Introduction to Electroweak Symmetry Breaking

— 3 —

M E Peskin
SSI 2001

In the previous lecture,

I described the Higgs boson sector of SUSY models in some detail

However, I did not discuss the most important question:

Why is electroweak symmetry spontaneously broken in SUSY models?

Analyze this with the following strategy:

Inoue et al
Ellis et al
Ibanez
Alvarez-Gaumé
Polchinski, Wise

Assume all scalar m^2 parameters, generated by SUSY breaking at a high scale M, are positive

Use RG equations to evolve to $Q \sim$ TeV

typical evolution:

$$\frac{d}{d\log Q^2} m_f^2 = - \sum_{i=1,2,3} \frac{2}{\pi} \underbrace{\alpha_i}_{\text{gauge coupling}} C_2(r_i) \underbrace{m_i^2}_{\text{gaugino mass}}$$

m_f increases toward IR

e.g. with grand-unified gaugino masses

$$m_{\bar{e}}^2 (Q \sim \text{TeV}) = m_0^2 + 0.2\, m_{\tilde{w}}^2$$

$$m_Q^2 (Q \sim \text{TeV}) = m_0^2 + 10.\, m_{\tilde{w}}^2$$

The answer to this question given in the previous lecture was

$$(M_2^2 + \mu^2) < 0$$

This equation raises a more general issue:

In the MSM, there is one scalar field ϕ;
we want to know why $m^2(\phi) < 0$.

In SUSY models, there are many scalar fields

In principle, any one could have a vacuum expectation value

Many of these possibilities are unpleasant or dangerous:

$\langle \tilde{\tau}^+ \rangle \neq 0$ breaks Q_{EM}

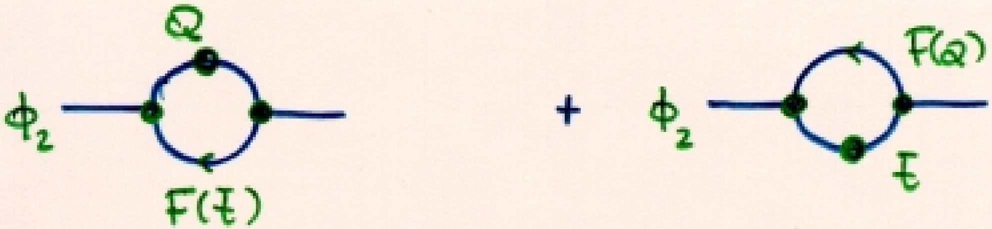
$\langle \tilde{\bar{u}} \rangle \neq 0$ breaks color SU(3)

To succeed, SUSY must imply that
the correct field condenses.

However, λ_t is almost as large as g_s.

Take this into account also:

$$W = \lambda_t \, \bar{t} \, \phi_2 \cdot Q$$



terms w. SUSY-breaking masses do not cancel
against fermion loops

$$\rightsquigarrow \quad \delta m^2 \sim -\lambda_t^2 \, m^2 \log \Lambda^2$$

Write these terms more precisely as terms in
RG equations

$$\frac{d}{d\log Q} m_Q^2 = +\frac{1}{8\pi^2} \cdot 1 \cdot \lambda_t^2 \cdot (m_Q^2 + m_{\bar{t}}^2 + m_{\phi_2}^2 + A_t^2)$$

$$\frac{d}{d\log Q} m_{\bar{t}}^2 = +\frac{1}{8\pi^2} \cdot 2 \cdot \lambda_t^2 \cdot (m_Q^2 + m_{\bar{t}}^2 + m_{\phi_2}^2 + A_t^2)$$

$$\frac{d}{d\log Q} m_{\phi_2}^2 = +\frac{1}{8\pi^2} \cdot 3 \cdot \lambda_t^2 \cdot (m_Q^2 + m_{\bar{t}}^2 + m_{\phi_2}^2 + A_t^2)$$

