_g	group velocity	Tp	pulse length
w	stored energy	s_{at}	normalized elastance	p_d	dissipated power
T_g	normalized time constant	ΔT	temperature rise	C	heat capacity
K	thermal conductivity	A	surface area	d	diffusion length

The stored energy is

$$w = G^2\lambda^2\left[\frac{(a/\lambda)^2}{s_{at}(1-\beta_g)}\right]$$

where s_{at}, the normalized elastance, is given by*

$$s_{at}(V-m/C) = 5.7\times10^{10}\beta_g^{0.4}$$

The dissipated power is

$$p_d = \frac{2wc^{1.5}}{T_g\lambda^{1.5}}$$

where T_g, the normalized time constant, for the structure is*

$$T_g(\mathrm{sec}) = 2.36 \times 10^8 (1 + 1.25\beta_g^{1.5}).$$

The temperature rise is given by

$$\Delta T = \frac{p_d T_p}{CAd}.$$

The surface area can be approximated as

$$A \approx 2\pi b = 2\pi\lambda \frac{b}{a}\frac{a}{\lambda}$$

where*

$$\frac{b}{a} = 1.04 - 0.29 \ln\beta_g + 0.068(\ln\beta_g)^2$$

and a/λ comes from inverting the equation*

$$\beta_g = \exp\left[3.1 - \frac{2.4}{\sqrt{a/\lambda}} - 0.9\frac{a}{\lambda}\right]$$

The diffusion length d is

$$d = \sqrt{\frac{KT_p}{C}}$$

Putting these equations together gives the temperature rise plotted at the beginning of this note.

These temperature rises are unacceptably large. There are a number of possible cures:

1. Lower the gradient since the temperature rise is proportional to G^2. This defeats the purpose of this research.
2. Reduce the pulse length since the temperature rise is proportional to $T_p^{1/2}$. This is a weak dependence.
3. Lower the operating temperature. The temperature rise is

$$\Delta T \propto \sqrt{\frac{\rho}{CK}}$$

where ρ is the resistivity, C is the heat capacity and K is the thermal conductivity. Compare room temperature with LN_2 temperature. The resistivity is proportional to temperature for this temperature range, so

$$\frac{\rho(77\ \mathrm{K})}{\rho(300\ \mathrm{K})} = \frac{77}{300} = 0.257$$

The American Institute of Physics Handbook (Third Edition) gives data on thermal conductivity and heat capacity. For the thermal conductivity (page **4**-154)

$$\frac{K(77\ \mathrm{K})}{K(300\ \mathrm{K})} = \frac{610\ \mathrm{W/m-K}}{401\ \mathrm{W/m-K}} = 1.521$$

For the heat capacity (page **4**-106)

$$\frac{C_p(77\ \mathrm{K})}{C_p(300\ \mathrm{K})} = \frac{3.00\ \mathrm{cal/mole-K}}{5.95\ \mathrm{cal/mole-K}} = 0.504$$

Combining these numbers

$$\frac{\Delta T(77\ \mathrm{K})}{\Delta T(300\ \mathrm{K})} = \sqrt{\frac{0.257}{1.521 \times 0.504}} = 0.58$$

4. Using a long wavelength.

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