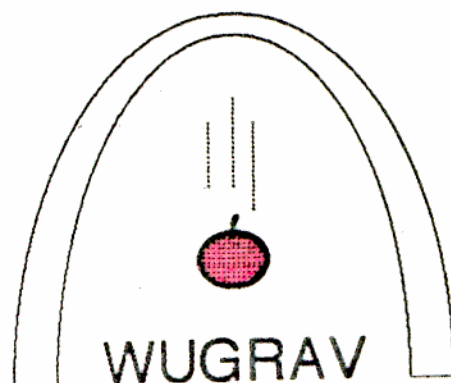# GRAVITATIONAL WAVES AND THE VALIDITY OF GENERAL RELATIVITY


WASHINGTON UNIVERSITY GRAVITATION GROUP

- The Binary Pulsar
- Gravitational Waves to High Post-Newtonian Order
    - Why?
    - DIRE: Direct Integration of the Relaxed Einstein Equations
- Gravitational-wave Tests of General Relativity
    - Polarization of Waves
    - Tests of Radiation Damping
    - Speed of Waves and a Bound on the Graviton Mass

SIGNIFICANCE OF INSPIRALLING COMPACT BINARIES

- Promising source for LIGO/VIRGO detectors
  - $3 - 100$ per year to 200 Mpc, $h \approx 10^{-21}$
  - possible correlation with gamma-ray bursts (2 NS)
- Motion and GW generation almost "pure" general relativity
  - "Point" masses (tidal and quadrupole effects small)
  - NS and BH can be treated the same (strong equivalence principle)
  - Spin effects calculable
  - Hydrodynamics only during coalescence
- Observations probe general relativity in new regimes
  - Dynamical, strong-field GR
  - Non-linear gravity important (e.g. tails)
  - Coalescence may reveal information about NS equation of state (finite radius effects) or "smoking gun" for black holes (quasinormal modes)
- Post-Newtonian methods applicable for most of inspiral
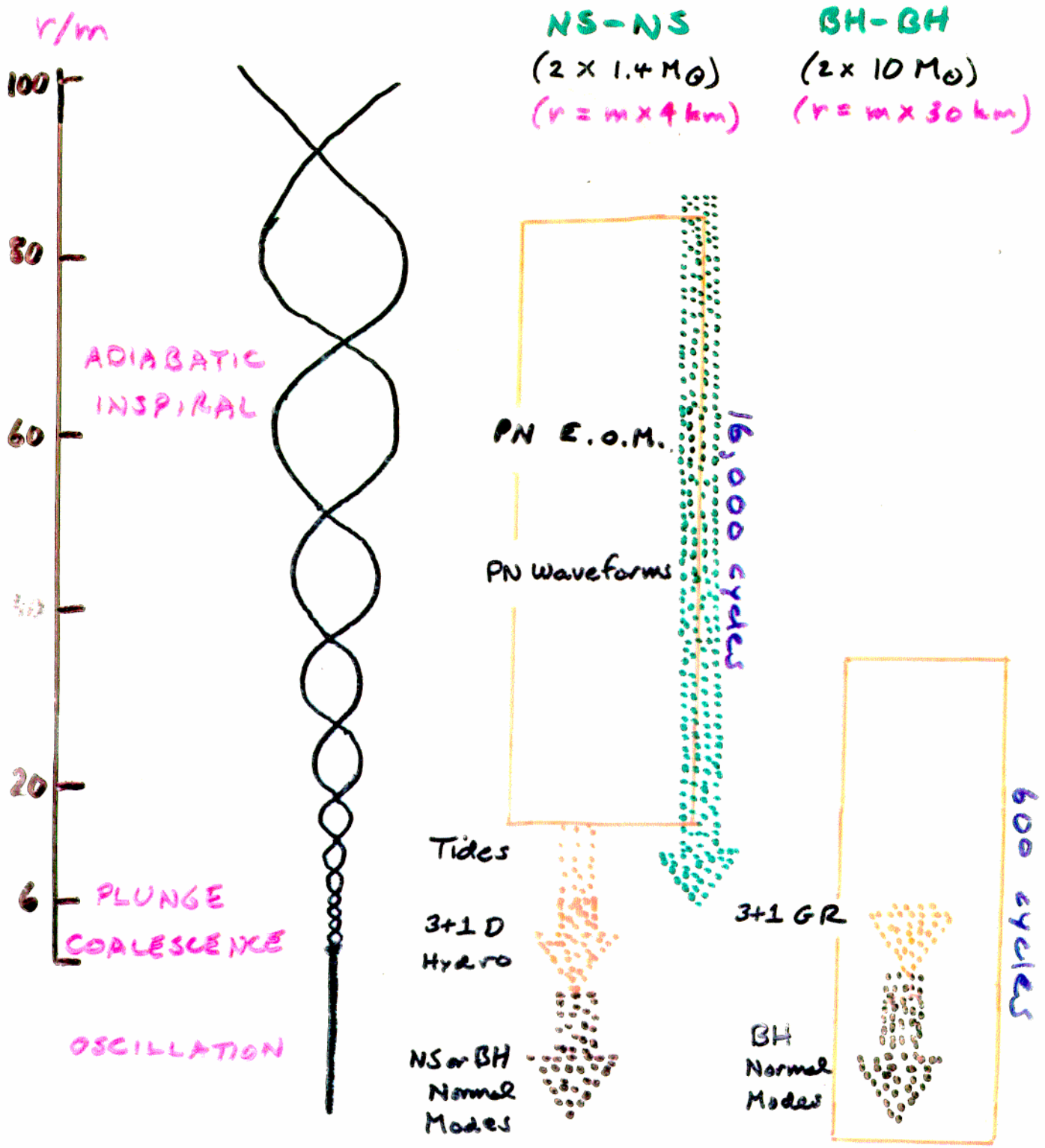 - 2 NS at 10 Hz: $v^2 \approx m/r \approx 5 \times 10^{-3}$

BINARY COALESCENCE,

r/m

100

80

ADIABATIC
INSPIRAL

60

40

20

6 PLUNGE
COALESCENCE

OSCILLATION
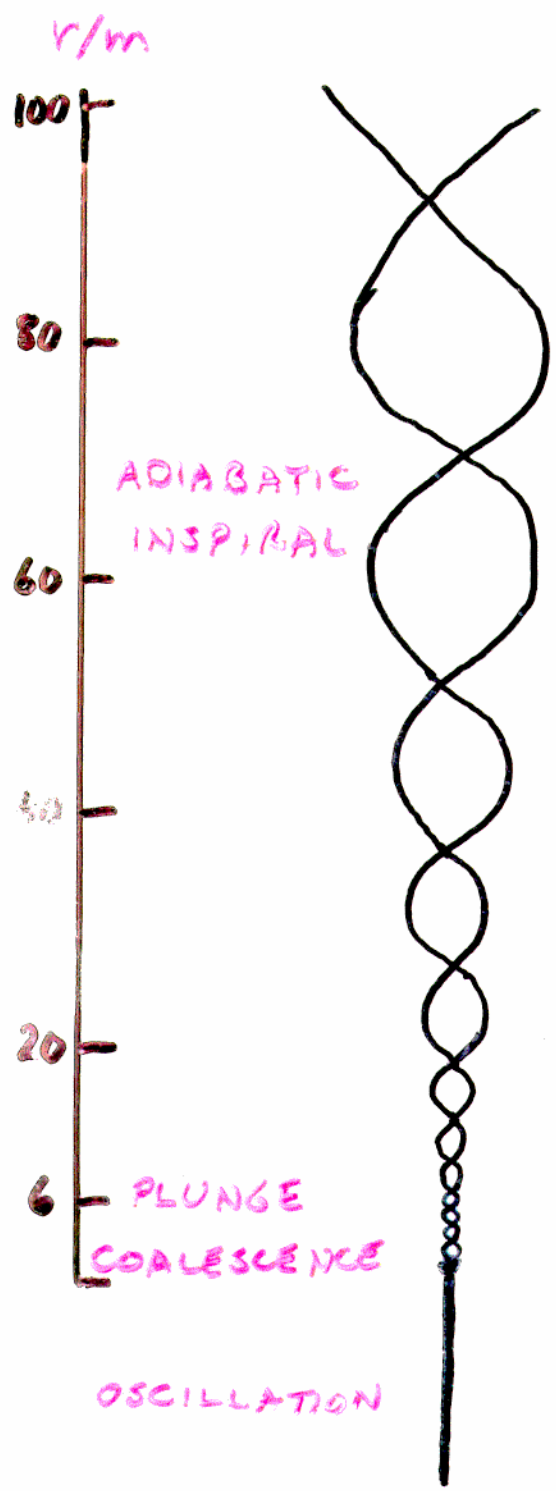
NS—NS
$(2 \times 1.4 \, M_\odot)$
$(r = m \times 4 \, km)$

BH—BH
$(2 \times 10 \, M_\odot)$
$(r = m \times 30 \, km)$

BINARY COALESCENCE, THEORETICAL METHODS, LIGO WINDOWS

r/m

NS-NS
$(2 \times 1.4 M_\odot)$
$(r = m \times 4 km)$

BH-BH
$(2 \times 10 M_\odot)$
$(r = m \times 30 km)$

100

80

ADIABATIC
INSPIRAL

60

PN E.O.M.

16,000 cycles

PN Waveforms

40

20

Tides

3+1 GR

600 cycles

6 PLUNGE

3+1 D
Hydro

COALESCENCE

NS or BH
Normal
Modes

BH
Normal
Modes

OSCILLATION

CHALLENGE TO THEORISTS

Matched filtering of data requires very accurate template waveforms, especially for extracting parameters ($O[(v/c)^6]$?)

