TESTING POST-NEWTONIAN GRAVITY

WUGRAV

WASHINGTON UNIVERSITY GRAVITATION GROUP

- The Parametrized Post-Newtonian Formalism
  - GR and Scalar-Tensor Gravity
- Measurement of PPN parameters
  - Light Deflection and Time Delay
  - Mercury's Perihelion Advance
  - Tests of the Strong Equivalence Principle
  - Is G constant?
  - Is Momentum Conserved?
- Bounds on the PPN Parameters
  - The Rise and Fall (and Rise) of Scalar-Tensor Gravity
- The Search for Gravitomagnetism
  - Gravitomagnetism, Frame-Dragging and Mach's Principle
  - Gravity Probe-B
  - LAGEOS Satellite Tracking
- Future Tests

Energy-Momentum Conservation: Three viewpoints

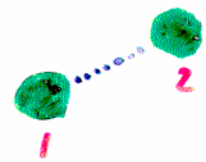
I. GENERAL

Conservation of E + P $\xLeftarrow{\text{Noether's Theorem}}$ Invariant Action $I = \int \mathcal{L}\, d^4x$

II. NEWTONIAN

$$\underset{\sim}{a} = \underset{\sim}{F}/m_i \qquad \underset{\sim}{F} = m_p \underset{\sim}{g} \qquad \underset{\sim}{g} = \underset{\sim}{\nabla} \frac{G m_a}{r}$$

$$\frac{d\underset{=}{P}}{dt} = F_1 + F_2 = 0$$



↑ Action = Reaction

$$\boxed{m_a = m_p}$$

III. POST-NEWTONIAN

$$T^{\mu\nu}{}_{;\nu} = 0$$

$$\Downarrow$$

$$\tau^{\mu\nu}{}_{,\nu} = 0$$

$$\Downarrow$$

$$P^{\mu} = \int \tau^{\mu 0} d^3x = \text{const}$$

$\Longleftrightarrow$

PPN Parameters

$$\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$$

(Will 1971)

Does Action Equal Reaction?

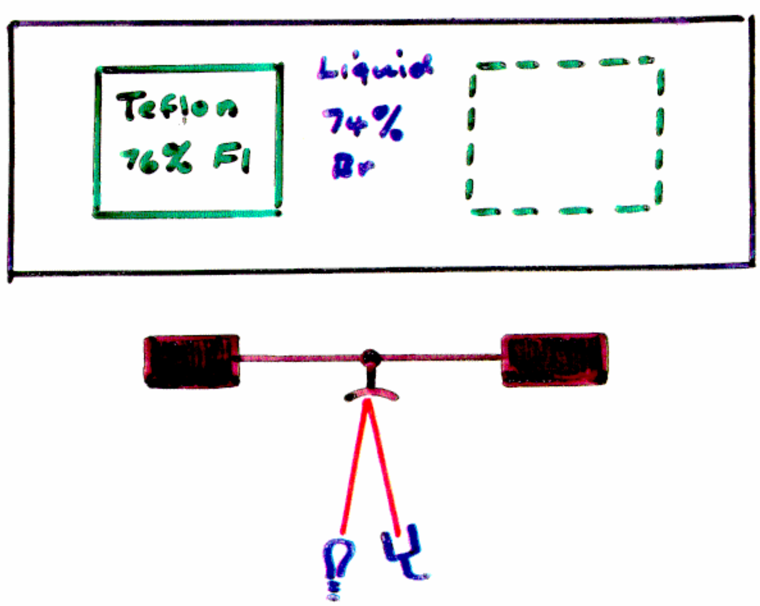
Newton's Third Law ⟷ Conservation of Momentum ⟷ Lagrangian based theories

⟷ $m_A = m_P$

Active mass : $U = G m_A / r$

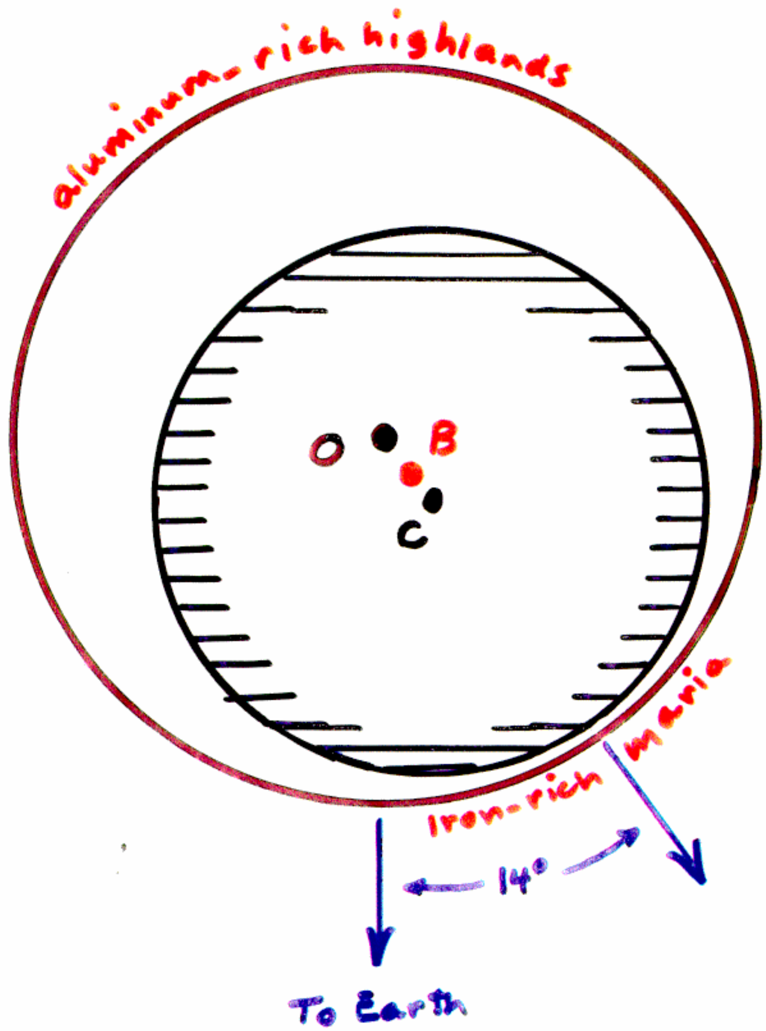
Passive mass : $\underline{F} = m_P \underline{\nabla} U$

