Introduction to Electroweak Symmetry Breaking

— 2 —

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SSI 2001

In the previous lecture, I discussed the minimal model of EWSB, the MSM

This model has many good features:

it naturally leads to $m_\gamma = 0$, $m_W/m_Z = \cos\theta_W$

it naturally removes flavor-changing transitions due to the symmetry-breaking fields

it is consistent with precision electroweak measurements, at least if $m_h \lesssim 200\,\mathrm{GeV}$

However, it has the unpleasant feature that

the reason that EWSB takes place cannot be understood in this model, even in principle

In this lecture, I will present two models
in which EWSB has a physical explanation

The reasons are different in the two models,
and this illustrates the extent of our ignorance
of the source of EWSB

However, there is a common feature:

EWSB is an epiphenomenon

The two models have interesting and complex
dynamics at TeV energies

They do not care that $SU(2) \times U(1)$ is broken

We care, but only because we view Nature
from much lower energies.

first idea: Φ is not a fundamental field

If we build Φ from scalars, we will bring back the problem of Λ^2-divergences in m^2

so

Build Φ as a fermion-antifermion bound state.

analogy to the theory of superconductivity:

$\Phi \rightarrow$ Landau-Ginzburg field

$f\bar{f} \rightarrow$ Cooper pairs

QCD can make bound states;

why not copy QCD and see what happens?

$\rightsquigarrow$ Technicolor

Weinberg
Susskind

An explicit model:

postulate a new gauge group, to be added to the SM

technicolor $SU(N_{TC})$

with fermions (T-quarks) (R_{TC}, I, Y)

$$Q_L^i = \begin{pmatrix} U^i \\ D^i \end{pmatrix}_L \qquad Q_R^i = \begin{cases} U_R^i \\ D_R^i \end{cases}$$

$$(N_{TC}, \tfrac{1}{2}, 0) \qquad\qquad (N_{TC}, 0, \pm\tfrac{1}{2})$$

with (postulate) zero bare mass

~ QCD w. 2 flavors

$$\mathcal{L} = \bar{Q}_L^i i \not{D} Q_L^i + \bar{Q}_R^i i \not{D} Q_R^i - \tfrac{1}{4}(F_{\mu\nu}^a)^2$$

$$Q_L \to e^{i\alpha\cdot\tau} Q_L \qquad Q_R \to e^{i\beta\cdot\tau} Q_R$$

$SU(2) \times SU(2)$ flavor symmetry

If this were QCD, then

1. T-quarks would be confined

   → mesons ($Q\bar{Q}$) and baryons ($\underbrace{Q \cdots Q}_{N_{TC}}$)

2. Attractive $Q\bar{Q}$ interactions would make it energetically favorable for the vacuum to fill with $Q\bar{Q}$ pairs (Cooper pairing)

3. As a sign of this, $\langle \bar{Q} Q \rangle \neq 0$

$$\langle \bar{Q}^i_L Q^j_R \rangle = \Delta \cdot \mathcal{U}^{ij}$$

$\mathcal{U}$ is an SU(2) matrix; $SU(2) \times SU(2) \xrightarrow[\text{sp. breaking}]{} SU(2)$

4. 3 spont. broken symmetries → 3 Goldstone bosons

$$\mathcal{U} = e^{i\, 2\pi^a \tau^a / F_\pi}$$

degenerate vacuum states ⇒ π^a are massless

5. there is a scalar (0^{++} $I=0$) bound state

   However, this is the σ or ϵ, a broad resonance

The most prominent T-hadronic resonance is the T-rho (1^{--}, $I=1$)

effective Lagrangian

$$\mathcal{L} = \frac{1}{4} F_\pi^2 \,\mathrm{tr}\, \partial_\mu \mathcal{U}^\dagger \partial^\mu \mathcal{U}$$

$$= \frac{1}{4} F_\pi^2 \,\mathrm{tr}\, (-2i \partial_\mu \pi \cdot \tau / F_\pi)(+2i \partial_\mu \pi \cdot \tau / F_\pi) +$$

$$= \frac{1}{2} (\partial_\mu \pi)^2 + \left(\frac{\pi^2 (\partial_\mu \pi)^2}{F_\pi^2} \text{ interactions} \right)$$

add back $SU(2) \times U(1)$

$$D_\mu \mathcal{U} = \partial_\mu \mathcal{U} - i g A_\mu^a \tau^a \mathcal{U} + i g' B_\mu \mathcal{U} \tau^3$$