- **Theoretical Methods**

- BDI Method (Blanchet, Damour, Iyer)

  - ⋆ Slow motion, PN approximation in near zone

  - ⋆ Non-linearity, "post-Minkowskiian" expansion in far zone

  - ⋆ Match & analytic continuation to control divergences

  - ⋆ Far-zone results to 2.5PN order $\rightarrow$ 3.5PN order

- BH Perturbations (Tagoshi, Nakamura, Sasaki, Poisson, Tanaka ...)

  - ⋆ Valid in limit $m_1 \ll m_2$ ($\eta = m_1 m_2/(m_1 + m_2)^2 \ll 1$)

  - ⋆ $\dot{E}$ through 5.5PN order $[(v/c)^{11}]$

- Extended Epstein-Wagoner Method (Will, Wiseman) ; DIRE

  - ⋆ Formal integral solution of relaxed Einstein equation

  - ⋆ Iterate repeatedly

  - ⋆ New trick for integrating over far-zone light cone

  - ⋆ Far-zone results to 2PN order $\rightarrow$ 3.5PN order

- **All three methods agree where they overlap**

THE RELAXED EINSTEIN EQUATIONS

Field Definition:

$$h^{\alpha\beta} = \eta^{\alpha\beta} - \sqrt{-g}g^{\alpha\beta}$$

Harmonic Coordinate Condition:

$$\partial_\beta h^{\alpha\beta} = 0$$

"Reduced" Field Equations:

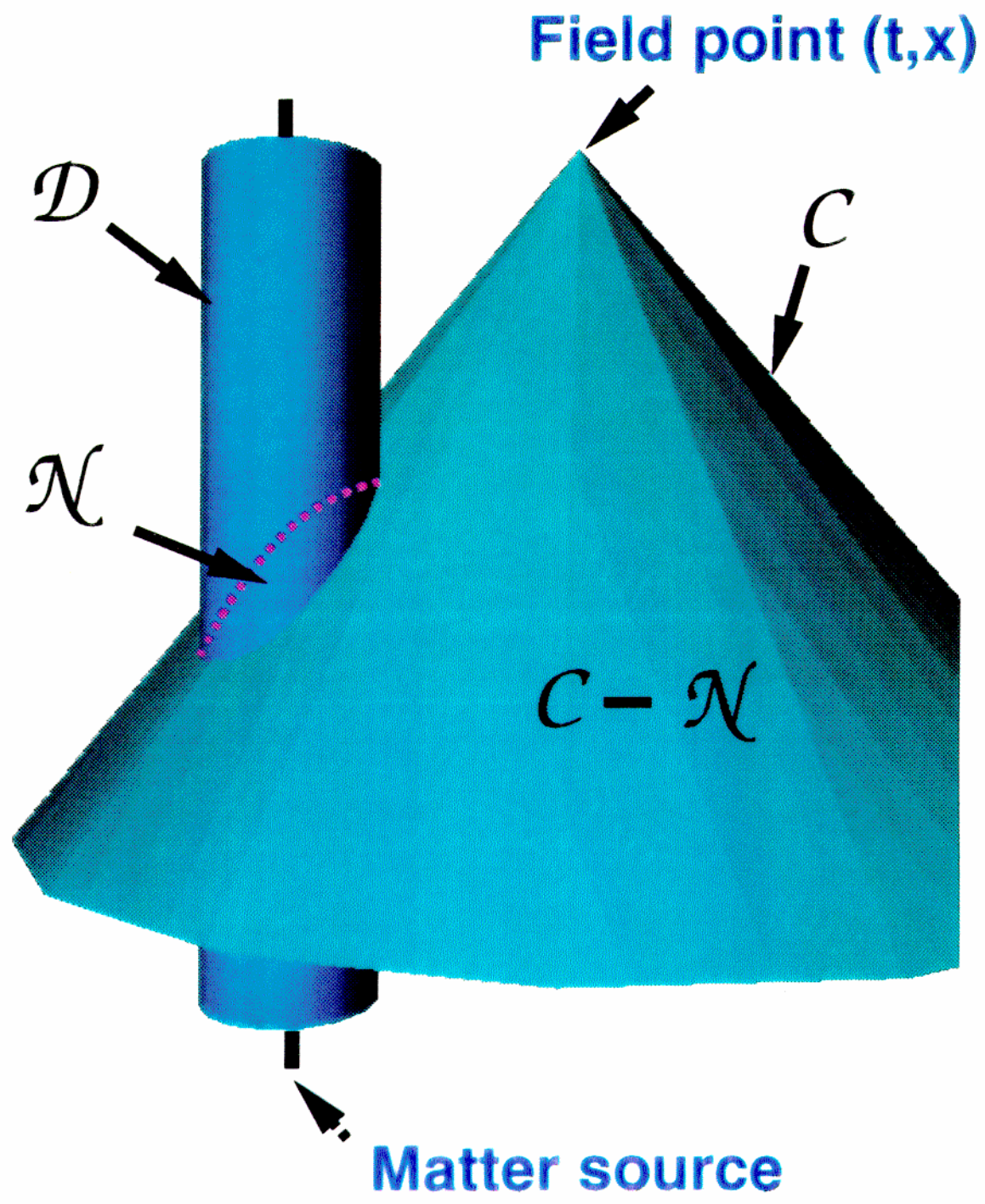$$\Box h^{\alpha\beta} = -16\pi(-g)T^{\alpha\beta} - \Lambda^{\alpha\beta}(h) \equiv -16\pi\tau^{\alpha\beta}$$
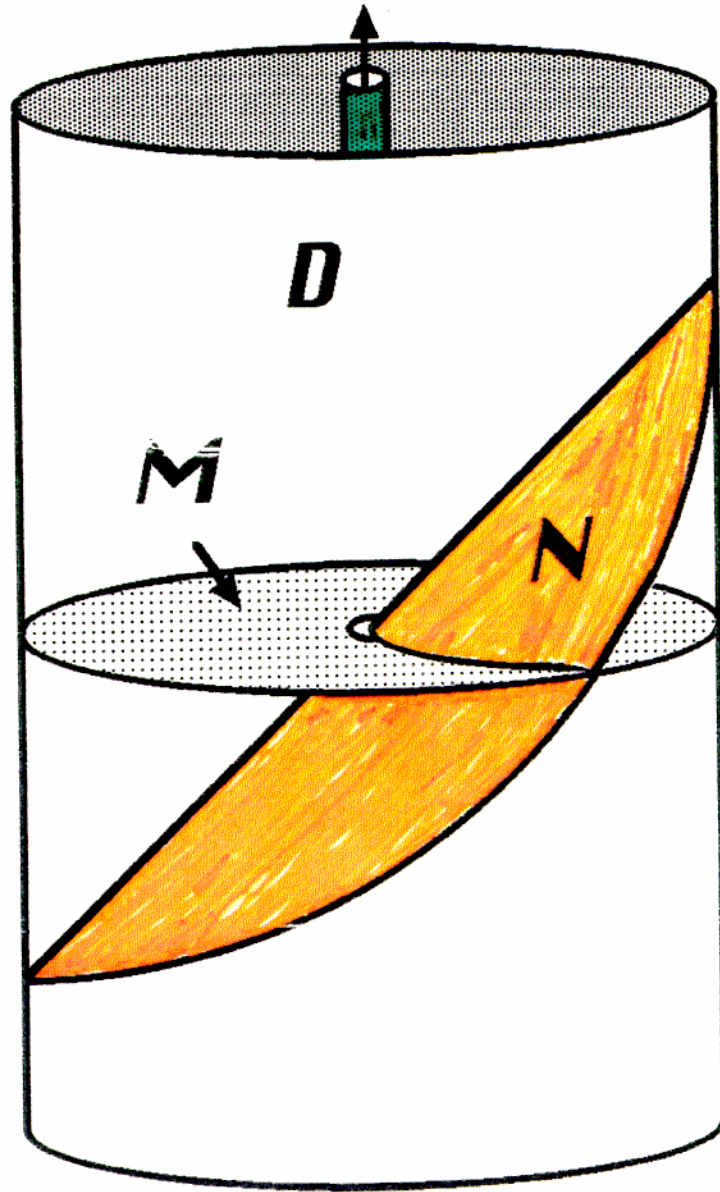
$$\partial_\beta \tau^{\alpha\beta} = 0$$

Formal Solution:

$$h^{\alpha\beta} = 4\int_{\mathcal{C}} \frac{\tau^{\alpha\beta}(t', \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\delta(t' - t + |\mathbf{x} - \mathbf{x}'|)d^4x'$$

Divide into Inner ($\mathcal{N} = \{|\mathbf{x}'| < \mathcal{R}\}$) and Outer ($\mathcal{C} - \mathcal{N} = \{|\mathbf{x}'| > \mathcal{R}\}$) integrals, where $\mathcal{R} \sim \lambda$.