Kreuzer's Experiment (1968)



$$\left| \frac{(m_A/m_P)_{Fl} - (m_A/m_P)_{Br}}{(m_A/m_P)_{Br}} \right| < 5 \times 10^{-5}$$

Action, Reaction and the Moon (1986) (Bartlett & van Buren)

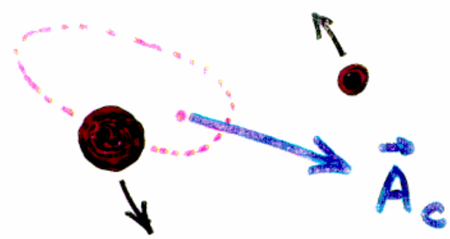


O = center of figure
B = center of mass
$OB \simeq 2$ km

$$\left| \frac{(m_A/m_P)_{Fe} - (m_A/m_P)_{Al}}{(m_A/m_P)_{Al}} \right| < 4 \times 10^{-12}$$

$$\boxed{|\zeta_3| < 10^{-8}}$$

Is Momentum Conserved?



$$\boxed{\begin{array}{c}\text{Conserved } E, \vec{P} \\ \Updownarrow \\ \text{Invariant Action}\end{array}}$$

$$\underset{\sim}{A}_c = \zeta_2 \left\langle A_N\left(\frac{Gm}{r_c^2}\right)\right\rangle \frac{\mu}{m}\,\frac{m_1-m_2}{m}\, e\, \hat{\underset{\sim}{n}}_p$$

↑ 0 in GR, any action-based theory

Test using the binary pulsar

$$\dot{P}/P = \vec{A}_c \cdot \vec{N}$$

$$\ddot{P}/P \simeq \dot{\vec{A}}_c \cdot \vec{N}$$

For the pulsar:

$$\ddot{P}_p = 10^{-25}\,\zeta_2 \ s^{-1}$$ (use GR values for m_1, m_2)

($\delta m/m = 3.8 \pm 0.1\,\%$)

$$|\ddot{P}_p|_{OBS} < 4\times 10^{-30} \ s^{-1}$$

$$\boxed{|\zeta_2| < 4\times 10^{-5}}$$

(C.W. 1992)

CURRENT LIMITS ON THE PPN PARAMETERS

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2×10^{-3}	Viking ranging
	light deflection	3×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	6×10^{-4}	$\eta = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging PSR J2317+1439
α_2	solar spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	2×10^{-20}	pulsar $\dot{P}$ statistics
η	Nordtvedt effect[1]	10^{-3}	Lunar laser ranging
ζ_1	–	2×10^{-2}	combined PPN bounds
ζ_2	binary self-acceleration	4×10^{-5}	$\ddot{P}$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	Lunar acceleration
ζ_4	–	–	not independent