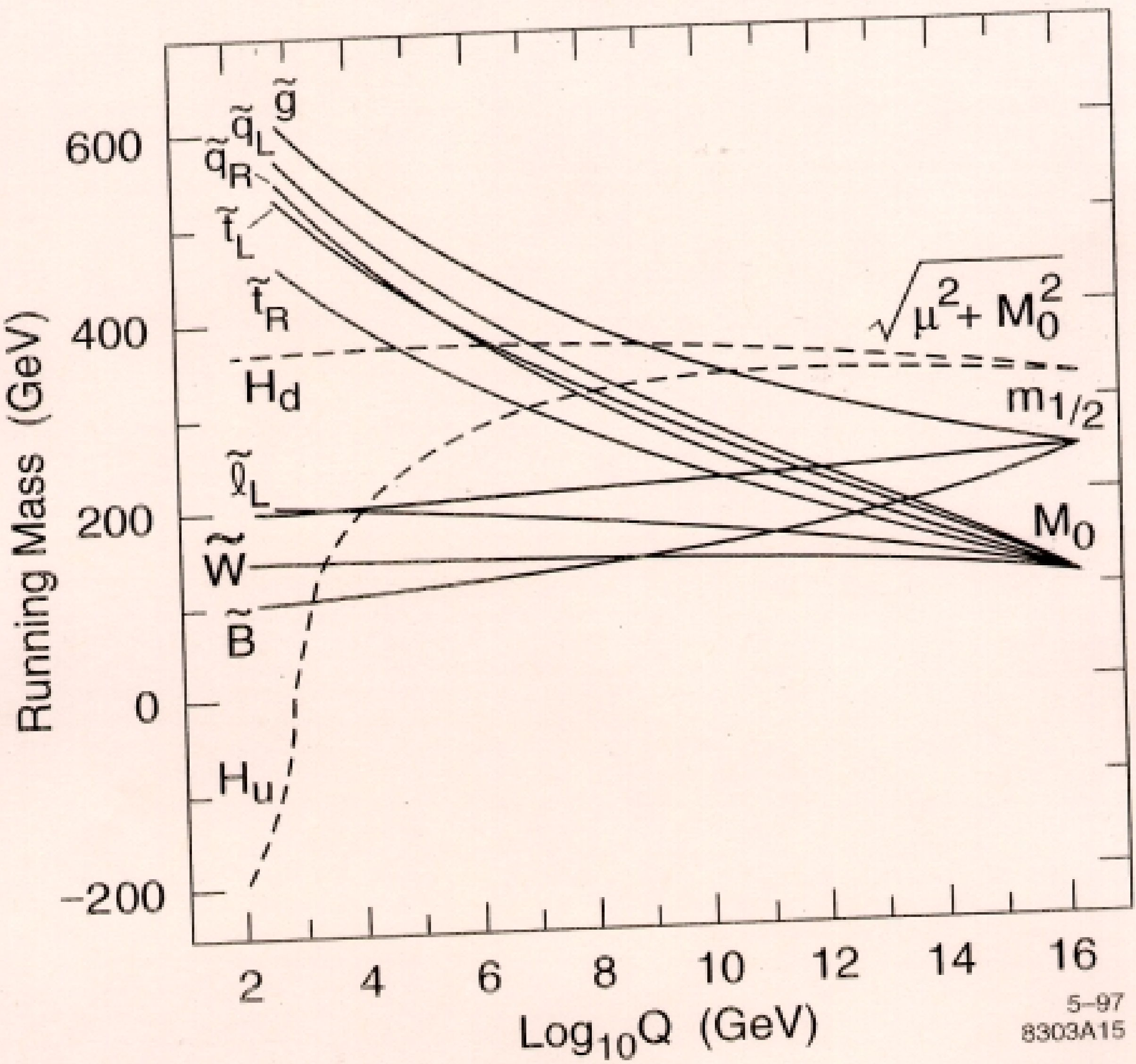
so ϕ_2 is most strongly driven to instability!

typically,

$m_{\tilde{Q}}^2$, $m_{\tilde{t}}^2$ are kept positive by the gluino renormalization

(however $\tilde{b}$, $\tilde{t}$ may be much lighter than the other squarks)

$m_{\Phi_2}^2$ is pulled negative $\rightsquigarrow$ EWSB !