Field point (t,x)

$\mathcal{D}$

$\mathcal{C}$

$\mathcal{N}$

$\mathcal{C} - \mathcal{N}$

Matter source

FAR ZONE SOLUTION: NEAR ZONE INTEGRAL

$$r > \mathcal{R}$$

$$h_{\mathcal{N}}^{\alpha\beta}(t, \mathbf{x}) = 4 \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{\partial^m}{\partial x^{k_1} \dots \partial x^{k_m}} \left(\frac{1}{r} M^{\alpha\beta k_1 \dots k_m} \right)$$

$$M^{\alpha\beta k_1 k_2 \dots k_m} \equiv \int_{\mathcal{M}} \tau^{\alpha\beta}(t - r, x') x^{k_1} \dots x^{k_m} d^3 x'$$

Radiative Fields: The Epstein-Wagoner Decomposition

Harmonic Identities ($\tau^{\alpha\beta}{}_{,\beta} = 0$):

$$\tau^{ij} = \frac{1}{2}(\tau^{00} x^i x^j)_{,00} + 2(\tau^{k(i} x^{j)})_{,k} - \frac{1}{2}(\tau^{kl} x^i x^j)_{,kl}$$

$$\tau^{ij} x^k = \frac{1}{2}(2\tau^{0(i} x^{j)} x^k - \tau^{0k} x^i x^j)_{,0} + (\tau^{l(i} x^{j)} x^k - \frac{1}{2}\tau^{lk} x^i x^j)_{,l}$$

Multipole Expansion of Radiative Field:

$$h_{\mathcal{N}}^{ij} = \frac{2}{R} \frac{d^2}{dt^2} \sum_{m=0}^{\infty} \hat{N}_{k_1} \dots \hat{N}_{k_m} I_{EW}^{ij k_1 \dots k_m}(u)$$

Epstein-Wagoner Moments:

$$I_{EW}^{ij} = \int_{\mathcal{M}} \tau^{00} x^i x^j d^3 x + I_{SURF}^{ij}$$

$$I_{EW}^{ijk} = \int_{\mathcal{M}} (2\tau^{0(i} x^{j)} x^k - \tau^{0k} x^i x^j) d^3 x + I_{SURF}^{ijk}$$

$$I_{EW}^{ijkl} = \int_{\mathcal{M}} \tau^{ij} x^k x^l d^3 x$$

$$I_{EW}^{ij k_1 \dots k_m} = \frac{2}{m!} \frac{d^{m-2}}{dt^{m-2}} \int_{\mathcal{M}} \tau^{ij} x^{k_1} \dots x^{k_m} d^3 x$$
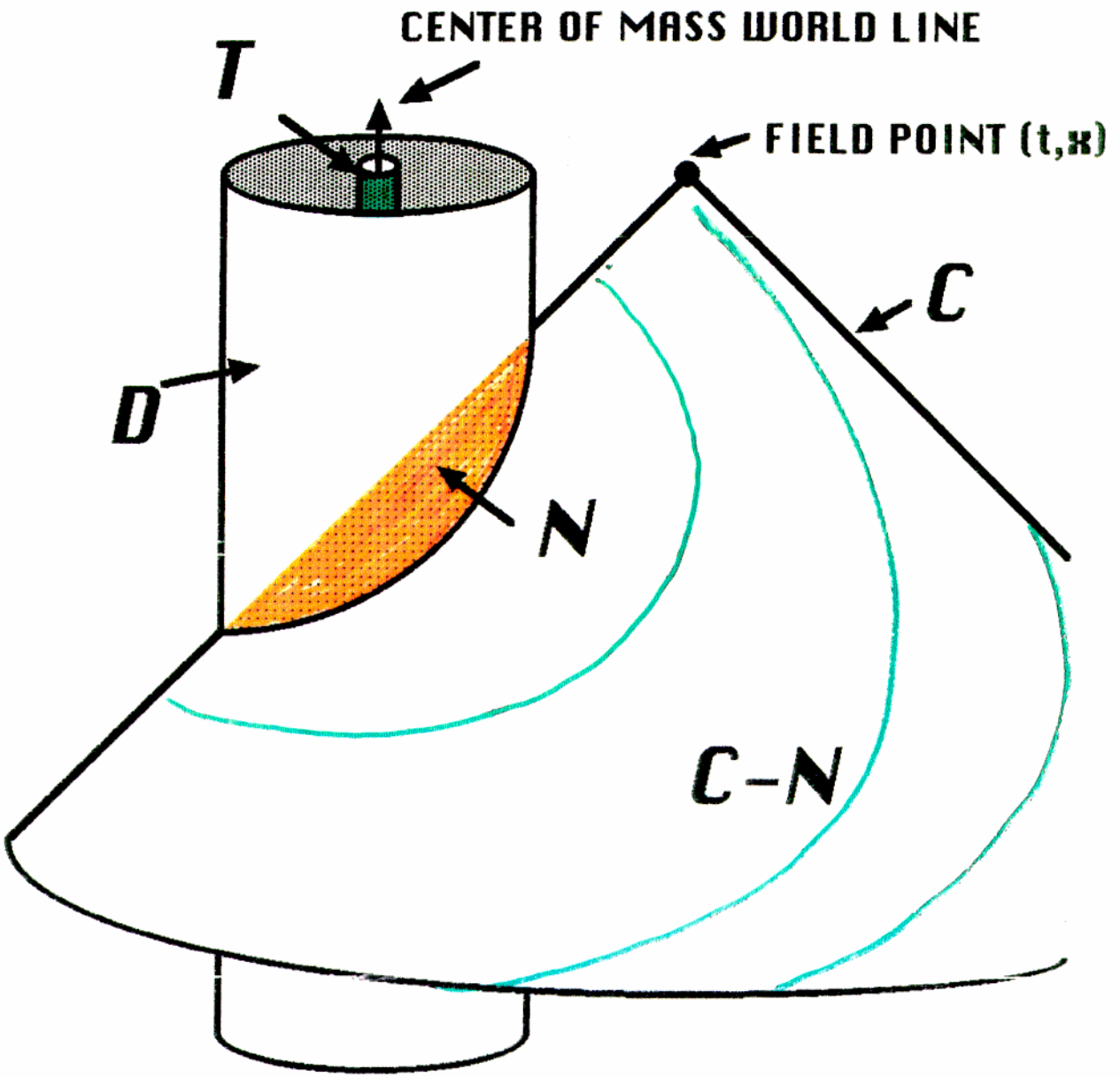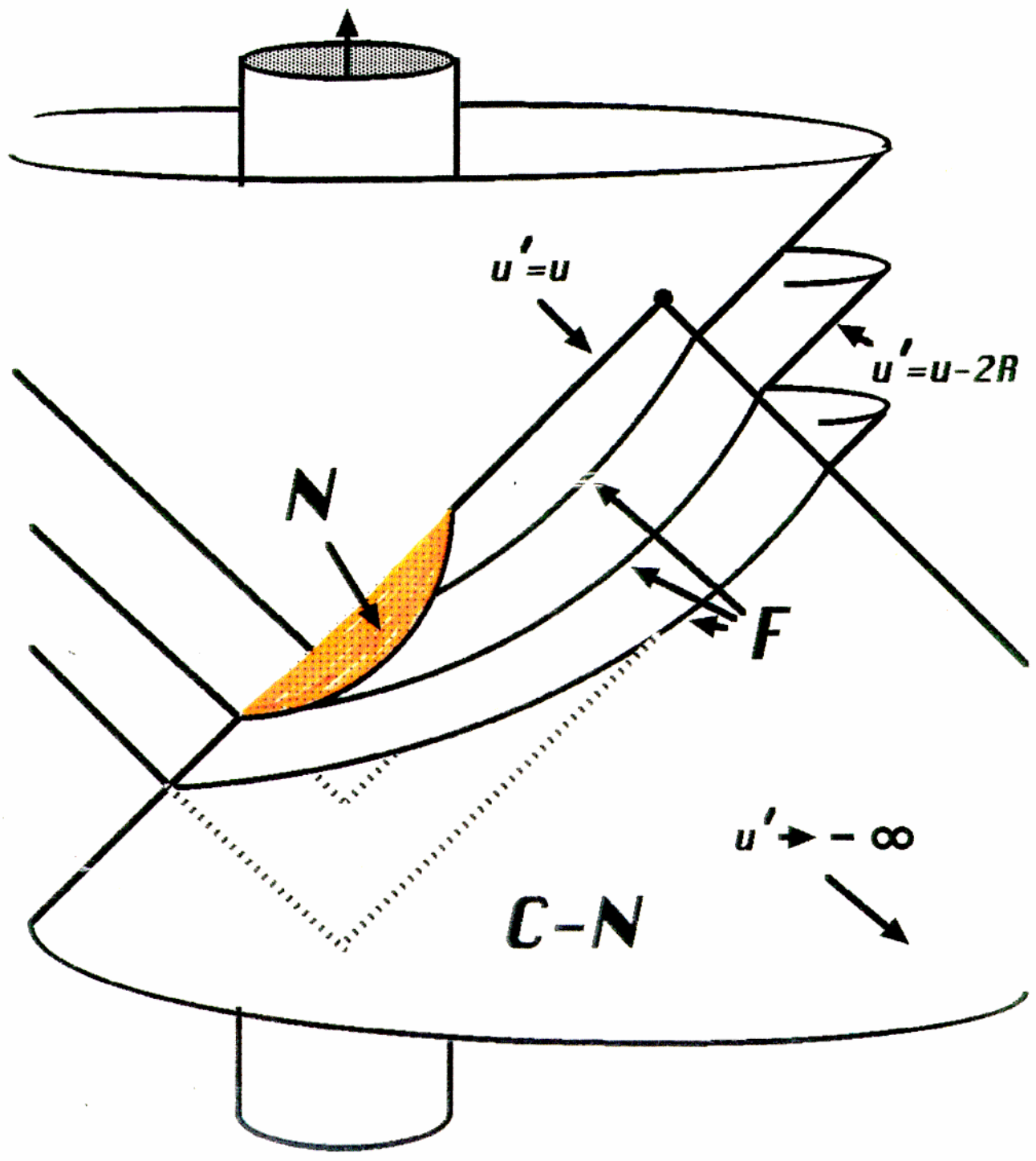
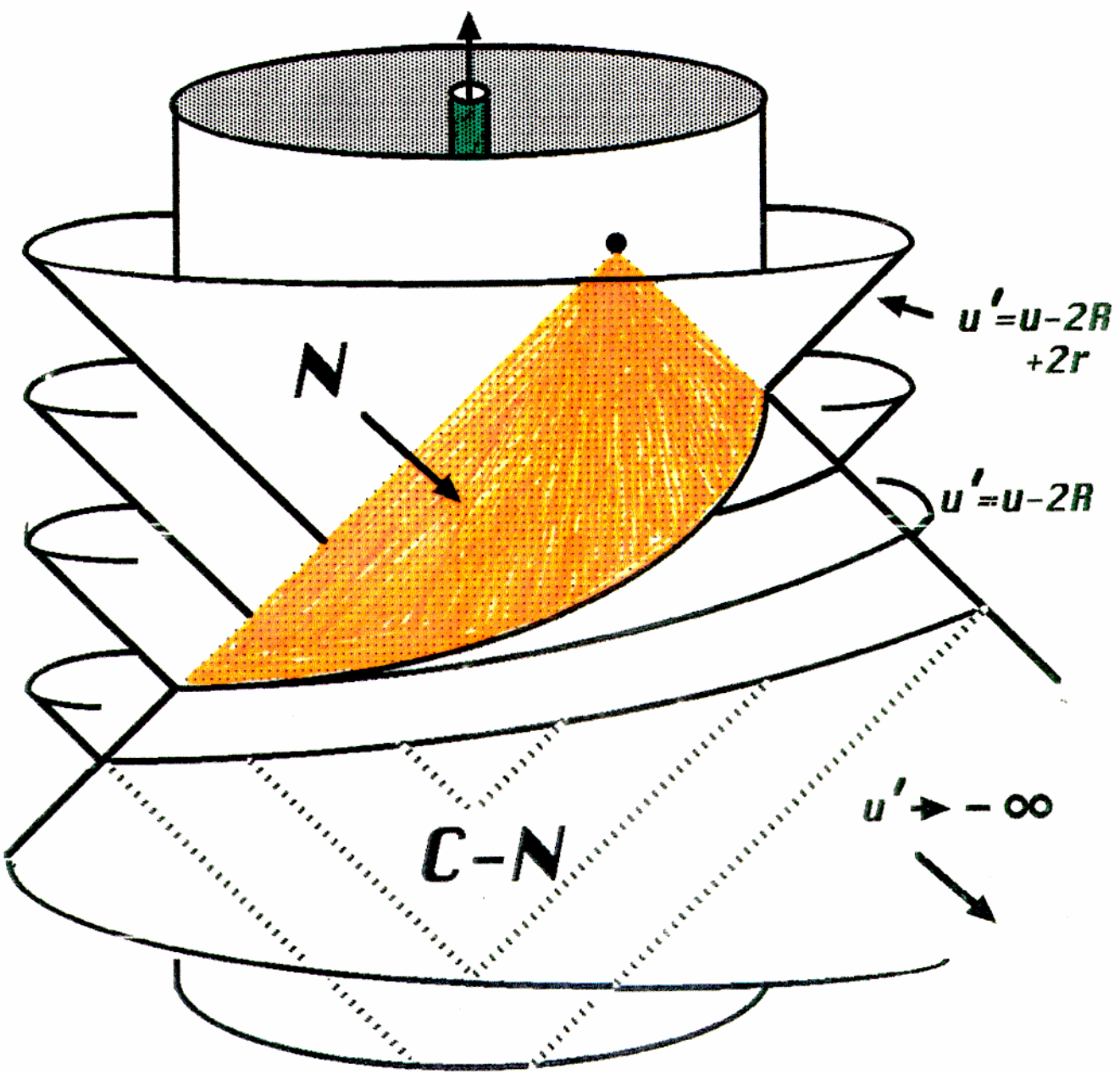# NEAR ZONE SOLUTION: NEAR ZONE INTEGRAL

