Introduction to Electroweak Symmetry Breaking

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SSI 2001

the electroweak fits lead to

$$m_h \lesssim 200 \text{ GeV} \qquad 95\% \text{ conf}$$

in the MSM, a if there is no other important contribution to S,T.

In Yang-Mills theory,

unbroken symmetry ⟹ massless vector bosons

more generally, there are serious problems in any Lorentz-invariant theory of massive vector bosons unless those particles are Yang-Mills bosons and the gauge symmetry is spontaneously broken.

Nambu, Anderson, ...

.. Higgs, Kibble, Guralnik, Hagen, Brout, Englert ...

.. 'tHooft, Veltman, Lee, Zinn-Justin ...

... Cornwall, Tiktopoulos ..

To understand why $W^{\pm}$ and Z^{0} are massive, we need to understand why $SU(2) \times U(1)$ is broken.

The Standard Model gauge theory does not break its own symmetry — the gauge couplings are too weak.

Some additional agent is needed. What is it?

From the precision electroweak experiments
of the 1990's, we know that

- the weak interactions are mediated by
vector bosons $W^{\pm}$ and Z^0

- these bosons have universal couplings to
all species of quarks and leptons

Z branching fractions, partial widths, asymmetries

- the W couplings to Z, γ are very close to
those of pointlike spin-1 particles

correctness of EW radiative corrections

direct measurement of TGC's at LEP2

These observations make a very strong case for
the idea of Glashow, Weinberg, Salam
that the weak interaction bosons are gauge bosons
of $SU(2) \times U(1)$.

I think this is the most important problem in high-energy physics.

Often (e.g. in the New York Times) you see another opinion:

Particle physics is almost finished. We only need to discover one more particle, the Higgs Boson.

The reason that the problem of electroweak symmetry breaking is so important is that this statement is so unlikely to be true.

In these lectures, I will review what is known about electroweak symmetry breaking and give you some tools to use in thinking about it. Then I will review models of EWSB, some conventional, some less so.

some nomenclature:

Standard Model (SM)

the $SU(3) \times SU(2) \times U(1)$ gauge theory with quarks & leptons, but without Higgs.

Minimal Standard Model (MSM)

SM + 1 Higgs field

Here are questions that a model of EWSB should answer:

1. What sets the mass scale of EWSB ?
   ('Gauge Hierarchy Problem')

2. What is the reason that EWSB occurs ?

3. Does EWSB contribute new sources of flavor violation ?

4. Does the top quark have a central role in EWSB ?

my prejudices:

1, 2 → must have physically sensible answers

3 → no

4 → yes

We'll see what different models say.

the simplest model of EWSB:

SM + 1 Higgs doublet field ϕ ("MSM")

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I = \tfrac{1}{2} \qquad Y = +\tfrac{1}{2}$$

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

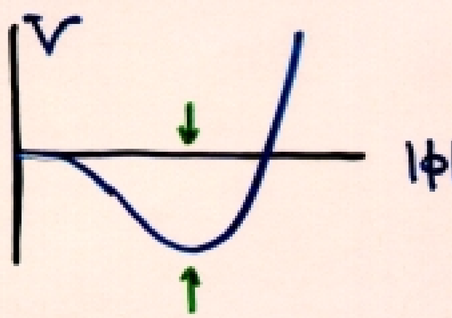
$$D_\mu \phi = (\partial_\mu - i g A_\mu^a \tau^a - i g' B_\mu Y)$$

$$\tau^a = \frac{\sigma^a}{2}$$

Assume $m^2 = -\mu^2 < 0$

then the potential energy of ϕ is

$$V = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$



minimized at

$$|\phi| = \frac{v}{\sqrt{2}}$$

one ground state is

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v = \text{real}$$

all other ground states are related by $SU(2)$ rotations

For this state

$$(D^\mu \phi)^* (D_\mu \phi) = \frac{1}{2} (0\ v) \left[g A^a_\mu \tau^a + g' B \cdot \frac{1}{2} \right]^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{2} (A^a_\mu, B)\, \mathcal{M}^2 \begin{pmatrix} A^a_\mu \\ B \end{pmatrix}$$