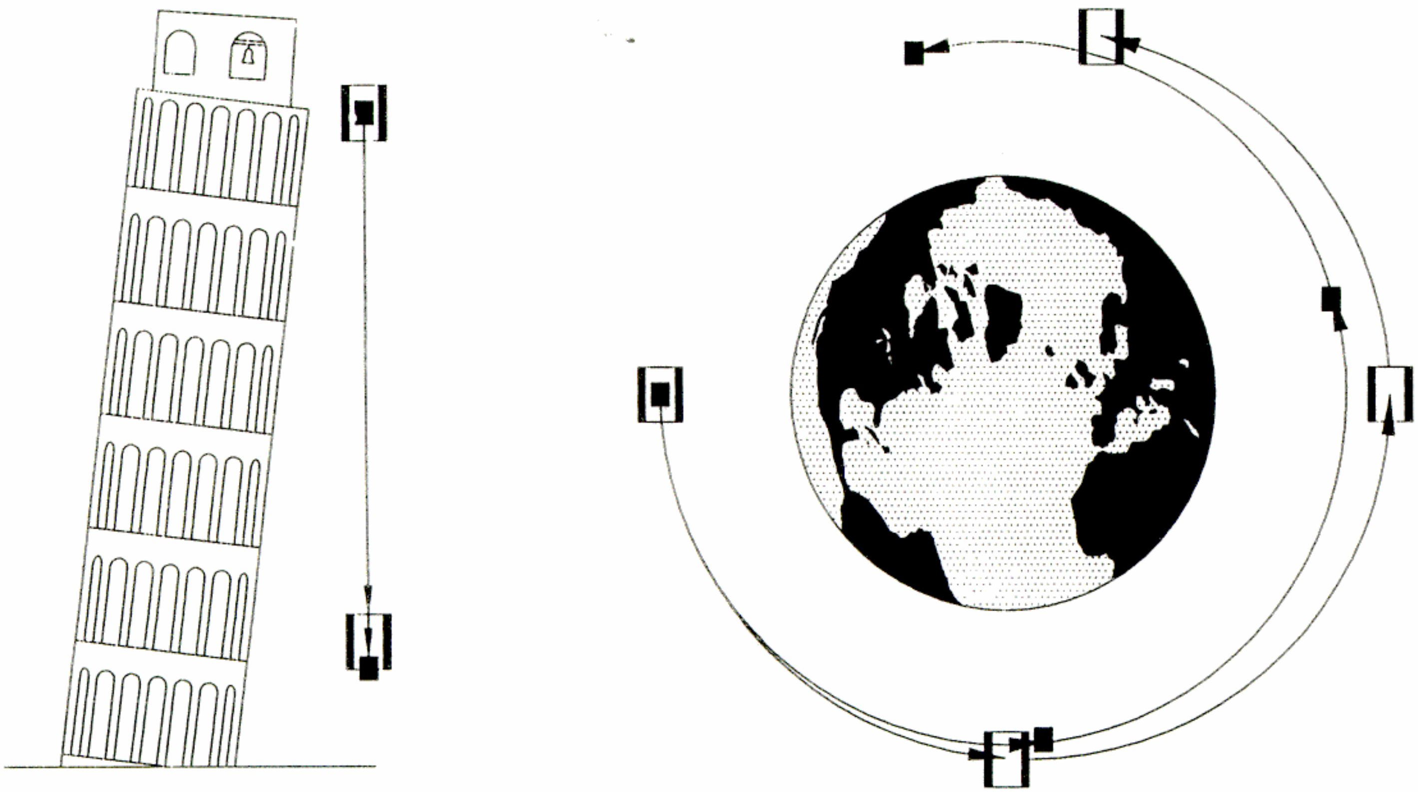
[1] Here $\eta = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$

Scalar-Tensor Gravity: $\omega > 3000, \alpha^2 < 10^{-4}$

EXPERIMENTAL GRAVITY:

LOOKING TO THE

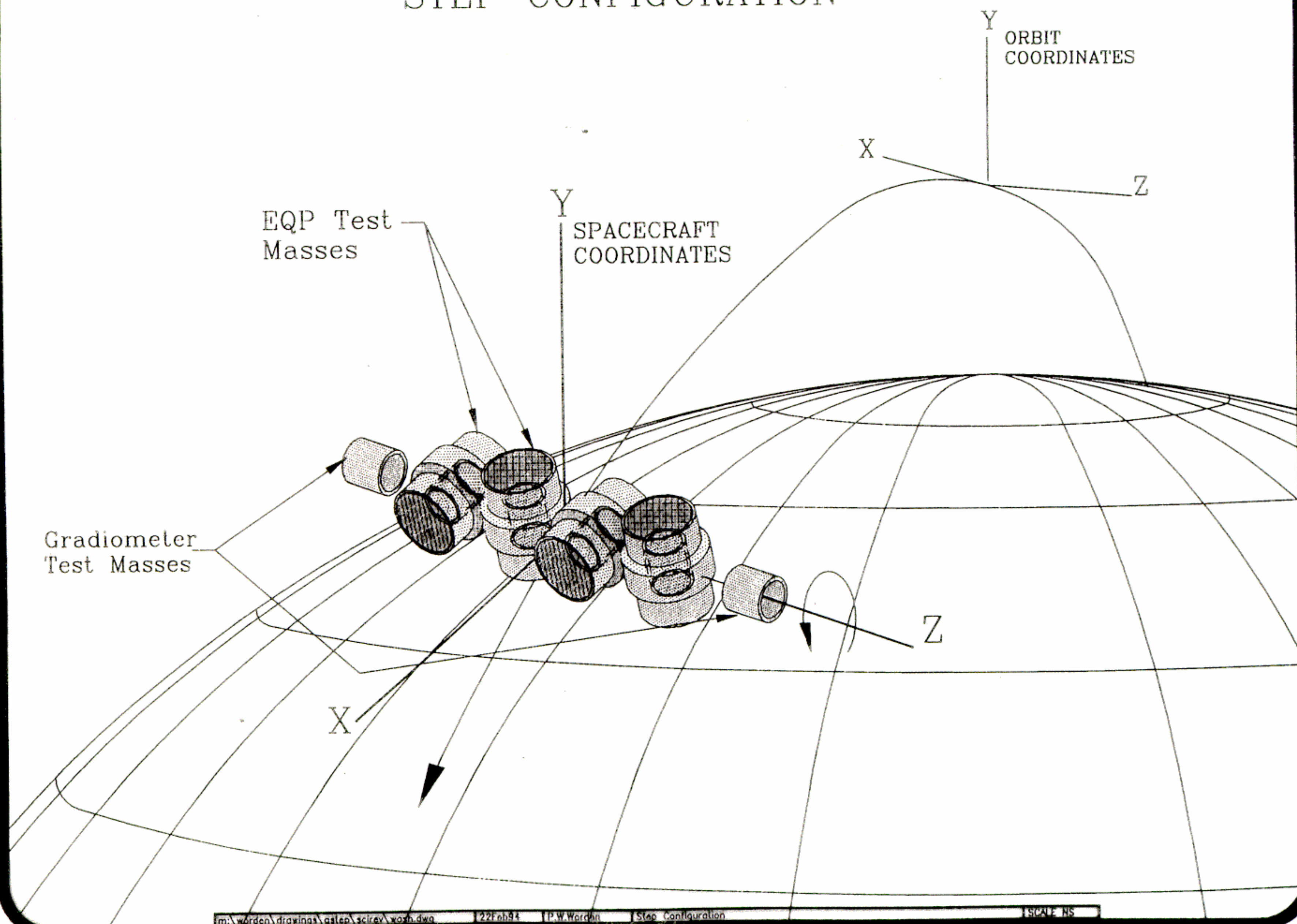
FUTURE . . . .