Kane, Kolda, Wells

What sets the scale of EWSB ?

1. magnitude of m_i^2 ← SUSY breaking

   e.g. in gravity-mediation $m^2 \sim \frac{F^2}{m_{PL}^2}$

   then $\sqrt{F} \ll m_{PL}$ implies $m \ll m_{PL}$.

2. magnitude of μ

   μ is apparently SUSY-preserving,
   but it is more attractive if μ also arises
   from SUSY breaking

   $W \sim \frac{1}{m_{pl.}} h_1 h_2 S^2 \Rightarrow W \sim \frac{\langle F_s \rangle}{m_{pl.}} h_1 h_2$ Nilles Kim

   $\mathcal{L} \sim \frac{F^* F h_1 h_2}{m_{pl}^2} \Rightarrow \mathcal{L} \sim \left| \frac{\langle F \rangle}{m_{pl}} \right|^2 h_1 h_2$ Giudice Masiero

3. scale where m_2^2 becomes negative

   this should be near the TeV scale

   fine tuning ? logarithmic evolution

supersymmetry turns out to be a theory that
explains EWSB and also retains
some advantages of the MSM

however,

it raises the question of how and at what scale
supersymmetry is broken

all explanations of the mass scale of v, m_W ..
eventually come back to this question.

In the remainder of this lecture, I will briefly review other approaches to EWSB

These are less well developed,

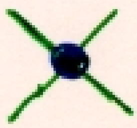
a playground courageous / foolhardy theorists

- more pointlike strong interactions
- extra dimensions

abstract from the study of technicolor

$$\frac{g^2}{p^2}\,\bar{Q}_L\gamma^\mu t^a Q_L\;\bar{Q}_R\gamma_\mu t^a Q_R$$

Fierz



$$\underbrace{\frac{g^2}{M^2}}\;\bar{Q}_L Q_R\;\bar{Q}_R Q_L$$

approximate by a pointlike interaction

Is a self-supported $\bar{Q}Q$ condensate possible?

 $= \Sigma$

 $=$  "gap equation"

$$\Sigma = \frac{g^2}{M^2}\int\frac{d^4p}{(2\pi)^4}\;2\,\frac{\Sigma}{p^2-M^2}$$

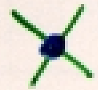
$$= \frac{g^2}{8\pi^2}\,\Sigma\,\left(1-\frac{\Sigma^2}{M^2}\log M^2/\Sigma^2\right)$$

There is a self-consistent solution if

Miransky
Yamawaki

$$\frac{g^2}{8\pi^2} > 1$$

$\leadsto$ (semi)-quantitative theory of "Composite Higgs" models

An attractive idea is that  is an interaction of the top quark.

top color   Hill

t, b couple to a different, more strongly coupled, version of QCD

$$[SU(3)]_t \times [SU(3)]_\ell \rightarrow \underset{QCD}{SU(3)}$$

problems with this idea:

Why is $m_t \gg m_b$?

b (observed in $Z \to b\bar{b}$), t (observed in $q\bar{q} \to t\bar{t}$) have no significant form factors

in technicolor

$$v = 246\ \text{GeV} \leftrightarrow m_Q \sim 1\ \text{TeV}$$

so can v be large enough for $m_t = 175$ GeV topcolor seesaw

Chivukula, Dobrescu, Georgi, Hill

$$[SU(3)]_t \times [SU(3)]_\ell \times SU(2) \times U(1)$$

t_L $\quad (3, 1, 2, \tfrac{1}{6})$ $\qquad$ t_R $\quad (1, 3, 1, \tfrac{2}{3})$

χ_R $\quad (3, 1, 1, \tfrac{2}{3})$ $\qquad$ χ_L $\quad (1, 3, 1, \tfrac{2}{3})$

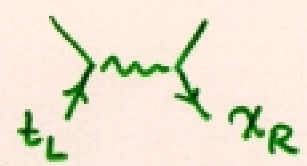
break $[SU(3)]_t \times [SU(3)]_\ell \to SU(3)_{QCD}$