where:

$$\mathcal{M}^2 = \left(\begin{array}{ccc|c} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ \hline & & -gg' & (g')^2 \end{array} \right) \cdot \frac{v^2}{4}$$

this gives the mass eigenstate

$$W^\pm = \frac{1}{\sqrt{2}} (A^1 \mp i A^2) \qquad m_W^2 = \frac{g^2}{4} v^2$$

$$Z = \frac{g A^3 - g' B}{\sqrt{g^2 + g'^2}} \qquad m_Z^2 = \frac{(g^2 + g'^2)}{4} v^2$$

$$A = \frac{g' A^3 + g B}{\sqrt{g^2 + g'^2}} \qquad m_A = 0$$

Why is $m_A = 0$?

$\mathcal{M}^2$ has a zero eigenvector if there is an unbroken gauge symmetry

proof: $D_\mu = (\partial_\mu - ig A^a_\mu T^a)$

$$\mathcal{M}^2_{ab} A^a_\mu A^{\mu b} = \langle\phi\rangle^T (g A^a_\mu T^a)(g A^b_\mu T^b) \langle\phi\rangle$$

$$\mathcal{M}^2_{ab} = \langle\phi\rangle^T T^a T^b \langle\phi\rangle$$

if $\xi_b T^b \langle\phi\rangle = 0$

$$(1 + i\xi_b T^b)\langle\phi\rangle = \langle\phi\rangle$$ unbroken symmetry

$$\mathcal{M}^2_{ab}\, \xi_b = 0$$ zero-mass vector boson

In the case at hand

$$(\tau^3 + Y)\langle\phi\rangle$$

$$= \left[\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\right] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$= 0 \quad \checkmark$$

the unbroken symmetry is exactly $Q = \underline{I^3 + Y}$

numerically :

$$\frac{g^2}{4\pi} = \frac{1}{29.6} \qquad \frac{g'^2}{4\pi} = \frac{1}{98.5}$$

$$\sin^2\theta_w = \frac{(g')^2}{g^2+(g')^2} = 0.231$$

$$\left.\begin{array}{l} m_W = 80.4 \\ m_Z = 91.19 \end{array}\right\} \qquad \begin{array}{l} v \cong 246 \text{ GeV} \\ m_W/m_Z \cong \cos\theta_w \ \checkmark \end{array}$$

from the formula for $V(\phi)$

$$v = (\mu^2/\lambda)^{\frac{1}{2}}$$

$$m_h = \sqrt{2}\mu = \sqrt{2\lambda}\, v$$

$$= \sqrt{\lambda} \cdot 350 \text{ GeV}$$

The MSM is not the only model that gives

$$m_A = 0 \qquad m_W/m_Z = \cos\theta_W$$

From the above, what we need is

(a) Unbroken gauge symmetry generated by

$$Q = I^3 + Y$$

(b) In ungauged theory, an unbroken O(3) or SU(2) symmetry among $A^1_\mu, A^2_\mu, A^3_\mu$

"custodial SU(2)" Sikivie, Susskind, Voloshin, Zakharov

We have seen that the MSM satisfies (a). It also automatically satisfies (b). Write

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^1 + i\phi^2 \\ \phi^0 + i\phi^3 \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}|\partial_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\phi^0)^2 + \frac{1}{2}(\partial_\mu\phi^1)^2 + \frac{1}{2}(\partial_\mu\phi^2)^2 + \frac{1}{2}(\partial_\mu\phi^3)^2$$

$\langle\phi^0\rangle = v$ leaves an O(3) symmetry among ϕ^1, ϕ^2, ϕ^3

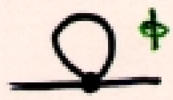
Other models of EWSB may satisfy (b) in other ways.

How does the MSM fare on the big questions?