$$r < \mathcal{R}$$

$$h_{\mathcal{N}}^{\alpha\beta}(t,\mathbf{x}) = 4 \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{\partial^m}{\partial t^m} \int_{\mathcal{M}} \tau^{\alpha\beta}(t,x')|\mathbf{x} - \mathbf{x}'|^{m-1} d^3 x'$$

Example to 3.5 PN order

$$h_{\mathcal{N}}^{00} = 4 \int_{\mathcal{M}} \frac{\tau^{00}(t,x')}{|\mathbf{x} - \mathbf{x}'|} d^3 x' - 4\frac{\partial}{\partial t} \int_{\mathcal{M}} \tau^{00}(t,x') d^3 x'$$

$$+ 2\frac{\partial^2}{\partial t^2} \int_{\mathcal{M}} \tau^{00}(t,x')|\mathbf{x} - \mathbf{x}'| d^3 x' - \frac{2}{3}\frac{\partial^3}{\partial t^3} \int_{\mathcal{M}} \tau^{00}(t,x')|\mathbf{x} - \mathbf{x}'|^2 d^3 x'$$

$$+ \frac{1}{6}\frac{\partial^4}{\partial t^4} \int_{\mathcal{M}} \tau^{00}(t,x')|\mathbf{x} - \mathbf{x}'|^3 d^3 x' - \frac{1}{30}\frac{\partial^5}{\partial t^5} \int_{\mathcal{M}} \tau^{00}(t,x')|\mathbf{x} - \mathbf{x}'|^4 d^3 x'$$

$$+ \frac{1}{180}\frac{\partial^6}{\partial t^6} \int_{\mathcal{M}} \tau^{00}(t,x')|\mathbf{x} - \mathbf{x}'|^5 d^3 x'$$

$$- \frac{1}{1260}\frac{\partial^7}{\partial t^7} \int_{\mathcal{M}} \tau^{00}(t,x')|\mathbf{x} - \mathbf{x}'|^6 d^3 x' + O(\epsilon^5)$$

CENTER OF MASS WORLD LINE

T

FIELD POINT (t,**x**)

D

C

N

C–N

$u'=u$

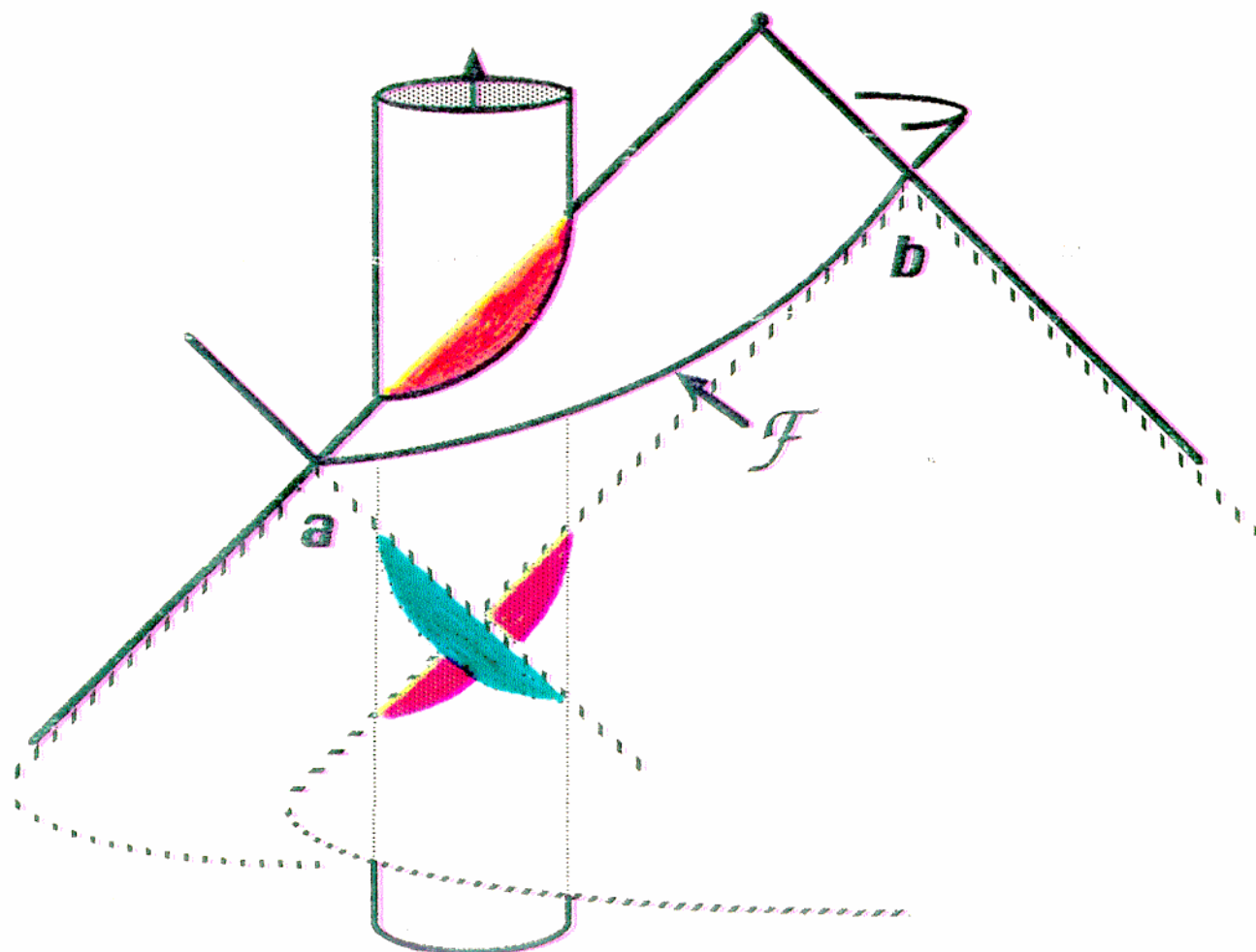$u'=u-2R$

N

F

$u' \to -\infty$

C-N

THE OUTER LIGHT-CONE INTEGRAL

Change Variables:

$$u' \equiv t' - r' \quad \text{or} \quad t - u' = r' + |\mathbf{x} - \mathbf{x}'|$$

$$r'(u', \Omega') = \frac{r^2 - (t - u')^2}{2(\mathbf{n}' \cdot \mathbf{x} - t + u')}$$

$$h^{\alpha\beta}_{\mathcal{C}-\mathcal{N}}(t, x) = 4 \int_{-\infty}^{u} du' \int \frac{\tau^{\alpha\beta}(u' + r', \mathbf{x}')}{t - u' - \mathbf{n}' \cdot \mathbf{x}} [r'(u', \Omega')]^2 d^2\Omega'$$