this allows a $\bar{\chi}_L \chi_R$ mass term

$$g_s^2 = \frac{g_t^2 g_\ell^2}{g_t^2 + g_\ell^2}$$

assume $g_t^2 \gg g_\ell^2 \sim g_s^2$, then, even broken,

g_t might be strong enough to induce a condensate



symmetry breaking expectation value $\langle \bar{t}_L \chi_R \rangle \neq 0$

$I = \tfrac{1}{2} \quad Y = \tfrac{1}{2}$

composite Higgs $(\bar{t}_L \chi_R)$

mass matrix

$$(t_L \ \chi_L) \begin{pmatrix} 0 & m \\ \mathcal{m} & M \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix}$$

take M large - scale of topcolor breaking (5 TeV)

m is an $SU(2)\times U(1)$ breaking parameter
at ~ scale of technicolor (500 GeV - 1 TeV)

m_t is the small eigenvalue of this matrix

$$m_t = m \frac{\mathcal{m}}{M}$$

so we can have $m_t \ll$ TeV

still lighter b ← larger seesaw ?

t is part of the mechanism of EWSB
and a constituent of ϕ
but sufficiently isolated from strong dynamics

another strategy for understanding the scale of EWSB:

instead of imposing symmetries on ϕ,

try to lower M.

to achieve this, consider models with additional space dimensions

Arkani-Hamed, Dimopoulos, Dvali

String theory predicts 7 extra dimensions; maybe some are experimentally accessible.

Write the gravitational potential in various dimensions:

4-d $\quad V = \frac{G_N m_1 m_2}{r} = 4\pi G_N \int \frac{d^3q}{(2\pi)^3} \frac{e^{iq\cdot r}}{|q|^2}$

5-d $\quad V = \frac{G_N^{(5)} m_1 m_2}{r^2}$

$\vdots$

for V fixed at some R,

V becomes large faster in higher d.

If we have a model with 4 extended dimensions

n dimensions of size $(2\pi R)$

fix V at R ; then for $r < R$

$$V = \frac{G_N m_1 m_2}{R} \cdot \left(\frac{R^{n+1}}{r^{n+1}} \right)$$

identify the true gravitational constant

$$G_N^{(4+n)} = G_N R^n \qquad [\text{mass}]^{-(n+2)}$$

The true Planck scale
— the scale of quantum gravity —
is then given by

$$M^{n+2} = \left[4\pi G_N R^n \right]^{-1}$$

Taking R large enough, we can have $M = 1$ TeV

$n = 6$ $\quad R = 30$ fm

$n = 4$ $\quad R = 0.12$ Å

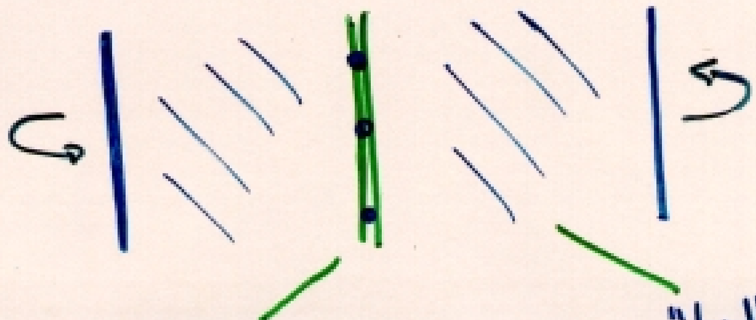
$n = 2$ $\quad R = 0.7$ mm

all of these distances are macroscopic to particle physicists

$e^+e^- \to e^+e^-$ is 4-d to $q > 200$ GeV

$q\bar{q} \to q\bar{q}$ is 4-d to $q > 500$ GeV

to realize large extra dimensions, we need the following picture



"brane"

$q\ l\ \gamma\ g\ W \cdots$ live here

"bulk"

gravity lives here

The brane may have reduced symmetry from boundary conditions on bulk fields

$$\phi = 0 \quad \text{or} \quad \partial_n \phi = 0$$

e.g.

$$\left(\frac{1+\gamma^5}{2}\right)\Psi = 0$$

In principle, there ought to be a theory
that starts from this picture
which implies $m_\phi^2 \sim M^2 \sim (\mathrm{TeV})^2$
and gives a reason why $m_\phi^2 < 0$.

