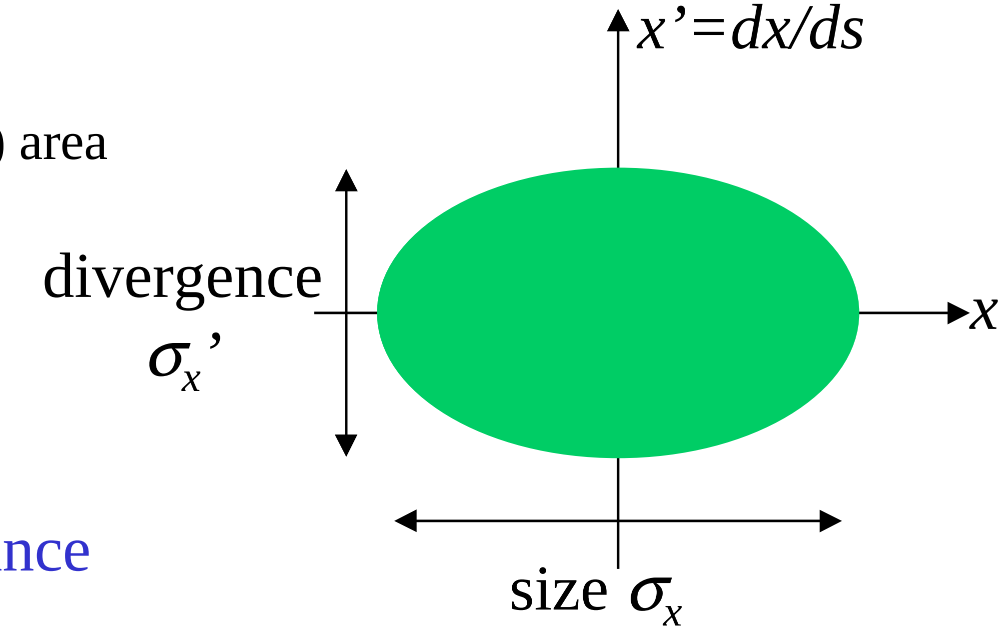
Digression on Emittance

- Beam emittance
  - Phase space (x, x') area

    at the waist

    $\varepsilon \sim \sigma_x \sigma_x'$



- Normalized emittance

  $$\varepsilon_N = \gamma\beta\varepsilon$$

  - Single particle invariant action J

  $$\varepsilon_N = \langle J \rangle / mc$$

  - Conserved in a linear system (no radiation)
  - Adiabatic invariant under acceleration
  - Determine the beam size and divergence

  $$\sigma_x = \sqrt{\varepsilon_N \beta_x / \gamma}, \qquad \sigma_x' = \sqrt{\varepsilon_N / \gamma\beta_x}$$

- Why are low emittance electron and positron beams necessary?
  - Linear colliders: very small IP spot size
    - SLC ~ 500 nm,
    - FFTB ~ 70 nm
    - Future linear colliders ~ a few nm
  - Some scales for ε_N

Devices	horizontal ε_N (m)	vertical ε_N (m)
RF Photocathode Gun	10^{-6}	10^{-6}
SLC Damping Ring	10^{-5}	10^{-6}
ATF Damping Ring	10^{-6}	10^{-8}

Radiative Cooling

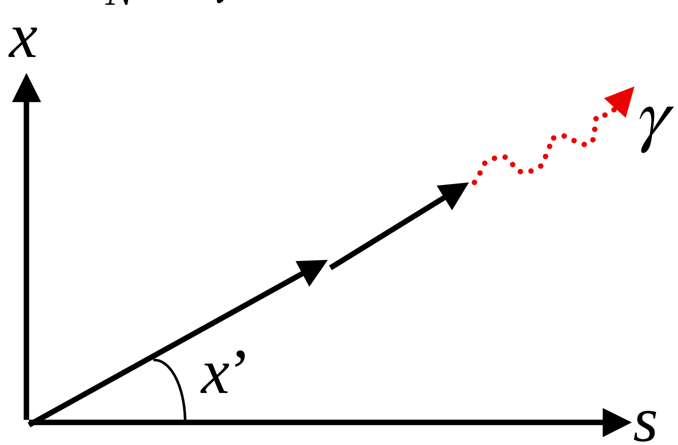
- Cooling <=> reduction of beam emittance,
- Radiative cooling: effective for electron beams
  - radiation damping: through synchrotron radiation occurs in all three degrees of freedom
  - quantum excitation: radiation not continuous, in quanta yields quantum noise!
- Equilibrium emittances <= Damping + Excitation

  Equilibrium emittances → smaller; Damping → increase; Excitation → reduce

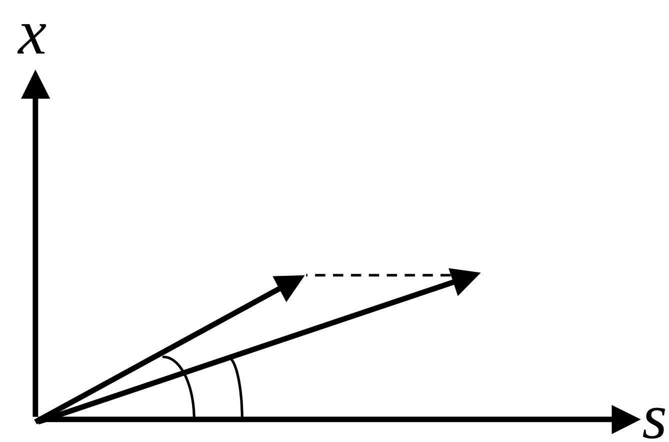
Radiation Damping

- Electron radiates momentum and energy, replenished by longitudinal acceleration