$$\mathcal{L} = \frac{1}{4} F_\pi^2 \,\mathrm{tr}\, D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U}$$

set $\mathcal{U} = 1$ $\qquad$ π^a are "eaten"

$$= \frac{1}{4} F_\pi^2 \,\mathrm{tr}\, [g A_\mu^a \tau^a - g' B_\mu \tau^3]^2$$

$$= \frac{1}{2} (A_\mu^a, B_\mu) (\mathcal{M}^2) \begin{pmatrix} A_\mu^b \\ B_\mu \end{pmatrix}$$

with

$$\mathcal{M}^2 = \left(\begin{array}{ccc|c} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ \hline & & -gg' & g'^2 \end{array}\right) \frac{F_\pi^2}{4}$$

This is exactly the MSM result, with

$$v \to F_\pi \qquad \text{(analogue of } f_\pi = 93 \text{ MeV)}$$

The structure should not be a surprise

$$Q = I^3 + Y \quad \text{is unbroken}$$

$$T\text{-isospin} \rightarrow \text{custodial } SU(2)$$

The scale of EWSB is set by

$$F_\pi = 246 \text{ GeV}$$

$$m_{T\text{-}\rho} \sim m_\rho \frac{F_\pi}{f_\pi} \sim 2000 \text{ GeV}$$

In QCD and in T-color (asymptotic freedom)

$$m_\rho = C \Lambda_{QCD} \qquad \Lambda_{QCD} = M e^{-2\pi/b_0 \alpha_s(M)}$$

$$b_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$$

this explains

$$\alpha_s(M) \ll 1 \Rightarrow f_\pi, m_\rho \ll M$$

$$\alpha_{TC}(M) \ll 1 \Rightarrow F_\pi, m_W \ll M$$

We are doing excellently on the big questions. But:

flavor ?

So far, $\langle \bar{Q}Q \rangle \neq 0$ does not generate masses for quarks and leptons

In our philosophy, we cannot use scalars to couple Q to q, ℓ, so

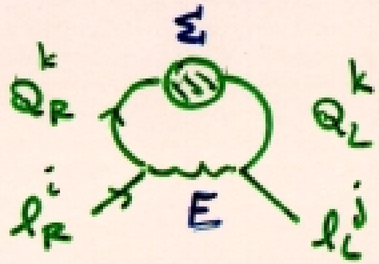
introduce a larger gauge symmetry

"extended technicolor"

Dimopoulos, Susskind

Eichten, Lane

which must be spontaneously broken to T-color at the scale m_E.



$\Rightarrow \quad \delta\mathcal{L} = (\Lambda_\ell^{ji}\phi)\, \bar{L}^j e_R^i$

where

$$(\Lambda_\ell^{ji}\phi) = \int \frac{d^4p}{(2\pi)^4}\, g_E^{jk} \frac{1}{p^2 - m_{Ek}^2}\, g_E^{ki} \frac{\Sigma(p)}{p^2 - \Sigma^2(p)}$$

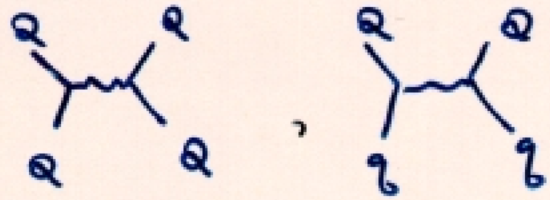
convergent ? $\Sigma(p) \sim \frac{\Delta}{p^2}$ in an asympt free gauge theory simple scaling estimates

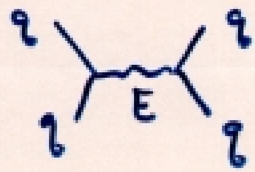
$$m_q \sim \frac{g_E^2}{m_E^2}\,\Lambda_{TC}^3$$

with

$\frac{g_E^2}{4\pi} \sim \frac{1}{10}$ $\Lambda_{TC} \sim \frac{1}{2} m_{T\text{-}\rho}$

$$m_E \sim \begin{cases} 100\text{ TeV} & s, \mu \\ 30\text{ TeV} & c \\ 3\text{ TeV} & t \end{cases}$$

A gauge group with 

must also contain 

This is a new source of flavor violation
not diagonalized with the quark mass matrix