I do not know of such a theory.

However, there are interesting examples in which
the special properties of extra dimensions
are used in other ways to produce EWSB.

In these examples $R \sim \hbar/\mathrm{TeV}$.

Antoniadis

Arkani-Hamed, Cheng, Dobrescu, Hall:

Like gravity, gauge interactions become strong at lower momentum scales in higher dimensions

e.g. 6-d, n = 2 $\qquad g^2 = g^2_{(4)} \cdot \frac{R^2}{r^2}$ for $r < R$

so

take $R \sim \hbar/\mathrm{TeV}$

allow the third generation to propagate in the bulk, with

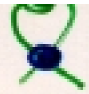
$\left(\frac{1+\gamma^5}{2}\right)(Q, L) = 0 \qquad \left(\frac{1-\gamma^5}{2}\right)(t, b, \tau) = 0$

on the brane

then the modes of these field with $p^4 = p^5 = 0$ are the conventional SM fermions

allow the SM gauge fields to propagate in the bulk.

At short distances in the bulk, the SM gauge couplings become strong.

Study the coefficient in the gap equation 

$\bar{Q}\, t$	$\frac{4}{3} g_s^2 + \frac{1}{6}\cdot\frac{2}{3} g'^2$	(1)
$\bar{Q}\, b$	$\frac{4}{3} g_s^2 - \frac{1}{6}\cdot\frac{1}{3} g'^2$	(0.93)
$\bar{Q}\, \bar{b}$	$\frac{2}{3} g_s^2 + \frac{1}{6}\cdot\frac{1}{3} g'^2$	(0.5)
$\bar{Q}\, \bar{t}$	$\frac{2}{3} g_s^2 - \frac{1}{6}\cdot\frac{1}{3} g'^2$	(0.43)
$\bar{L}\, \tau$	$\frac{1}{2}\cdot 1\, g'^2$	(0.21)
$\bar{L}\, u$	$\frac{1}{2}\cdot\frac{2}{3} g'^2$	(0.14)
⋮	⋮	⋮

even if $g_1 = \sqrt{\frac{5}{3}}\, g'$ $g_2 = g_w$ $g_3 = g_s$

become unified, $\bar{Q}t$ $\bar{Q}b$ are more strongly bound. $\uparrow$

so it is plausible that $\bar{Q}t$ $\bar{Q}b$ only form condensates.

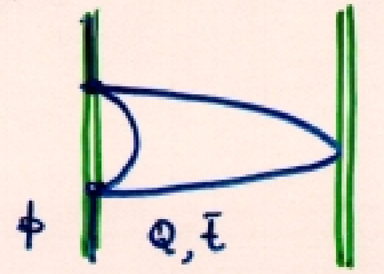
This gives EWSB from extra dimensions

Arkani-Hamed, Hall, Nomura, Smith, Weiner

construct a SUSY model w. 5-d and
2 branes, separated by $R \sim \hbar/\mathrm{TeV}$

assume SUSY is broken by a boundary condition
on brane 2 $\left(\to m_{\tilde f}^2 \sim \frac{1}{R^2}\right)$
and that Higgs bosons live on brane 1

Higgs masses arise from



$$m_h^2 \sim -\frac{\lambda_t^2}{(4\pi)^2}\frac{1}{R^2}$$

a naturally negative contribution, as previously
in SUSY.