EFFECT OF EQUIVALENCE PRINCIPLE VIOLATION

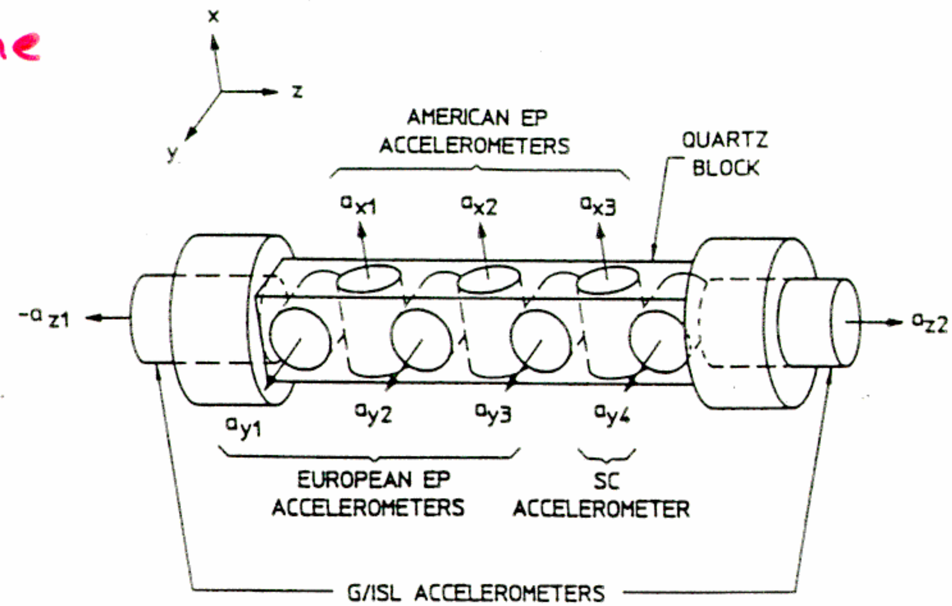
m:\worden\drawings\gstep\scirev\wash2.dwg	22Feb94	P.W.Worden	Effect of EP Violation	SCALE NS