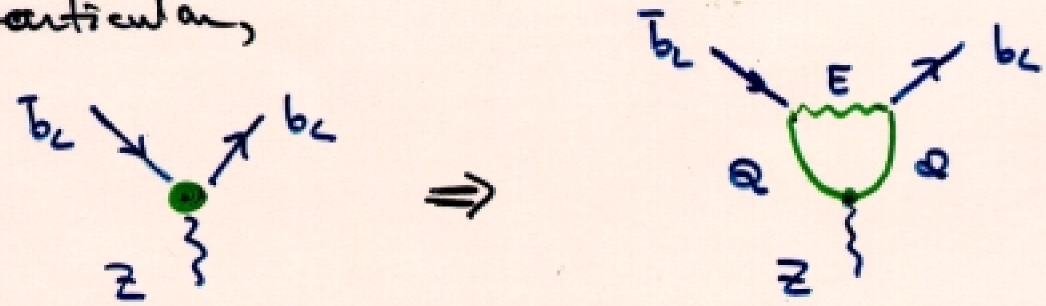
$m_E \sim 100$ TeV is too low to be
invisible in $K\bar{K}$ mixing
dangerously low for $\mu \to e\gamma$ $\mu \to e$ conversion

top?

the ETC bosons which give mass to t are not much heavier than Λ_{TC}.

So, expect TC effects in t dynamics?

In particular,



$$\delta g_{Zbb} \sim \frac{e}{cs} \cdot \left(\frac{g_E^2}{m_E^2} F_\pi \Lambda_{TC} \right)$$

$$\sim g_{Zbb} \cdot \left(\frac{m_t}{\Lambda_{TC}} \right)$$

Chivukula
Simmons
Selipsky

a ~5% correction

This is a huge effect:

$$R_b = R_b(SM)\,(1 \pm 0.3\%)$$

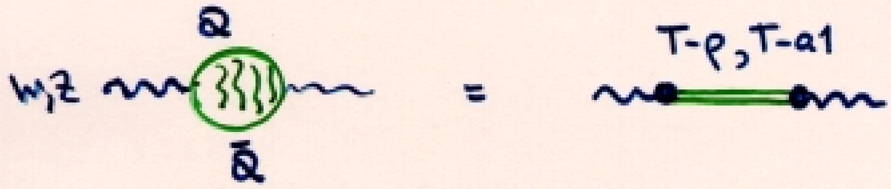
precision electroweak?

The scalar Higgs boson is heavy, so

W, Z self-energy diagrams give large effects

$\sim \quad \alpha_w \log m_h^2/m_W^2$

and, there are new contributions to the vacuum polarization



Holdom + Terning
Golden + Randall
Takeuchi + MP

$$S = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right]$$

$$R(s) = \sum_{R = \rho \text{ or } a1} 12\pi^2 F_R^{\,2}\, \delta(s - m_R^2)$$

in 2-flavor QCD

$\approx +0.3$ (cf. 2 free quarks

$S = \frac{N_c}{6\pi} \sim 0.16$)

Total contribution to S:

$$S = 0.1 + 0.3 = +0.4$$

Can technicolor be saved ?

Hope that this gauge theory has dynamics different from that of QCD

Conformally invariant UV fixed point — Holdom

[present in supersymmetric QCD for $N_f > N_c + 1$]

Slow running of $\alpha_{TC}(Q)$ "walking technicolor" — Appelquist, Karabali, Wijewardhana

$\Downarrow$

Keep $\Sigma(p) \sim$ constant for large p, allowing a higher ETC scale

$\Sigma(p) \sim$ const is much like a pointlike effective scalar $Q\bar{Q}$

second idea : Φ is a fundamental scalar field

To understand and eventually compute m^2,
we need a symmetry that will control the radiative corrections
to the scalar mass term

relate Φ to a higher-spin field

Dimopoulo + Georgi Sakai

$\rightsquigarrow$ Super<u>sym</u>metry

I will first explain how supersymmetry solves
the problem of Λ^2 divergences in
scalar masses.

Only after some further analysis will it
be clear that SUSY can also
<u>explain</u> EWSB

For an introduction to the formulae of SUSY,
see J. Feng's lectures
Here I will only pick up topics associated
with EWSB.