1. scale of EWSB?

the scale is set by m^2 or μ^2, that is, by hand.

actually, there is a worse problem:



$$\rightarrow \quad \delta m^2 = \frac{\lambda}{4\pi}\Lambda^2$$

$$\rightarrow \quad \delta m^2 = \frac{g^2}{4\pi}\Lambda^2$$

so $$m^2 = m^2_{bare} + \frac{g^2}{4\pi}\Lambda^2 + \cdots$$

if $\Lambda = m_{Pl.} = 10^{19}$ GeV, $m \sim 300$ GeV

then $m_{bare} \sim 10^{18}$ GeV

m_{bare} and δm cancel almost completely

m comes from a residue in the 31st decimal place !

2. reason for EWSB?

this residue is <u>negative</u>

For these reasons, the MSM does not let us understand the origin of EWSB.

However, it is important to study this model in more detail, and not only because it is the simplest.

Consider a model of EWSB that produces a light Higgs field ϕ (perhaps as a bound state) and has masses $\geq M$ for all other states.

Then we can write an effective Lagrangian describing physics at energies below M

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + (\text{most general coupl. of } \phi \text{ to the SM}) + O\left(\frac{1}{M^2}\right)$$

models of this type have the MSM as their low-energy limit, up to (small ?) corrections so, continue with the big questions

(4) top quark ? no special role (but see below)

(3) flavor ? here there is an interesting story:

Write the most general renormalizable coupling of ϕ to quarks and leptons:

$$Q^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L \quad u_R^i \quad d_R^i \qquad L^i = \begin{pmatrix} \nu^i \\ e^i \end{pmatrix}_L \quad e_R^i$$

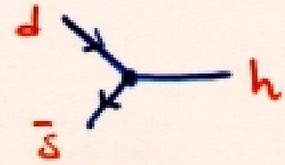
$$\mathcal{L} = -\Lambda_\ell^{ij}\, \bar{L}^i \cdot \phi\, e_R^j \;-\; \Lambda_d^{ij}\, \bar{Q}^i \cdot \phi\, d_R^j$$

$$-\Lambda_u^{ij}\, \epsilon^{ab}\, \bar{Q}_a^i \phi_b^\dagger u_R^j$$

each term is a singlet of SU(2) with $\Sigma Y = 0$ ✓

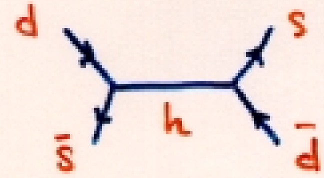
Λ^{ij} are 3×3 complex-valued matrices flavor-changing terms from the Higgs coupling are extremely dangerous

e.g. allow



$$= \delta_{ds}\, \gamma^5$$

$$\Delta m_K^2 \sim \delta_{ds}^2 \left(\frac{f_K m_K^2}{m_s}\right)^2 \frac{1}{m_h^2}$$



$$\Delta m_K \sim \delta_{ds}^2 \left(\frac{f_K m_K}{m_s m_h}\right)^2 m_K$$

$$\sim \delta_{ds}^2 \cdot \left(\frac{100\text{ GeV}}{m_h}\right)^2 \cdot 5\times 10^{-3}\text{ MeV}$$

compare to

$$m_{K_L} - m_{K_S} = 3.5\times 10^{-12}\text{ MeV}$$

the naive estimate $\delta_{ds} \sim \sin\theta_c = V_{us}$

requires $m_h > 800$ TeV !

$B\bar{B}$ mixing, $D\bar{D}$ mixing, $\mu \to e$ conversion, $b \to s\gamma$ give weaker, but also important, constraints

How does the MSM evade this problem?