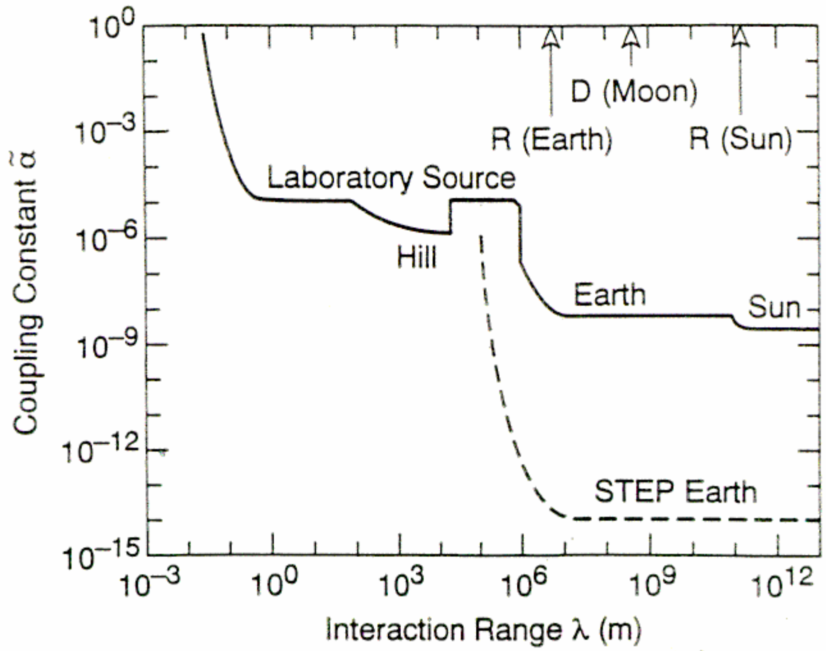
STEP CONFIGURATION

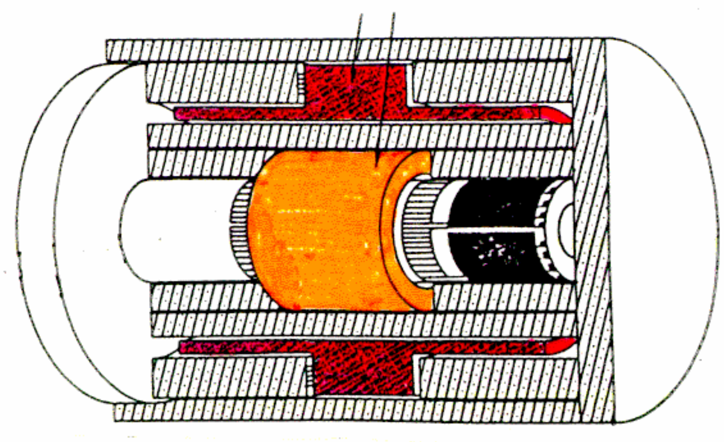


Satellite Test of the Equivalence Principle

$$\frac{\Delta a}{a} \sim 10^{-17}$$







$$G_{eff} = G_0 (1 + \alpha e^{-r/\lambda})$$

Lunar Laser Ranging

- 1-2 cm accuracy (can improve)
- $ for telescope time, equipment, data anal.

Science results

- $\Delta a/a = (-3 \pm 4) \times 10^{-13}$
  - * better than lab tests
  - * $|4\beta - \gamma - 3| < (-7 \pm 10) \times 10^{-4}$
- $\dot{G}/G < 3 \times 10^{-12}\ yr^{-1}$
  - * "grand fit" could do better
- Geodetic (de Sitter) Precession
  - * $(2\gamma+1)/3$ to 0.5%
  - * could rival deflection / time delay
- Geo- and Selenophysics results

Clocks Near the Sun

- $4 R_\odot$
- 10^{-16} stability (trapped ion, cryo H-maser)

3 Scenarios (Nordtvedt, MG8)

No drag-free No tracking 2 clocks	$(\alpha_p - \alpha_{p'}) \frac{GM}{c^2 R(t)}$	$\lvert \alpha_p - \alpha_{p'} \rvert < 10^{-9}$ or 10^{-10} from non-metric coupled scalar field
10^{-7} or 10^{-8} c/s^2 Maybe drag-free 1 & 2 way tracking 1 clock		from vector or non-metric scala $\lvert \alpha_p - 1 \rvert < 10^{-9}$ or 10^{-10} $\lvert J_2 \rvert < 10^{-8}$ $\lvert \beta - 1 \rvert < 10^{-3}$
10^{-10} c/s^2 drag-free 1 & 2 way tracking 1 clock		$\lvert \alpha_p - 1 \rvert < 10^{-10}$ $\lvert J_2 \rvert < 10^{-9}$ $\lvert \dot{T}_s \rvert < 10^{-11}$

$|\gamma - 1| < 10^{-6}$ or 10^{-7} $\quad |\beta - 1| < 10^{-6}$

$|\vec{J}| < 10^{-2}$

- MERCURY ORBITER
  - Possible ESA mission ca 2010; US proposals coming
  - Ka and X-band tracking proposed
  - Could improve $\dot{\omega}$, $\dot{G}$ and γ by factor ~ 10.
- RELATIVISTIC CELESTIAL MECHANICS
  - Global analyses of large data sets
  - Earth satellites (LAGEOS, ...), LLR, planetary tracking (Mercury, Mars ...), interplanetary S/C (magellan, galileo, cassini, ...)
  - Yukawa-like gravity effects
  - Modest improvements in PPN parameters, $\dot{G}$.
  - Interplanetary laser ranging
- OPTICAL INTERFEROMETERS IN SPACE
  - $3 - 30$ μas accuracy
  - γ to 1×10^{-5}
  - Barely sensitive to 2nd order term (Λ)
  - Ongoing development effort at NASA for stellar and galactic astronomy, search for planets, proper motions, binaries
- DEDICATED TIME DELAY MEASUREMENTS
  - Piggyback accurate ranging on other missions that go behind the sun

GRAVITATIONAL WAVES AND THE VALIDITY OF GENERAL RELATIVITY

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- The Binary Pulsar
- Gravitational Waves to High Post-Newtonian Order
  - Why?
  - DIRE: Direct Integration of the Relaxed Einstein Equations
- Gravitational-wave Tests of General Relativity
  - Polarization of Waves
  - Tests of Radiation Damping
  - Speed of Waves and a Bound on the Graviton Mass

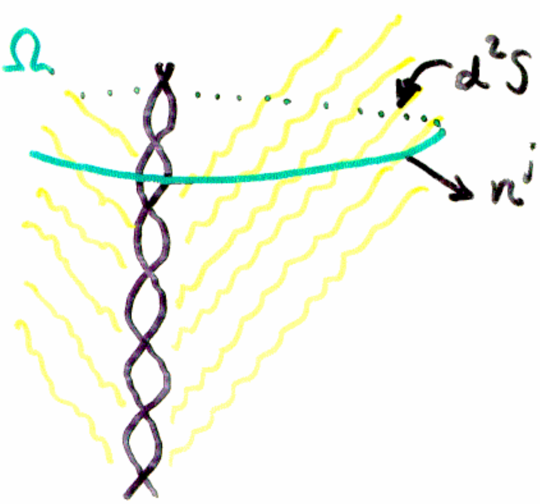
Quadrupole GW in General Relativity

$$h^{ij} = \frac{2}{R}\ddot{I}^{ij}(t-R) + \cdots$$

Energy conservation

$$\dot{E} = \frac{d}{dt}\int \tau^{00}\, d^3x = -\oint \tau^{0j} n_j\, d^2S$$

$$= -\frac{R^2}{32\pi}\oint \dot{h}^{ij}_{TT}\dot{h}^{ij}_{TT}\, d\Omega$$



$$\boxed{\dot{E} = -\frac{1}{5}\dddot{I}^{ij}\dddot{I}^{ij}}$$

"The GR Quadrupole Formula"