- Photon emission
  $\Delta x = 0,\ \Delta x' = 0$
  ε remains constant
  $\varepsilon_N = \gamma\varepsilon$ decreases



- Adiabatic acceleration
  $\Delta x = 0,\ \Delta x' < 0$
  ε decreases
  $\varepsilon_N = \gamma\varepsilon$ is constant



- Rates: $\frac{1}{\varepsilon}\frac{d\varepsilon}{dt} = \vartheta\ \frac{1}{E}\frac{dE}{dt}$ and $\vartheta_x + \vartheta_y + \vartheta_s = 4$ (Robinson)

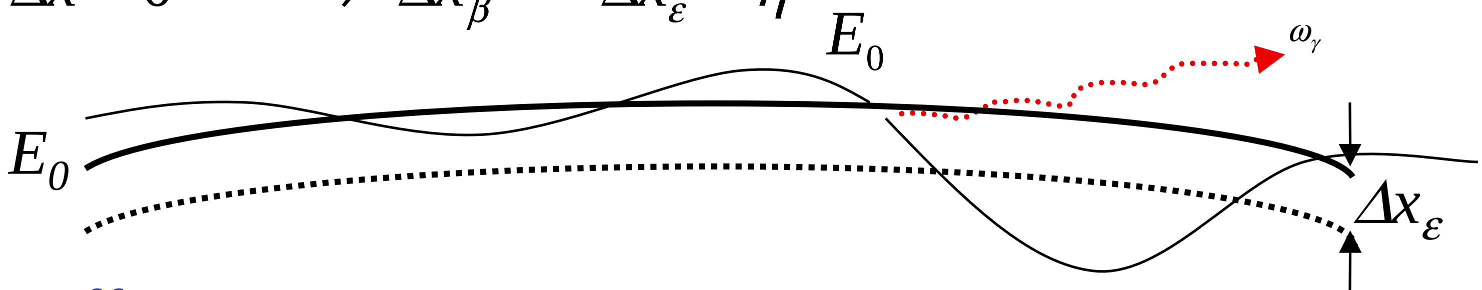
Quantum Excitation

- Quasiclassical model (Sands) $\begin{cases} \text{emission is } \textbf{instantaneous} \\ e^- \textbf{ recoils} \text{ against photon.} \end{cases}$

- Horizontal: dispersion effect

$$x = x_\beta + x_\varepsilon, \qquad x_\varepsilon = \frac{E - E_0}{E_0}\eta \text{ (dispersion)}$$

$$\Delta x = 0 \qquad \Rightarrow \Delta x_\beta = -\Delta x_\varepsilon = \eta \frac{\hbar\omega_\gamma}{E_0}$$



- Diffusion rate

$$\frac{d\varepsilon_N^x}{dt} \sim \frac{\gamma}{\beta_x}\frac{d(\Delta x_\beta)^2}{dt} \sim \frac{\gamma\eta^2}{\beta_x}\frac{(\hbar\omega_\gamma)^2}{E_0^2} \times \text{rate of photon emitted.}$$

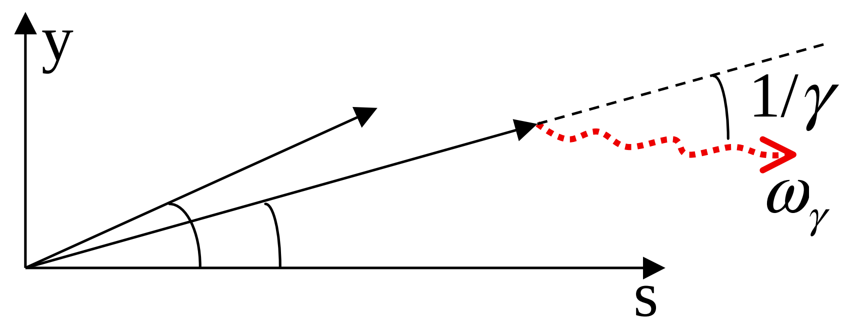
Equilibrium Emittances

- Damping rate = Diffusion rate =>
- Equilibrium horizontal emittance

$$\varepsilon_N^x \sim \bar{\lambda}_c \left(\frac{\beta_x}{\rho / \gamma} \right)^3$$

- Vertical: opening angle effect

$$\Delta y = 0, \;\; \Delta y' \sim \frac{1}{\gamma} \frac{\hbar \omega_\gamma}{E_0}$$



- Equilibrium vertical emittance
  - much smaller than horizontal one
  - usually determined by x-y coupling

$$\varepsilon_N^y \sim \bar{\lambda}_c \left(\frac{\beta_y}{\rho / \gamma} \right)$$