Decompose:

$$\Lambda_\ell = U_\ell \lambda_\ell W_\ell^\dagger \qquad \Lambda_d = U_d \lambda_d W_d^\dagger$$

$$\Lambda_u = U_u \lambda_u W_u^\dagger$$

U W unitary $\qquad \lambda$ diagonal, real, positive

Compensate U, W by

$$L \to U_\ell L \qquad e_R \to W_\ell e_R \qquad \text{etc.}$$

U_u, U_d reappear in the weak interaction current:

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \gamma^\mu d_L^i \quad \to \quad \frac{g}{\sqrt{2}} \bar{u}_L^i \gamma^\mu \left[U_u^\dagger U_d \right]^{ij} d_L^j$$

so

$$U_u^\dagger U_d = V_{CKM} \quad !$$

U_ℓ disappears in the MSM but can reappear in the ν mass matrix

(which is due to dimension - 5 operators $L \cdot L \cdot \phi \cdot \phi$)

W_u, W_d, W_ℓ completely disappear in the SM.

They can reappear in a theory w. right-handed currents.

$\lambda_\ell, \lambda_u, \lambda_d$ give the fermion masses

$$m_f = \frac{\lambda_f}{\sqrt{2}} v$$

v sets the scale, but

$$\lambda_f = \begin{cases} \sim 1 & \text{for } t \\ \sim 3\times 10^{-6} & \text{for } e^- \end{cases}$$

All explanations of fermion masses, ν masses, CKM angles, CP violation start from the Higgs field — or whatever replaces it.

If we cannot predict the mass of the Higgs boson with the MSM, can we at least get limits?

limits come from three sources:

- direct searches
- renormalization group analysis
- precision electroweak measurements

I'll discuss the last two of these;

for the first, see the lectures of P. Janot.

Renormalization group analysis

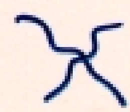
since $m_h = \sqrt{2\lambda}\, v$

a bound on λ gives a bound on m_h

Naively, $\lambda < 4\pi \rightarrow m_h < 1800$ GeV

More carefully, tree-level unitarity of

$$W^+W^- \rightarrow W^+W^-$$

 + crosses +  +  + crosses

implies $m_h < \left(\frac{16 m_W^2}{3\alpha_W}\right)^{1/2} \cong 1000$ GeV

Lee Quigg Thacker

However, a stronger constraint on λ may apply:

the RG evolution of λ is:

$$\frac{d\,\lambda(Q)}{d\log Q} = + \frac{3}{2\pi^2}\lambda^2 + \cdots$$

self-renormalization gives + for all couplings except non-Abelian gauge couplings

With this sign, fixed λ_0 at high Q_0

$\rightsquigarrow$ bounded λ at lower Q

$$\lambda(v) = \frac{\lambda_0}{1 - \frac{3\lambda_0}{2\pi^2}\log\frac{v}{Q_0}} = \frac{2\pi^2/3}{\log(M/v)}$$

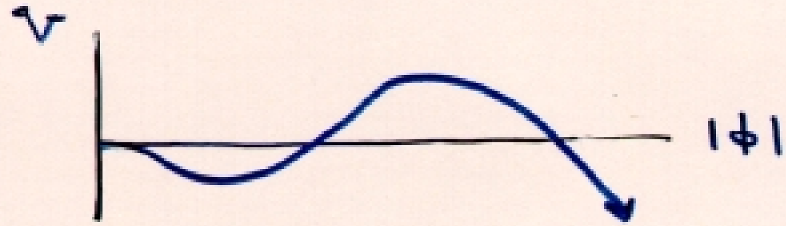
then if $\lambda(Q)$ is defined up to the mass scale M

$M = 10^{19}$ GeV	$\lambda < 0.17$	$m_h < 145$ GeV
10^{16}	0.21	160
10^{10}	0.38	210
10^{5}	1.1	360
10^{4}	1.8	460

the top quark also affects the running of λ

$$\frac{d\lambda}{d\log Q} = \frac{1}{8\pi^2}\left\{ 12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 + \cdots \right\}$$