For binary systems:

$$I^{ij} = \mu\left(x^i x^j - \tfrac{1}{3}\delta^{ij} r^2\right)$$

$$\dot{E} \sim \mu^2 r^4 \omega^6 \qquad E = m - \frac{1}{2}\frac{\mu m}{a}$$

PARAMETERS OF THE HULSE-TAYLOR BINARY PULSAR (J1915+1606)

Parameter	Symbol (units)	Value
(i) **'Physical' Parameters**		
Right Ascension	α	$19^h15^m28.^s00018(15)$
Declination	δ	$16^o06'27''.4043(3)$
Pulsar Period	P_p (ms)	59.029997929613(7)
Derivative of Period	$\dot{P}_p$ (10^{-18})	8.62713(8)
(ii) **'Keplerian' Parameters**		
Projected Semimajor Axis	$a_p \sin \imath$(light-s)	2.3417592(19)
Eccentricity	e	0.6171308(4)
Orbital Period	P_b (day)	0.322997462736(7)
Longitude of Periastron	ω_0 (o)	226.57528(6)
Julian Ephemeris Date of Periastron	T_0 (MJD)	46443.99588319(3)
(iii) **'Post-Keplerian' Parameters**		
Mean Rate of Periastron Advance	$\langle\dot{\omega}\rangle$ (^{o}yr^{-1})	4.226621(11)
Redshift and Time Dilation	γ' (ms)	4.295(2)
Orbital Period Derivative	$\dot{P}_b$ (10^{-12})	−2.422(6)

*Numbers in parenthesis denote errors in last digit(s)

Post-Keplerian Effects in Binary Systems

$$\boxed{\begin{array}{c}\text{Kepler's 3rd Law}\\ \left(\dfrac{P_b}{2\pi}\right) = \left(\dfrac{a^3}{\mathcal{G} m}\right)^{1/2}\end{array}}$$

Periastron shift

$$\dot{\omega} = \left[\frac{6\pi m}{P_b a(1-e^2)}\right] \mathcal{P}\mathcal{G}^{-1} = 2.^{\circ}10 \left(\frac{m}{m_\odot}\right)^{2/3} \mathcal{P}\mathcal{G}^{-4/3} \, \mathrm{yr}^{-1}$$

[no tidal effects]

Gravitational redshift / Time dilation

$$\gamma = 2.93 \left(\frac{m_2}{m}\right)\left(\frac{m}{m_\odot}\right)^{2/3} \mathcal{G}^{-1/3} \left(\alpha_2^* + \mathcal{G}\frac{m_2}{m} + \mathcal{K}\eta_2^*\right) \, \mathrm{ms}$$