Vanishing of Λ^2 divergences

look first at the simplest model of a self-interacting boson

"chiral multiplet": complex boson + L-handed fermion

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \overline{\psi}_L i\not{\partial} \psi_L + F^* F$$

$$+ \left(F \frac{\partial W}{\partial \phi} + \frac{1}{2} \psi_L^T C \psi_L \frac{\partial^2 W}{\partial \phi^2} \right) + h.c.$$

$W(\phi)$ = "superpotential" $\qquad C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

choose $\qquad W = \frac{1}{2} m \phi^2 + \frac{1}{3} \lambda \phi^3$

$$= \partial_\mu \phi^\dagger \partial^\mu \phi + F^* F + F(m\phi + \lambda \phi^2) + h.c.$$

$$+ \overline{\psi}_L i\not{\partial} \psi + \frac{1}{2} \psi_L^T C \psi_L (m + 2\lambda\phi) + h.c.$$

eliminate F: $\qquad \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$

m is also the Majorana mass of ψ_L

$$\overline{\phi\,\phi^*} = \frac{i}{p^2} \qquad \overline{\psi\,\overline{\psi}} = \frac{i\sigma \cdot p}{p^2} \qquad \overline{F F^*} = i$$

$$\sigma^\mu = (1, \vec{\sigma})$$

divergent radiative corrections to the ϕ mass term

$$\int \frac{d^4p}{(2\pi)^4} (2i\lambda)^2 \cdot \frac{i}{p^2} \cdot i \quad + (-1) \cdot \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} (2i\lambda)^2 \, \mathrm{tr}\Big[c \, \frac{i\sigma \cdot p}{p^2} \, c \, \Big(\frac{-i\sigma \cdot}{p}$$

$$4\lambda^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \quad + \quad 4\lambda^2 \left(-\frac{1}{2}\right) \int \frac{d^4p}{(2\pi)^4} \, \mathrm{tr} \, \frac{c \, \sigma \cdot p \, c \, (-\sigma \cdot p)^T}{p^4}$$

$$= \quad 0 \qquad !$$

this had to happen:

a fermion mass cannot have an additive renormalization, since if $m=0$ chiral symmetry prohibits a mass from appearing

by SUSY, the scalar and the fermion must have the same mass

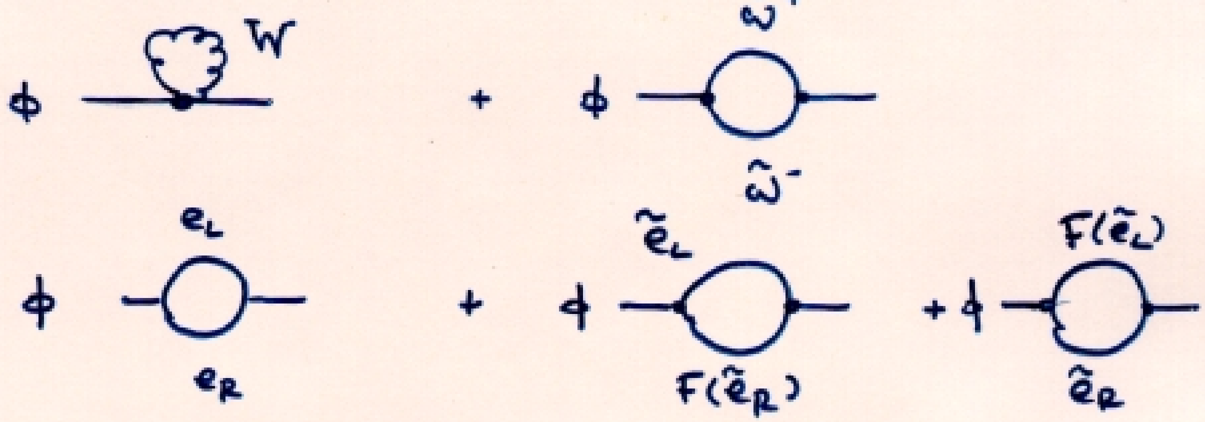
there is even a more powerful theorem:

$W(\phi)$ is renormalized only by field rescaling. — Iliopoulos, Zumino

Can we supersymmetrize only the Higgs part of $\mathcal{L}$,

avoid SUSY partners of q, l, $W^\pm$, ... ?