RESULTS OF DIRE

Far Zone Field to 2PN Order (Wiseman & Will, 96)

$$h_{\mathcal{N}}^{ij}(t, \mathbf{x}) = \mathcal{R} - \text{independent terms} + 1/\mathcal{R} \text{ terms}$$

$$-\frac{1912}{315}\frac{m}{R}\,^{(4)}Q^{ij}(u)\mathcal{R}$$

$$h_{C-\mathcal{N}}^{ij}(t, \mathbf{x}) = \frac{4m}{R}\int_0^\infty ds\,^{(4)}Q^{ij}(u-s)\left[\ln\left(\frac{s}{2R+s}\right) + \frac{11}{12}\right]$$

$$+ \frac{4m}{3R}\hat{N}^k\int_0^\infty ds\,^{(5)}Q^{ijk}(u-s)\left[\ln\left(\frac{s}{2R+s}\right) + \frac{97}{60}\right]$$

$$- \frac{16m}{3R}\epsilon^{(i|ka}\hat{N}^k\int_0^\infty ds\,^{(4)}J^{a|j)}(u-s)\left[\ln\left(\frac{s}{2R+s}\right) + \frac{7}{6}\right]$$

$$+ \frac{1912}{315}\frac{m}{R}\,^{(4)}Q^{ij}(u)\mathcal{R}$$

⋆ No $\mathcal{R}$ dependence

⋆ Propagation along "true" null cones

 o R dependence in tail integral gives circular orbit waveform phase

$$\psi = \omega\{t - R - 2m\ln R - 2m[\gamma + \ln(4\omega e^{-11/12})]\}$$

⋆ Outer integral also gives Christodoulou memory (Wiseman & Will 91)

⋆ Energy flux and TT waveform agree with BDI and GI

• BIWW 2PN waveform now incorporated in GRASP: Gravitational Radiation Analysis and Simulation Package (B. Allen), a software toolkit publicly available. See

http://www.ligo.caltech.edu/LIGO_web/Collaboration/manual.pdf

CANCELLATION OF BOUNDARY-DEPENDENT TERMS

$\star$ Through 3.5PN order for the near zone and 2PN order for the far zone,

we can decompose $\tau^{\alpha\beta}$ outside the source into terms of the form

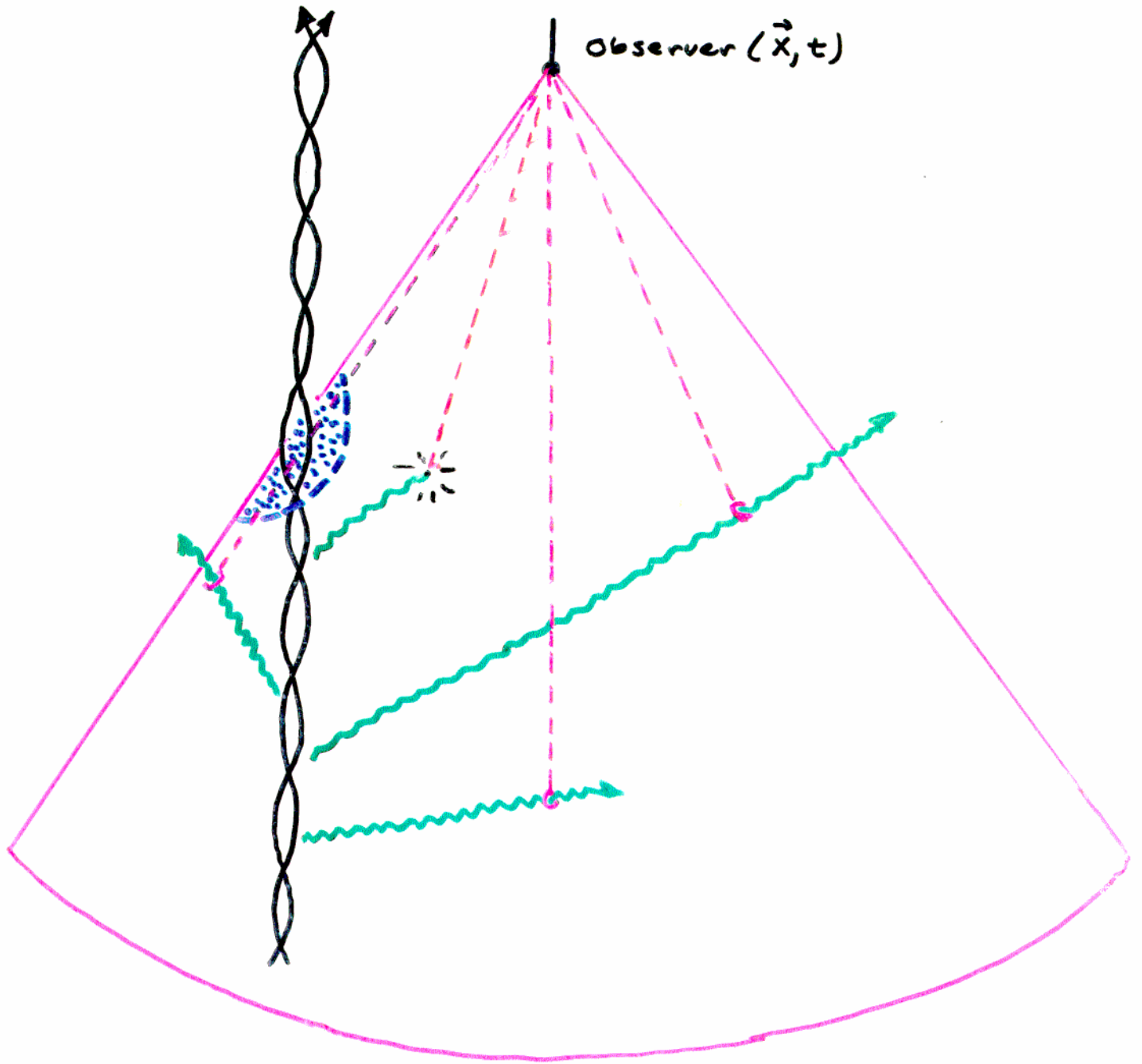$$\tau^{\alpha\beta} \sim \frac{\hat{n}^{<L>}}{r^N} f(u)$$

$\star$ For each (N, L):

$$\int_{\mathcal{M}}^{\mathcal{R}} \frac{\tau^{\alpha\beta}(t - |\mathbf{x} - \mathbf{x}'|, x')}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \to a_p(N, L)\mathcal{R}^p + b(N, L)ln\mathcal{R}$$

$$+ \mathcal{R}- \text{ independent terms}$$

$\star$ Using Maple, for each (N, L):

$$\int_{\mathcal{C-N}} \frac{\tau^{\alpha\beta}(t - |\mathbf{x} - \mathbf{x}'|, x')}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \to - a_p(N, L)\mathcal{R}^p - b(N, L)ln\mathcal{R}$$

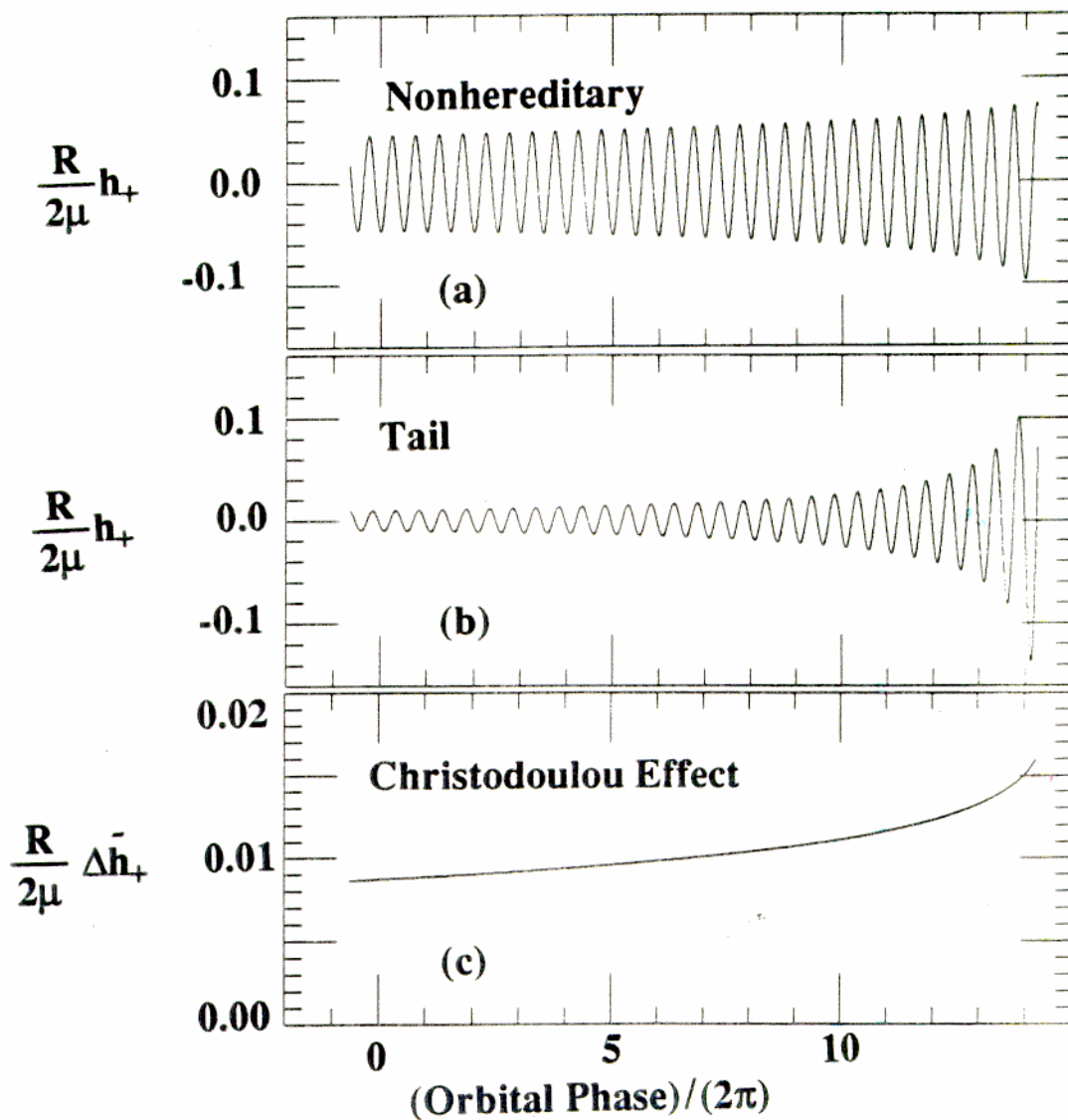$$+ \mathcal{R}- \text{ independent terms}$$