Gravitational Radiation Damping

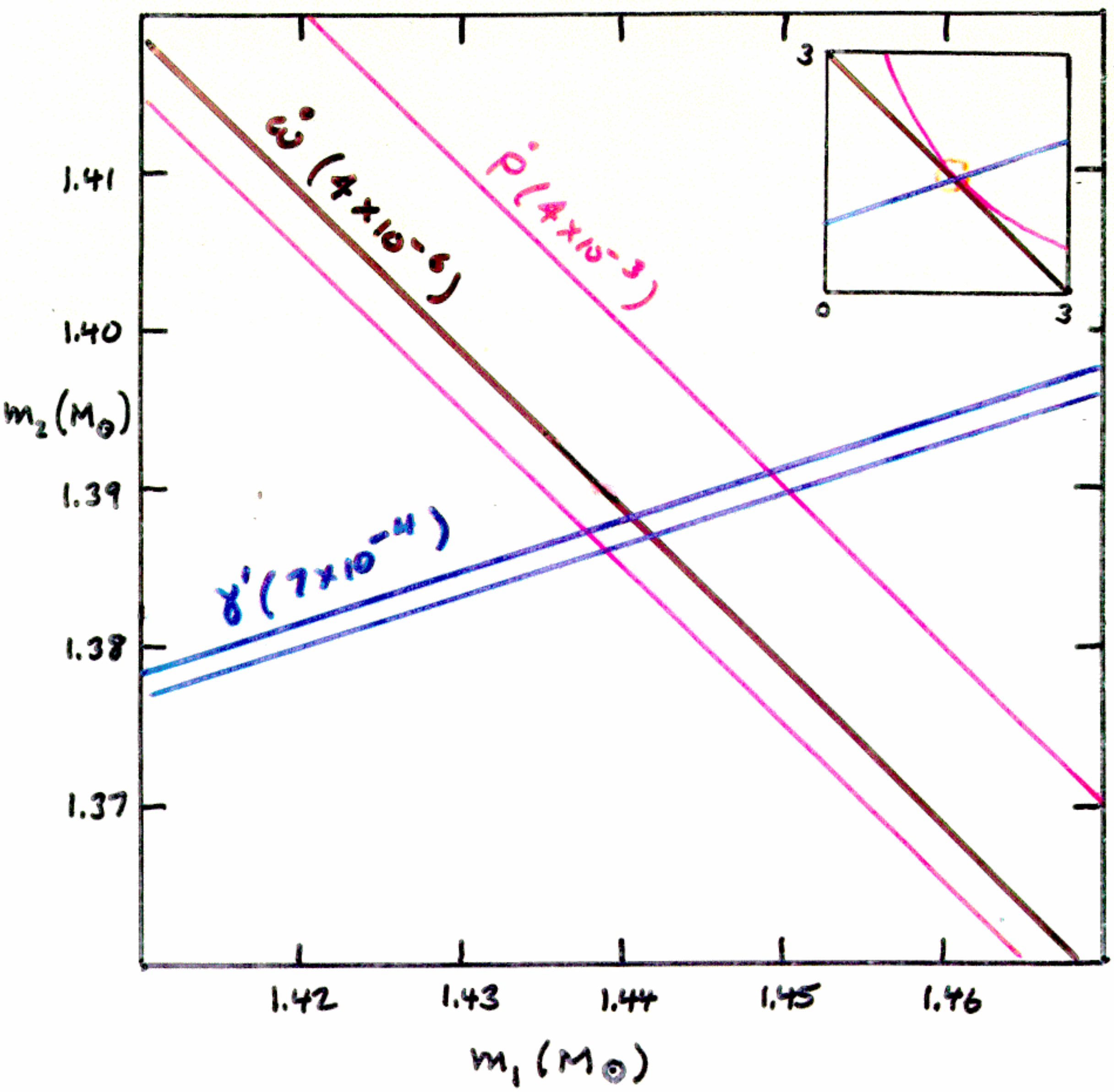
$$\frac{\dot{P}}{P} = -\frac{3}{2}\frac{\dot{E}}{E}$$

$$\dot{P}_b = -\frac{192\pi}{5}\left(\frac{2\pi m}{P_b}\right)^{5/3}\left(\frac{\mu}{m}\right) \mathcal{G}^{-4/3} \mathcal{F}(e) \qquad \left\{\begin{array}{l}\text{QUAD}\\ \text{MONO}\end{array}\right.$$

$$- 4\pi\left(\frac{2\pi m}{P_b}\right)\left(\frac{\mu}{m}\right) \xi_D \mathcal{S}^2 G(e) \qquad \text{DIPOLE}$$

$$\boxed{\begin{array}{ll}\text{GR} & \mathcal{G} = \mathcal{P} = \alpha_2^* = 1, \quad \eta^* = 0, \; \xi_D = 0 \\ & \mathcal{F}(e) = \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)(1-e^2)^{-7/2}\end{array}}$$

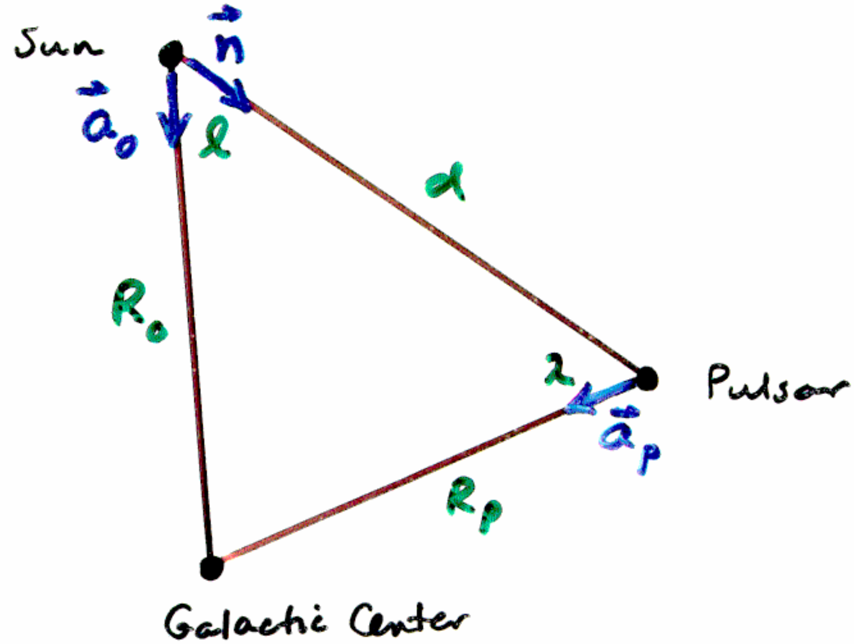
The Binary Pulsar m_1-m_2 Plane



$m_1 = 1.4411(7)$ $m_2 = 1.3874(7)$

Galactic Acceleration

(Damour + Taylor 1990)



