Figures of Merit for Materials

The following is a comparison of materials for short wavelength accelerators assuming the pulse length, frequency, geometry, etc. are held fixed.

The cyclic thermal stress is

$$\sigma_{cyc} \approx \frac{1}{2}\alpha E\,\Delta T$$

where α is the thermal expansion coefficient and E is the Elastic Modulus.* The temperature rise is

$$\Delta T = \frac{p_d T_p}{CAd}$$

where p_d is the dissipated power, T_p is the pulse length, C is the heat capacity, A is the surface area. The diffusion length d is given by

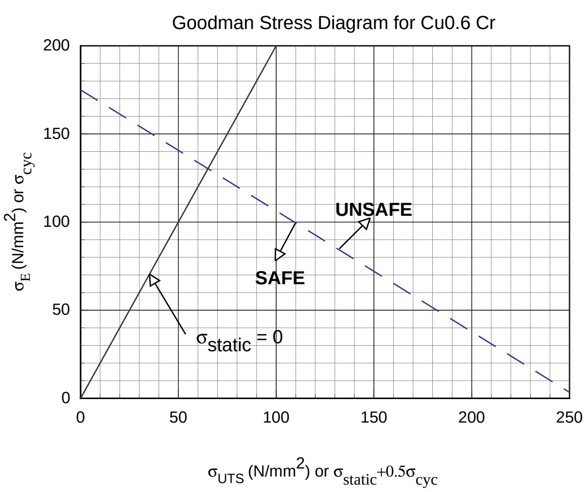
$$d = \sqrt{\frac{KT_p}{C}}$$

where K is the thermal conductivity. The dissipated power in terms of the rf frequency, stored energy and Q is

$$p_d = \frac{\omega_{rf} W}{Q} \propto \omega_{rf} W R_{surf} \propto \omega_{rf} W \frac{1}{\delta\sigma} \propto \frac{1}{\sqrt{\sigma}} = \sqrt{\rho}$$

where δ, σ, and ρ are the skin depth, conductivity and resistivity, respectively. Combining these equations, the temperature rise and cyclic thermal stress have the following dependences on material properties

$$\Delta T \propto \sqrt{\frac{\rho}{CK}};\ \ \sigma_{cyc} \propto \alpha E \sqrt{\frac{\rho}{CK}}$$



The right-hand sides of these two equations are two figures of merit for materials.

The third figure of merit comes from Goodman stress diagram which Mike Seidel discusses in his paper.* It is illustrated for Cu0.6Cr. The failure curve is the dashed line. The two coordinates of the failure curve are $\{0, \sigma_E\}$ where σ_E is the endurance limit and $\{\sigma_{UTS}, 0\}$ where σ_{UTS} is the Ultimate Tensile Strength. The operating line for the application is the line $\{\sigma_{static} + 0.5\sigma_{cyc}, \sigma_{cyc}\}$. The line in the figure is drawn for $\sigma_{static} = 0$. It leads to the limit $\sigma_{cyc} < 130$ N/mm^2 and $\Delta T < 148$K.

Sticking with the criterion $\sigma_{static} = 0$, the intercept of the operating and Goodman stress curves is

* M. Seidel, DESY Print TESLA 95-18.

$$\sigma_{cyc} = \frac{2\sigma_E \sigma_{UTS}}{2\sigma_{UTS} + \sigma_E}$$

The right hand side of this equation is the third figure of merit.

The tables below list some initial looks at materials

Material	thermal expansion (/K)	Elastic Modulus (N/m^2)	Resistivity (ohm-m)	Thermal Conduct (W/m-K)	Density (kg/m**3)	Specific Heat (J/kg-K)	Specific Heat (J/m^3-K)	Endurance Limit (N/m^2)	Tensile Strength (N/m^2)
OFHC Copper	1.76E-05	1.17E+11	1.71E-08	3.91E+02	8.95E+03	3.85E+02	3.44E+06	7.58E+07	2.21E+08
Be-Cu	1.67E-05	1.31E+11	5.00E-08	1.73E+02	8.23E+03	4.18E+02	3.44E+06	2.07E+08	4.83E+08
Be	1.15E-05	3.03E+11	5.00E-08	1.51E+02	1.86E+03	1.88E+03	3.49E+06	2.02E+08	4.10E+08
Gold	1.42E-05	8.27E+10	2.35E-08	2.98E+02	1.93E+04	1.30E+02	2.51E+06	3.17E+07	1.31E+08
6061 Alum	2.34E-05	6.90E+10	3.80E-08	1.71E+02	2.71E+03	9.61E+02	2.61E+06	6.21E+07	1.24E+08
Approx Ti15Mo	8.60E-06	8.69E+10	9.10E-07	1.16E+01	4.74E+03	4.93E+02	2.34E+06	3.45E+08	8.39E+08
Silver	1.96E-05	7.58E+10	1.59E-08	4.19E+02	1.05E+04	2.34E+02	2.46E+06	0.00E+00	1.52E+08

Material	diffusion Length (relative)	Temp Rise (relative)	Cyclic Stress (relative)	Max cyclic stress (N/m^2)	Cyclic Stress/Max (relative)
OFHC Copper	1.07E-02	3.57E-09	7.37E-03	6.47E+07	1.14E-10
Be-Cu	7.09E-03	9.17E-09	2.01E-02	1.70E+08	1.18E-10
Be	6.57E-03	9.76E-09	3.41E-02	1.62E+08	2.10E-10
Gold	1.09E-02	5.61E-09	6.61E-03	2.83E+07	2.33E-10
6061 Alum	8.10E-03	9.22E-09	1.49E-02	4.96E+07	3.00E-10
Approx Ti15Mo	2.23E-03	1.83E-07	1.37E-01	2.87E+08	4.78E-10
Silver	1.31E-02	3.93E-09	5.85E-03	0.00E+00	#DIV/0!

Open issues at this time are searches for better materials and understanding of the physics behind the Goodman stress diagram.