$\star$ Cancellation valid for field points in far zone **and** in near zone

$\star$ At $> 2PN$ order in the far zone, $\ln r$ terms arise (tails of tails) in $\tau^{\alpha\beta}$.

These must be treated separately.

observer $(\vec{x}, t)$

Hereditary Contributions to Waveform

$\Theta = 90°$ $m_1 = m_2$

$\frac{R}{2\mu} h_+$ 0.1   Nonhereditary

0.0

-0.1   (a)

$\frac{R}{2\mu} h_+$ 0.1   Tail

0.0

-0.1   (b)

$\frac{R}{2\mu} \Delta\bar{h}_+$ 0.02   Christodoulou Effect

0.01

(c)

0.00

0   5   10

(Orbital Phase)/(2π)

THE STF MOMENTS TO 2PN ORDER

$$\mathcal{I}_{STF}^{ij} = \mu r^2 \Big\{ \hat{n}^i \hat{n}^j + \frac{1}{42} \Big[\hat{n}^i \hat{n}^j \left(29(1-3\eta)v^2 - 6(5-8\eta)\frac{m}{r} \right)$$

$$-24(1-3\eta)\dot{r}\hat{n}^{(i}v^{j)} + 22(1-3\eta)v^i v^j \Big]$$

$$+\frac{1}{1512}\hat{n}^i \hat{n}^j \Big[3(253 - 1835\eta + 3545\eta^2)v^4 - 6(355 + 1906\eta - 337\eta^2)\left(\frac{m}{r}\right)^2$$

$$+2(2021 - 5947\eta - 4883\eta^2)\frac{m}{r}v^2 - 2(131 - 907\eta + 1273\eta^2)\frac{m}{r}\dot{r}^2 \Big]$$

$$+\frac{1}{378}v^i v^j \Big[2(742 - 335\eta - 985\eta^2)\frac{m}{r}$$

$$+3(41 - 337\eta + 733\eta^2)v^2 + 30(1 - 5\eta + 5\eta^2)\dot{r}^2 \Big]$$

$$-\frac{1}{378}\hat{n}^{(i}v^{j)}\dot{r}\Big[(1085 - 4057\eta - 1463\eta^2)\frac{m}{r} + 12(13 - 101\eta + 209\eta^2)v^2 \Big] \Big\}_{STF}$$

$$+\mathcal{I}_{TAIL}^{ij},$$

$$\mathcal{I}_{STF}^{ijk} = -\mu\frac{\delta m}{m}r^3 \Big\{ \hat{n}^i \hat{n}^j \hat{n}^k \Big[1 + \frac{1}{6}(5 - 19\eta)v^2 - \frac{1}{6}(5 - 13\eta)\frac{m}{r} \Big]$$

$$+(1-2\eta)(v^i v^j \hat{n}^k - \dot{r}v^i \hat{n}^j \hat{n}^k) \Big\}_{STF} + \mathcal{I}_{TAIL}^{ijk},$$

$$\mathcal{I}_{STF}^{ijkl} = \mu r^4 \Big\{ \hat{n}^i \hat{n}^j \hat{n}^k \hat{n}^l \Big[(1-3\eta)$$

$$+\frac{1}{110}(103 - 735\eta + 1395\eta^2)v^2 - \frac{1}{11}(10 - 61\eta + 105\eta^2)\frac{m}{r} \Big]$$

$$+\frac{6}{55}(1 - 5\eta + 5\eta^2)(13v^i v^j \hat{n}^k \hat{n}^l - 12\dot{r}v^i \hat{n}^j \hat{n}^k \hat{n}^l) \Big\}_{STF},$$

$$\mathcal{I}_{STF}^{ijklm} = -\mu\frac{\delta m}{m}r^5 \Big\{ (1 - 2\eta)\hat{n}^i \hat{n}^j \hat{n}^k \hat{n}^l \hat{n}^m \Big\}_{STF},$$

$$\mathcal{I}_{STF}^{ijklmn} = \mu r^6 \Big\{ (1 - 5\eta + 5\eta^2)\hat{n}^i \hat{n}^j \hat{n}^k \hat{n}^l \hat{n}^m \hat{n}^n \Big\}_{STF},$$

$$\mathcal{J}_{STF}^{ij} = -\mu\frac{\delta m}{m}r \Big\{ (\mathbf{x} \times \mathbf{v})^i \Big[\hat{n}^j \left(1 + \frac{1}{28}(13 - 68\eta)v^2 + \frac{3}{14}(9 + 10\eta)\frac{m}{r} \right)$$

$$+\frac{5}{28}(1 - 2\eta)\dot{r}v^j \Big] \Big\}_{STF} + \mathcal{J}_{TAIL}^{ij},$$

$$\mathcal{J}_{STF}^{ijk} = \mu r^2 \Big\{ (\mathbf{x} \times \mathbf{v})^i \Big[\hat{n}^j \hat{n}^k \Big(1 - 3\eta$$

$$+\frac{1}{90}(41 - 385\eta + 925\eta^2)v^2 + \frac{2}{9}(7 - 8\eta - 43\eta^2)\frac{m}{r} \Big)$$

$$+\frac{1}{45}(1 - 5\eta + 5\eta^2)(10\dot{r}v^j \hat{n}^k + 7v^j v^k) \Big] \Big\}_{STF},$$

$$\mathcal{J}_{STF}^{ijkl} = -\mu\frac{\delta m}{m}r^3 (1 - 2\eta)\Big\{ (\mathbf{x} \times \mathbf{v})^i \hat{n}^j \hat{n}^k \hat{n}^l \Big\}_{STF},$$

$$\mathcal{J}_{STF}^{ijklm} = \mu r^4 (1 - 5\eta + 5\eta^2)\Big\{ (\mathbf{x} \times \mathbf{v})^i \hat{n}^j \hat{n}^k \hat{n}^l \hat{n}^m \Big\}_{STF},$$

1

EVOLUTION OF GRAVITY-WAVE FREQUENCY

$$\frac{df}{dt} = \frac{96}{5\pi\mathcal{M}^2}(\pi\mathcal{M}f)^{11/3}\left[1\right.$$

ϵ

$$-\left(\frac{753}{336} + \frac{11}{4}\eta\right)(\pi M f)^{2/3} \qquad (\textit{Wagoner \& Will '76})$$

$\epsilon^{3/2}$

$$+ (4\pi - \beta)(\pi M f) \qquad \begin{array}{l}(\textit{Poisson '93, Wiseman '93} \\ \textit{Blanchet \& Schafer '93})\end{array}$$

ϵ^2

$$+ \left(\frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 + \sigma\right)(\pi M f)^{4/3} \qquad \begin{array}{l}(\textit{Blanchet,} \\ \textit{Damour, Iyer} \\ \textit{Will} \\ \textit{Wiseman '95})\end{array}$$

$\epsilon^{5/2}$

$$\left. + O[(\pi M f)^{5/3}]\right] \qquad (\textit{Blanchet '96})$$

$$M \equiv m_1 + m_2 \quad , \qquad \eta \equiv m_1 m_2/M^2$$

$$\mathcal{M} \equiv \eta^{3/5} M$$

$$\beta \equiv \frac{1}{12}\sum_{i=1}^{2}(113\frac{m_i^2}{m^2} + 75\eta)\hat{\mathbf{L}}\cdot\chi_i \qquad \begin{array}{l}(\textit{Kidder,} \\ \textit{Wiseman \&} \\ \textit{Will '93})\end{array}$$
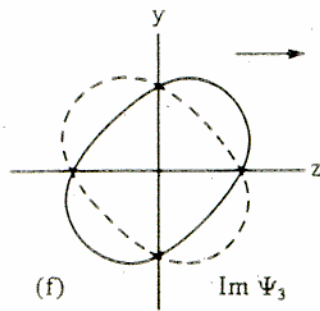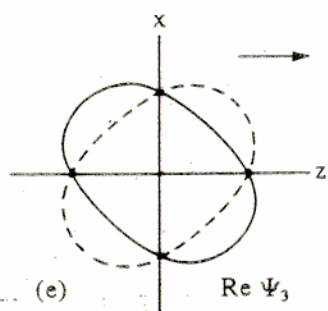
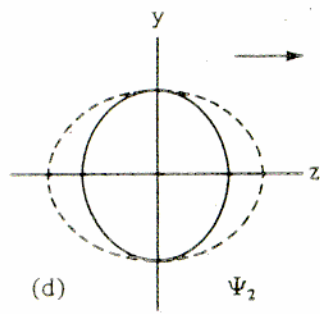$$\sigma \equiv -\frac{\eta}{48}\left(247\chi_1\cdot\chi_2 - 721\hat{\mathbf{L}}\cdot\chi_1\hat{\mathbf{L}}\cdot\chi_2\right)$$

• Test body BH perturbation theory $\qquad$ Sasaki, Tagoshi,

$(\eta = 0)$ to $O(\epsilon^{11/2})$ $\qquad$ Tanaka, ...