No! eventually, the ϕ mass correction reaches into all sectors of the theory



SUSY must be (spontaneously) broken

⇝ partial cancellations $\quad \Lambda \leftarrow M(\text{SUSY partner})$

We have more freedom to all $M(\tilde{e})$, $M(\hat{u})$ to be heavy ...

to preserve Q_{EM}, VEV's of ϕ_1, ϕ_2 must align

$$\phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

define: $\tan\beta = v_2/v_1$

physical states of a 2-Higgs-field system:

eaten Goldstone bosons	$\pi^+ \ \pi^- \ \pi^0$
physical charged bosons	$H^+ \ H^-$
physical CP-odd	A^0
physical CP-even	$h^0 \ H^0$

$$\phi_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}[v_1 + \cos\alpha\, H^0 - \sin\alpha\, h^0] + \frac{i}{\sqrt{2}}[\cos\beta\, \pi^0 - \sin\beta\, A \\ \cos\beta\, \pi^- - \sin\beta\, H^- \end{pmatrix}$$

$$\phi_2 = \begin{pmatrix} \sin\beta\, \pi^+ + \cos\beta\, H^+ \\ \frac{1}{\sqrt{2}}[v_2 + \sin\alpha\, H^0 + \cos\alpha\, h^0] - i/\sqrt{2}[\sin\beta\, \pi^0 + \cos\beta\, A \end{pmatrix}$$

place π by acting $\pi \cdot \tau \langle\phi\rangle$

$h^0 \ H^0$ mixing introduces a second angle α

Because the Higgs - fermion coupling is through the superpotential,
<u>two</u> Higgs doublet fields are required

chiral multiplet: complex boson ↔ L-handed fermion

W must be an <u>analytic</u> function of boson fields

so: $Q_L / \bar{Q}_R \to \tilde{Q}$ $\quad u_R/\bar{u}_L \to \tilde{\bar{u}}$ $\quad d_R/\bar{d}_L \to \tilde{\bar{d}}$

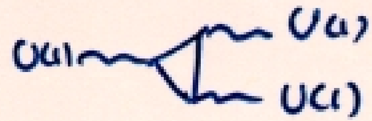
$$W = \Lambda_\ell^{ij}\, \tilde{\bar{e}}^i \phi_1 \cdot \tilde{L}^j + \Lambda_d^{ij}\, \tilde{\bar{d}}^i \phi_1 \tilde{Q}^j + \Lambda_u^{ij}\, \tilde{\bar{u}}^i \epsilon^{ab} \phi_{2a} \tilde{Q}^j$$

$I = \frac{1}{2}$ $Y = -\frac{1}{2}$ $\qquad$ $I = \frac{1}{2}$ $Y = +\frac{1}{2}$

(not ϕ_1^* !)

a cross-check:

the fermionic partners of ϕ_1 ϕ_2 have triangle anomalies:



which cancel when we include <u>both</u> ϕ_1, ϕ_2.

the minimal SUSY extension of the SM,

with 2 Higgs doublets $\phi_1\ \phi_2$

is called the MSSM.

add more doublet?

this disrupts the apparent grand unification of SM couplings

add SM singlets?

this is allowed, adds freedom/complexity

flavor?

In a general model with 2 Higgs doublets, flavor-changing effects of the Higgs field cannot be rotated away.

However, with the specific structure

$$e, d \rightarrow \phi_1 \qquad u \rightarrow \phi_2$$

there is no obstruction to removing flavor violation as in the MSM

Glashow-Weinberg

The various elements of the MSSM combine to give a well-defined scalar potential:

SUSY-breaking mass terms:

$$V_{soft} = M_1^2\left(|\phi_1^0|^2 + |\phi_1^-|^2\right) + M_2^2\left(|\phi_2^+|^2 + |\phi_2^0|^2\right)$$
$$- B\mu\left(\phi_1^0\phi_2^0 - \phi_1^-\phi_2^+\right) + h.c.$$

Superpotential terms: $W = \mu\, \varepsilon^{ab}\phi_{1a}\phi_{2b}$

$$V_F = \mu^2\left(|\phi_1^0|^2 + |\phi_1^-|^2 + |\phi_2^+|^2 + |\phi_2^0|^2\right)$$

D-terms $(\phi^\dagger\phi)^2$ terms from gauge couplings

$$V_D = \frac{(g')^2}{2}\left(-\tfrac{1}{2}\left(|\phi_1^0|^2 + |\phi_1^-|^2\right) + \tfrac{1}{2}\left(|\phi_2^+|^2 + |\phi_2^0|^2\right)\right)^2$$
$$+ \frac{g^2}{2}\left(\tfrac{1}{2}|\phi_1^0|^2 - \tfrac{1}{2}|\phi_1^-|^2 + \tfrac{1}{2}|\phi_2^+|^2 - \tfrac{1}{2}|\phi_2^0|^2\right)^2$$
$$+ \frac{g^2}{2}\left(\phi_1^{0\dagger}\phi_1^- + \phi_2^{+\dagger}\phi_2^0\right)\left(\phi_1^{-\dagger}\phi_1^0 + \phi_2^{0\dagger}\phi_2^+\right)$$