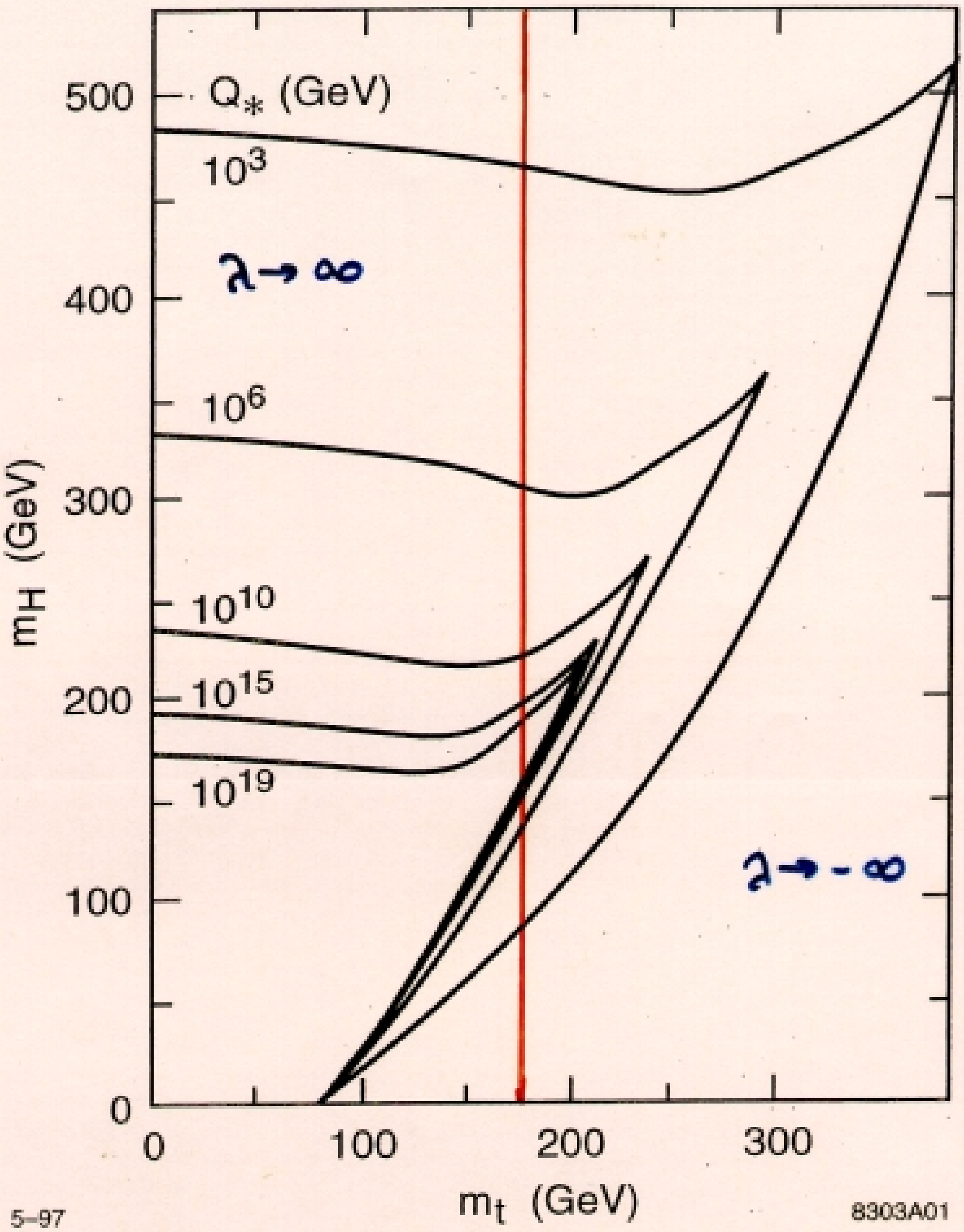
so if $\lambda_t > 2\lambda$, λ can run to $-\infty$ at large Q



this leads to a stability region for the MSM depending on m_h, m_t, M

with M = highest scale where the MSM is valid

Lindner



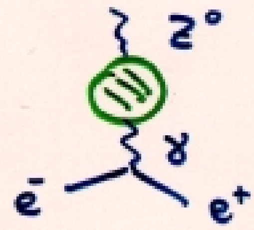
$m_t = 175.$

Most electroweak corrections involve light quarks and leptons in the external states.