POLARIZATION OF GRAVITATIONAL WAVES

(a) Re Ψ_4

(b) Im Ψ_4

(c) Φ_{22}

(d) Ψ_2

(e) Re Ψ_3

(f) Im Ψ_3
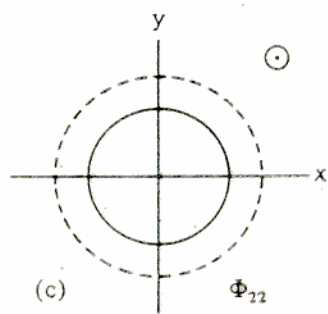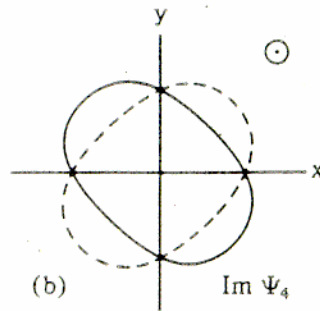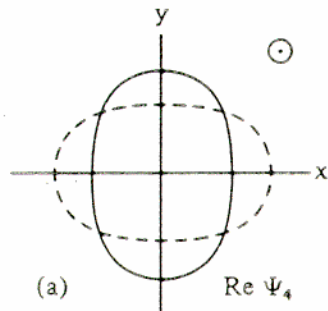
DIRECTION KNOWN?	# "INDEPENDENT" DETECTORS
YES	6
NO	8

GRAVITATIONAL RADIATION REACTION

- Gravitational radiation back-reaction controls inspiral
- Accurate measurement of phasing of waves can test GR and other theories

EXAMPLES

- **Testing Scalar-Tensor Gravity**
  - ⋆ Dipole gravitational radiation can change damping significantly
  - ⋆ Difference in $s \sim E_G/M$ between bodies crucial ($s_{BH} = 0.5$, $s_{NS} \approx 0.2$)
  - ⋆ Strong bounds on ω possible *(Will, 1994, 1997)*

- **Testing GR Non-Linearities**
  - ⋆ For strong enough signal, can test GR Tail term *(Blanchet & Sathyaprakash 1995)*
  - ⋆ For 2 BH @ 10 $M_\odot$, need $S/N \sim 50$

- **Testing BH Exterior Spacetime**
  - ⋆ Inspiral into massive black hole could map external spacetime, via waveform & phasing
  - ⋆ Test no-hair theorems; look for exotic supermassive objects
  - ⋆ LISA *(Ryan, Thorne, Finn, Poisson, ...)*

SPEED OF GRAVITATIONAL WAVES

WHY SPEED COULD BE DIFFERENT FROM LIGHT SPEED

- Massive graviton: $v_g^2 = 1 - \left(\frac{m_g}{E_g}\right)^2$
- Gravity coupling to "background" fields: $v_g = F(\phi, \vec{K}, \mathbf{H}, \ldots)$

EXAMPLES

- *General Relativity.* For $\lambda \ll \mathcal{R}$, GW move on null geodesics of background spacetime, as do photons. $v_g \equiv 1$.
- *Scalar Tensor Gravity.* Tensor waves have have $v_g \equiv 1$, even if scalar is massive.
- *Theories with background flat metric.* *Eg.* Rosen's Bimetric Theory (1970s). GW follow null cones of η, light follows cones of $\mathbf{g}$.

POSSIBLE LIMITS

$$1 - v_g \approx 5 \times 10^{-17} \left(\frac{200\text{Mpc}}{D}\right) [\Delta t_a - (1 + Z)\Delta t_e]$$
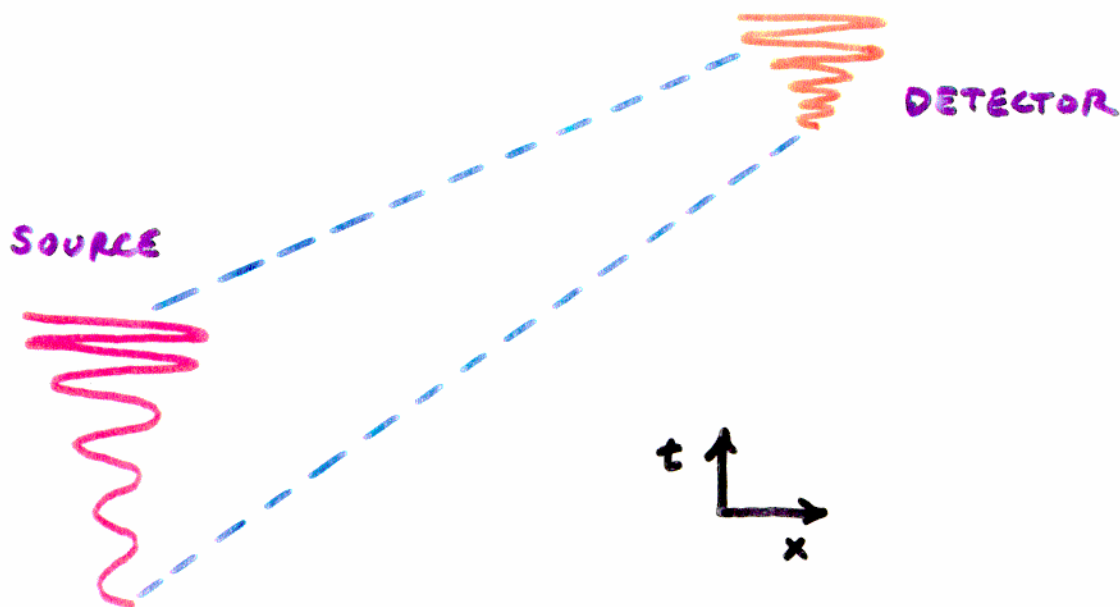
D = distance of source Z = redshift of source

$\Delta t_a \ (\Delta t_e)$ = time difference, in seconds

$$\lambda_g > 3 \times 10^{12} \text{ km} \left(\frac{D}{200\text{Mpc}}\right)^{1/2} \left(\frac{100\text{Hz}}{f}\right)^{1/2} \left(\frac{1}{f\Delta t}\right)^{1/2}$$

$$3 \times 10^{12} \text{ km} = (4 \times 10^{-22} \text{ eV})^{-1}$$

BOUNDING THE GRAVITON MASS
USING INSPIRALING COMPACT BINARIES

SOURCE

DETECTOR

t

x

m_1	m_2	Distance (Mpc)	Bound on λ_g (km)
Ground-based (LIGO/VIRGO)			
1.4	1.4	300	4.6×10^{12}
1.4	10	630	5.4×10^{12}
10	10	1500	6.0×10^{12}
Space-based (LISA)			
10^7	10^7	3000	6.9×10^{16}
10^6	10^6	3000	5.4×10^{16}
10^5	10^5	3000	2.3×10^{16}
10^4	10^4	3000	0.7×10^{16}

OTHER LIMITS ON THE GRAVITON MASS

$\lambda_g > 3 \times 10^{12}$ km   Solar system $1/r^2$ law

$\lambda_g > 6 \times 10^{19}$ km   Galaxy&Cluster Dynamics

Will (1997) gr-qc 9709011
PRD 1/98

Physics Launches New Business

Global Positioning System

A system of 24 satellites broadcasts GPS location coordinates to special receivers anywhere on Earth. GPS helped Persian Gulf troops navigate the desert at night. Automakers are developing GPS systems for on-screen road maps to guide drivers, and GPS is used to monitor earthquake zones and observe shifts in the earth's geological plates. GPS can help find people lost or stranded anywhere from mountaintops to caves. This Defense Department investment has resulted in a powerful tool for military applications and has spawned a multi-billion dollar consumer industry. And it all became possible because of the atomic clock--funded by the U.S. government in the 1940s to help scientists test Einstein's general theory of relativity. In 1995, the global GPS market was estimated at $2.3 billion, with 70,000 units produced per month.

CLIMBING MT. EVEREST

- GPS was used in 1993 on the 40th anniversary of the first climb to the peak.

- 100,000 new GPS-related jobs will be created by the year 2000.

- Position accuracy from typical commercial GPS is 15-30 feet.

NAVIGATION

- America's Cup sailor Gary Jobson and crew navigate the Straits of Magellan using GPS.

- Military applications include gathering intelligence, landing missions, and alignment of antennae and radar.

- GPS is used in some rental cars to help tourists in unfamiliar cities.