add these up, set

$$\phi_1^0 = \frac{v_1}{\sqrt{2}} \qquad \phi_2^0 = \frac{v_2}{\sqrt{2}}$$

this gives

$$V = \tfrac{1}{2} M_1^2 v_1^2 + \tfrac{1}{2} M_2^2 v_2^2 - B\mu\, v_1 v_2$$

$$+ \frac{\mu^2}{2}(v_1^2 + v_2^2) + \frac{1}{32}(g^2 + g'^2)(v_1^2 - v_2^2)^2$$

if, e.g. $(M_2^2 + \mu^2) < 0$

there is an instability to $v_2 \neq 0$

then if $B\mu \neq 0$, we also find a nonzero v_1

$$v_1 \sim \frac{B\mu}{M_1^2 + \mu^2} v_2 \quad \text{or} \quad \tan\beta \sim \frac{M_1^2 + \mu^2}{B\mu}$$

It is convenient to define

$$m_A^2 = \frac{B\mu}{\sin\beta\cos\beta}$$

Now work out the mass matrix of CP-even bosons

$$\phi_1^0 = \frac{(v_1 + h_1)}{\sqrt{2}} \qquad \phi_2^0 = \frac{(v_2 + h_2)}{\sqrt{2}}$$

$$m^2 \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 V}{\partial v_1^2} & \frac{\partial^2 V}{\partial v_1 \partial v_2} \\ \frac{\partial^2 V}{\partial v_1 \partial v_2} & \frac{\partial^2 V}{\partial v_2^2} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$= \begin{pmatrix} M_1^2 + \mu^2 + \frac{1}{2}\frac{m_Z^2}{v^2}(3v_1^2 - v_2^2) & -B\mu - \frac{m_Z^2}{v^2} v_1 v_2 \\ -B\mu - \frac{m_Z^2}{v^2} v_1 v_2 & M_2^2 + \mu^2 + \frac{1}{2}\frac{m_Z^2}{v^2}(3v_2^2 - v_1^2) \end{pmatrix}$$

using minimization conditions for V

$$= \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta & -(m_A^2 + m_Z^2)\sin\beta\cos\beta \\ -(m_A^2 + m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2 \sin^2\beta \end{pmatrix}$$

then eg. for $\sin\beta = 1 \ \cos\beta = 0$

$$m^2 = \begin{pmatrix} m_A^2 & \\ & m_Z^2 \end{pmatrix} \qquad \text{eigenvalues!} \quad m_Z^2 \ \ m_A^2$$

in general, let $|v\rangle = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}$

$$\langle v | m^2 | v \rangle = m_Z^2 (\sin^2\beta - \cos^2\beta)^2$$

so there is one Higgs boson with

$$m_{h^0} \leq m_Z \cos 2\beta$$

up to radiation corrections

1-loop radiative corrections turn out to give a substantial addition to this bound

Haber Hempfling
Okada, Yamaguchi, Yanagida
Ellis, Ridolfi, Zwirner

$$m_{h^0}^2 \leq m_Z^2 + \frac{3\alpha_W}{2\pi m_W^2}\, m_t^4 \log \frac{m_{\tilde{Q}}^2}{m_t^2}$$

$$\sim (135 \text{ GeV})^2 \quad \text{for} \quad m_{\tilde{Q}} \sim 1 \text{ TeV}$$

Nevertheless, this is still a very stringent bound on the mass of the lightest Higgs boson

This bound is not valid in models with extra Higgs singlets. However, as long as ϕ_1, ϕ_2 are elementary fields up to $M \sim M_{GUT} \sim 2\times10^{16}$ GeV, one finds

$$m_{h^0} < 205 \text{ GeV}$$

Quiros
Espinosa

the rest of the Higgs spectrum (tree level)

A^0 $\qquad m_A$

$H^\pm$ $\qquad m_{H^+}^2 = m_A^2 + m_W^2$

H^0 $\qquad m_H^2 + m_h^2 = m_A^2 + m_Z^2$

(take trace of m^2)

if $m_A \gg m_Z$,

A^0 $H^\pm$ H^0 are approximately degenerate

h^0 is isolated

Properties of h^0 closely approximate those of the MSM Higgs boson, up to corrections of order m_Z^2 / m_A^2.