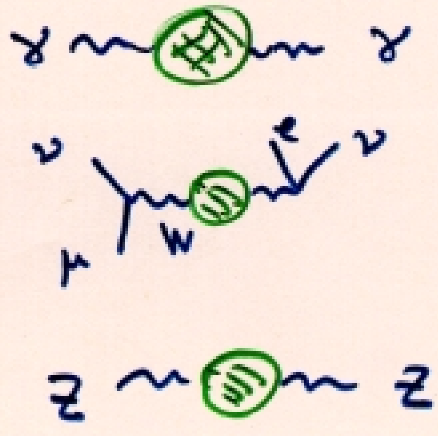
Ignore the direct couplings of these particles to new physics

The remaining effects come from vacuum polarization diagrams

e.g. correction to $A_{LR}(e)$



To pursue this systematically, remember that the precision calculations of EW observables depend on α, G_F, m_t, and that these also receive oblique corrections:



Study vacuum polarizations of the currents J^1_L, J^2_L, J^3_L, J^Q

Effects of heavy particles are suppressed by q^2/M^2, so Taylor expand

$$\Pi_{11}(q^2) = \Pi_{11}(0) + q^2\,\Pi'_{11}(0) + \cdots$$

$$\Pi_{33}(q^2) = \Pi_{33}(0) + q^2\,\Pi'_{33}(0) + \cdots$$

$$\Pi_{3Q}(q^2) = q^2\,\Pi'_{3Q}(0) + \cdots$$

$$\Pi_{QQ}(q^2) = q^2\,\Pi_{QQ}(0) + \cdots$$

there are 6 coefficients; eliminate 3 in favor of α, G_F, m_Z.

This leaves:

$$S = 16\pi\left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right] \qquad q^2/M^2$$

$$T = \frac{4\pi}{s^2c^2 m_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right] \qquad \text{custodial } SU(2) \text{ violation}$$

$$U = 16\pi\left[\Pi'_{11}(0) - \Pi'_{33}(0)\right] \qquad \text{both !}$$

(ignore U from here on)

EW parameters can be expanded in terms of S, T

$$m_W^2 = m_W^2(*) + \frac{\alpha c^2}{c^2-s^2}\, m_Z^2 \left[-\tfrac{1}{2} S + c^2 T\right]$$

$$\sin^2\theta_{w\,\mathrm{eff}} = \sin^2\theta_w(*) + \frac{\alpha}{c^2-s^2} \left[\tfrac{1}{4} S - s^2c^2 T\right]$$

$\vdots$ [(*) = MSM at ref. values.]