Tight connection of scales

$$|m_h| \sim \frac{1}{10}|m_f| \sim \mathrm{TeV}$$

Arkani-Hamed, Cohen, Georgi

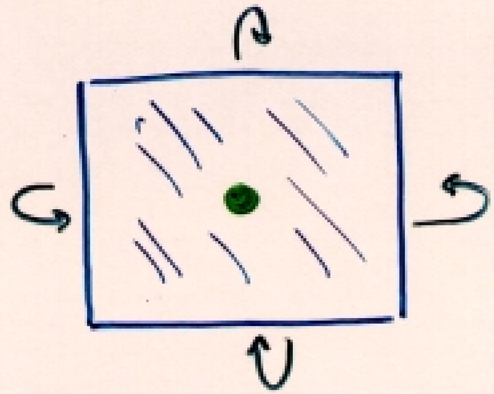
In the previous lecture, I motivated SUSY by arguing that,
to eliminate Λ^2 divergences in m^2,
a symmetry must relate ϕ to a higher-spin field

Why not
$$\delta\phi = \xi^\mu A_\mu \quad ?$$

This is naturally realized in higher dimensions.

Choose $n=2$ $(d=6)$

SU(3) gauge theory, broken to SU(2)×U(1) on a brane



$$\underbrace{A_M}_{012345} = (\underbrace{A_\mu}_{0-3}, \underbrace{A_4, A_5}_{\text{4-d scalars in adjoint rep. of SU(3)}})$$

$$A_4 = \left(\begin{array}{c|c} & h_4 \\ \hline h_4^\dagger & \end{array}\right) \qquad A_5 = \left(\begin{array}{c|c} & h_5 \\ \hline h_5^\dagger & \end{array}\right) \qquad Y = \frac{1}{6}\left(\begin{array}{cc|c} -1 & & \\ & -1 & \\ \hline & & 2 \end{array}\right)$$

h_4 h_5 are fields with $I = \frac{1}{2}$ $Y = \frac{1}{2}$

Assume that the brane carries an interaction

$$\delta\mathcal{L} = \int d^4x \; f\mu^2 \cdot \mathrm{tr}\left[Y \cdot F_{45} \right]$$

$$= \int d^4x \; f\mu^2 \cdot \frac{g}{6} \cdot \mathrm{tr} \begin{pmatrix} -1 & & \\ & -1 & \\ & & 2 \end{pmatrix} [A_4, A_5]$$

$$= \int d^4x \; \frac{f\mu^2 \cdot g}{6} \; \mathrm{tr} \left(\begin{array}{cc|c} -1 & & \\ & -1 & \\ \hline & & 2 \end{array}\right) \left(\begin{array}{c|c} h_4 h_5^\dagger - h_5 h_4^\dagger & \\ \hline & h_4^\dagger h_5 - h_5^\dagger h_4 \end{array}\right)$$

$$= \int d^4x \; \frac{gf}{6} \mu^2 \left(h_4^\dagger h_5 - h_5^\dagger h_4 \right)$$

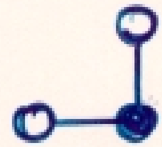
write $\quad h_4 = \dfrac{h_1 + h_2}{\sqrt{2}} \qquad h_5 = \dfrac{h_1 - h_2}{\sqrt{2}}$

$$= \int d^4x \; \frac{gf\mu^2}{6} \left(h_1^\dagger h_1 - h_2^\dagger h_2 \right)$$

Whatever the sign of f, either h_1 or h_2 condenses!

Actually, we do not need continuous extra dimensions to realize the physics of this model.

A single lattice will do:



Alternatively, a 4-d gauge theory of

$$SU(3) \times SU(2) \times U(1) \times SU(3)$$

suffices.

In this case, the Higgs mass is protected because h_4, h_5 are approximate Goldstone bosons

Kaplan
Georgi

Extra dimensions only provide the inspiration to write down the model.

So, what actually is the mechanism of EWSB ?

We have seen that models which explain why EW symmetry is broken are complex and interesting

Many possibilities are known, and there are more to be imagined.

Most importantly,

$v = 246$ GeV sets the scale

so we do not have long to wait before experiments reveal the answer.