GPS SATELLITE
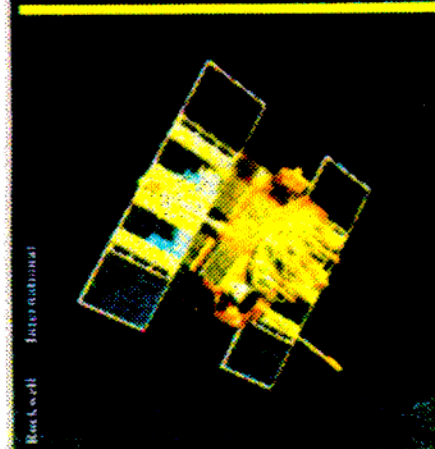
- Each satellite is equipped with 4 atomic clocks.

- The accuracy of atomic clocks has increased 1000% every decade since 1950.

- The 24 satellites circle the earth twice a day, sending precisely timed signals to ground-based receivers.

GPS Research

- Initial research: 1930s-40s Office of Naval Research, AEC (now DOE)
- Applied research: 1950s-90s DOD, NIST, Navy, Air Force

Another in a Series of Physics Success Stories from the

AMERICAN INSTITUTE OF PHYSICS
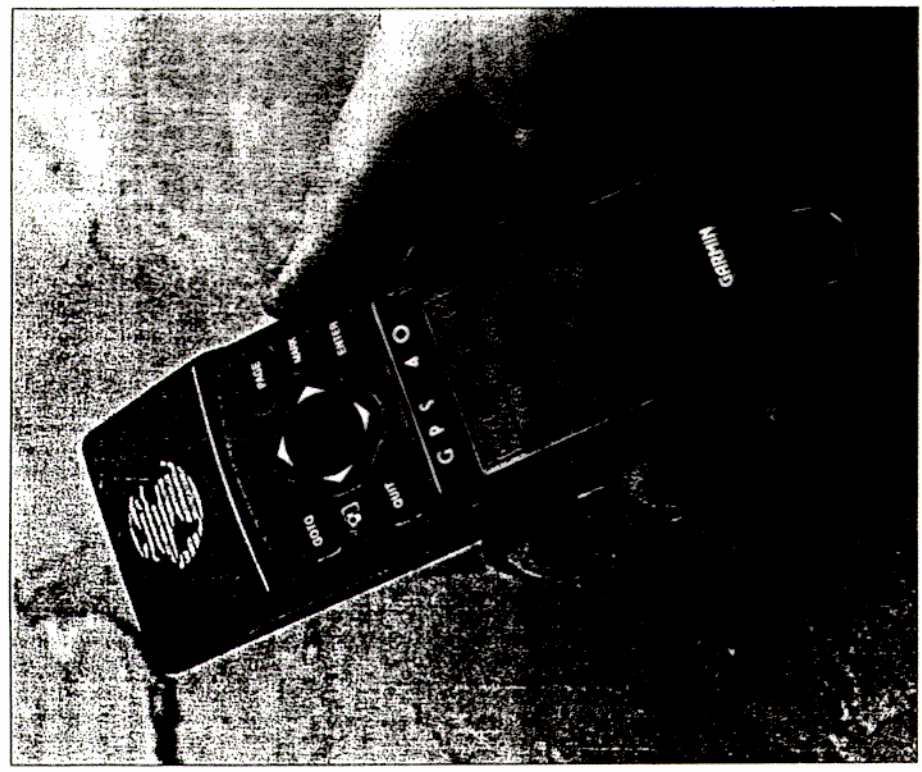
New! Garmin GPS 40

If you're looking for a compact GPS receiver, you won't find one any smaller than Garmin's GPS 40. It's as small as a television remote control and weighs just under 10 oz., providing you with a truly pocket-size GPS receiver! But, don't let its size fool you. It's still powerful enough to perform many of the basic GPS functions of larger, more expensive GPS receivers. For example, it will inform you of your present latitude, longitude, and altitude coordinates; guide you to your pre-defined destination, displaying your distance, bearing, and estimated time enroute; and plot your location from nearby way-points (user-defined) with easy-to-interpret distance rings*. These unique rings display areas to scale from 1/5 mile to 320 miles, allowing you to zoom in, zoom out, and pan around your location.

To ensure easy operation, the GPS 40 is ergonomically designed to fit in one hand and features a specially arranged keypad that can be easily operated with your thumb. Special function keys allow you to mark up to 30 waypoints in a route, scroll to different navigational displays, and backlight the LCD screen during poorly lit conditions or after dark. Best of all, a specially developed training video is included to walk you through every step of using the GPS 40. Other standard accessories include a carrying case, owner's manual, quick-reference card, and lanyard.

Tracking: up to 8 GPS satellites. **Accuracy:** 15 meters**. **Waypoints (max.):** 250. **Routes (max.):** 20 containing up to 30 waypoints each. **Display:** backlit LCD. **Operating temperature:** 5°F to 158°F (-15°C to 70°C). **Housing:** plastic with waterproof seals. **Power:** 4 "AA" batteries. **Battery life:** 10 hours normal mode, 20 hours battery saving mode. **Weight:** 10 oz. **Dimensions (approx.):** 6"L x 2"W x 1-1/4"H. 37891 (1 lb.)..$499.00
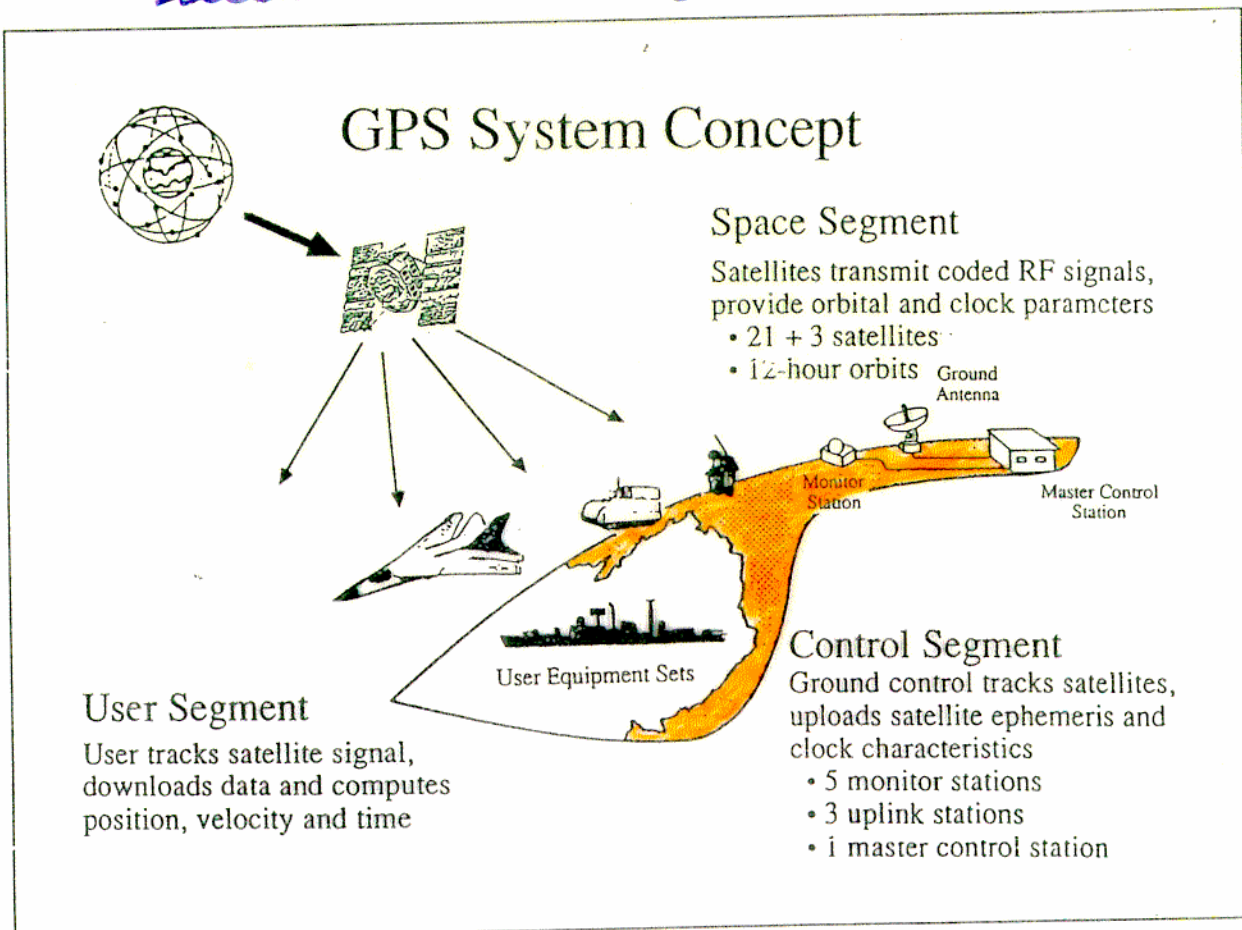
*Distance rings are displayed in 12 scales ranging from 1/5 mile to 320 miles.

**Accuracy subject to U.S. Department of Defense 100-meter Selective Availability policy.

GENERAL RELATIVITY, NATIONAL DEFENSE, AND EVERYDAY LIFE

The Global Positioning System (GPS)

GPS System Concept

Space Segment
Satellites transmit coded RF signals, provide orbital and clock parameters
- 21 + 3 satellites
- 12-hour orbits

Ground Antenna

Monitor Station

Master Control Station

Control Segment
Ground control tracks satellites, uploads satellite ephemeris and clock characteristics
- 5 monitor stations
- 3 uplink stations
- 1 master control station

User Equipment Sets

User Segment
User tracks satellite signal, downloads data and computes position, velocity and time

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Accuracy Requirement : 15 m ⇒ 50 ns

Relativity Effects : 40,000 ns per day