Various types of heavy particles give characteristic contributions to S, T

heavy fermion doublet
$|m_N - m_E| \ll m_N, m_E$

$$S \cong \frac{1}{6\pi}$$

$$T = \frac{1}{12\pi s^2c^2}\,\frac{(\Delta m)^2}{m_Z^2}$$

top quark

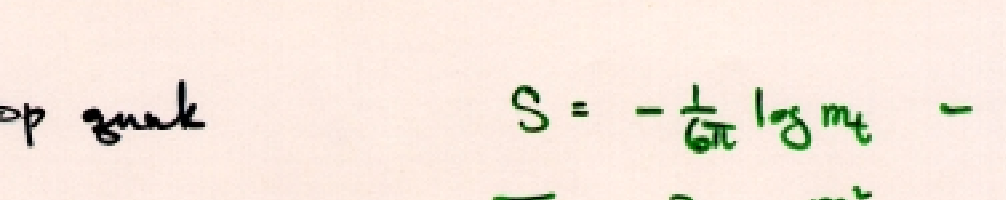
$$S = -\frac{1}{6\pi}\log m_t - (*)$$

$$T = \frac{3}{16\pi s^2c^2}\,\frac{m_t^2}{m_Z^2} - (*)$$

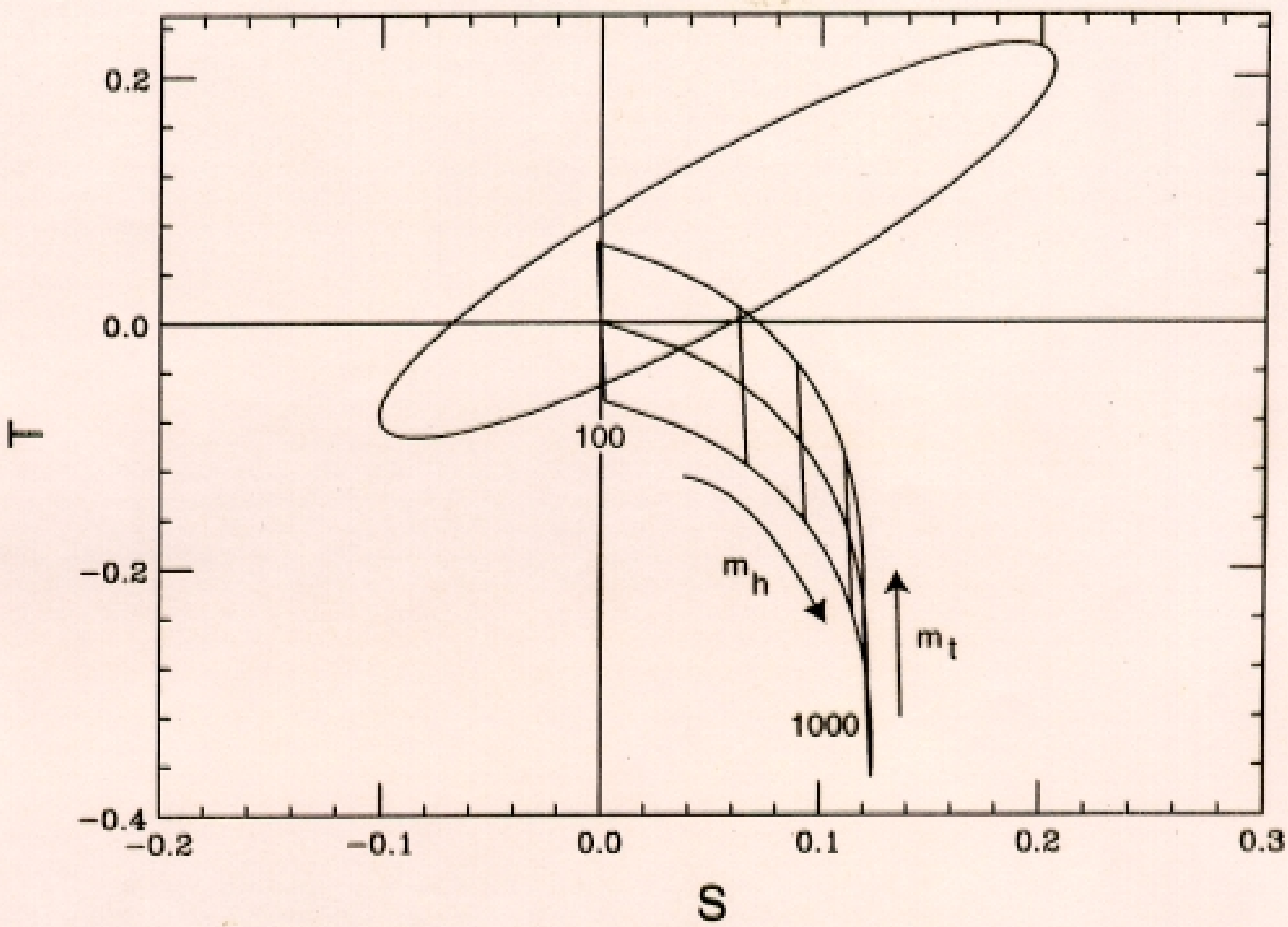
Higgs boson

$$S = \frac{1}{12\pi}\log m_h^2 - (*)$$

$$T = -\frac{3}{16\pi c^2}\log m_h^2 - (*)$$



comparison of EW observables (summer 2000)
to the MSM expectation



$*$ = $m_h = 100$ GeV
$m_t = 173 \pm 5$ GeV