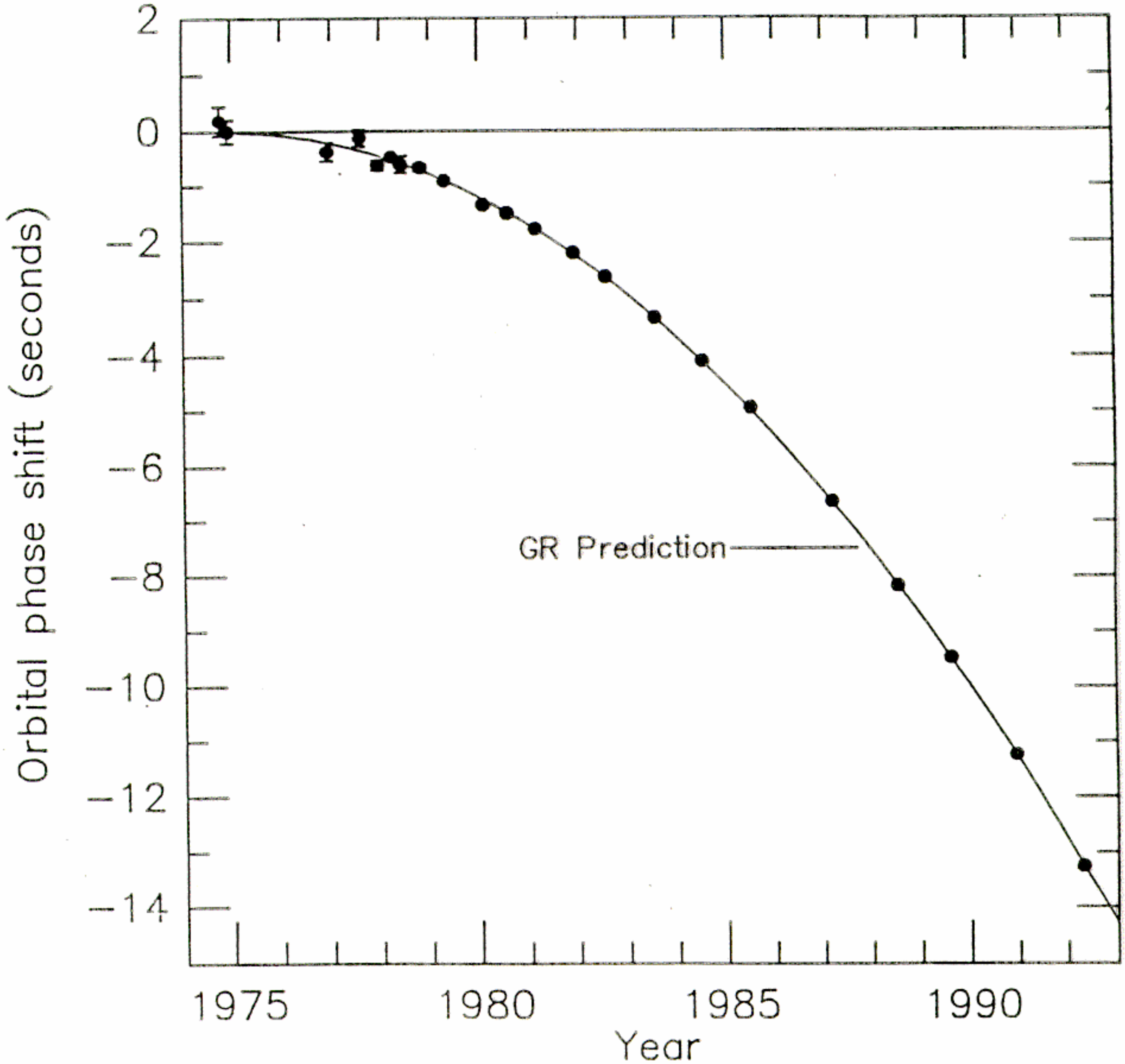
$$\frac{\dot{P}}{P} = \vec{n}\cdot(\vec{a}_p - \vec{a}_0) + v_T^2/d$$

$$= -\frac{v_0^2}{R_0}\cos\ell - \frac{v_p^2}{R_p}\cos\lambda + v_T^2/d$$

$$\boxed{\begin{aligned} \dot{P}_{Gal} &= -0.017 \pm 0.005 \times 10^{-12} \\ \dot{P}_{obs} &= -2.439 \pm 0.018 \times 10^{-12} \\ \dot{P}_{GW} &= -2.422 \pm 0.019 \times 10^{-12} \\ \dot{P}_{GR} &= -2.403 \times 10^{-12} \end{aligned}}$$

$d \sim 8$ kpc

C. Will, SSI 1998



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$\Downarrow$

$m_p = 1.4411(7)\, M_\odot \quad m_c = 1.3874(7)\, M_\odot$

Sensitivity factors in Brans-Dicke Theory

$$\left\{ \text{first order in } \xi = \frac{1}{2+\omega} \quad ; \quad s = -\left(\frac{\partial(\ln m)}{\partial \ln G}\right)_N \right\}$$

$$\mathcal{G} = 1 - \xi(s_1 + s_2 - 2s_1 s_2)$$

$$\mathcal{P} = \mathcal{G}\left[1 - \frac{2}{3}\xi + \frac{1}{3}(s_1 + s_2 - 2s_1 s_2)\right]$$

$$\alpha_2^* = 1 - \xi s_2$$

$$\eta_2^* = (1 - 2s_2)\xi \qquad \left\{ \kappa = -\left(\frac{\partial \ln I}{\partial \ln G}\right)_N \right\}$$

$$\mathcal{F}(e) = \frac{1}{12}(1-e^2)^{-7/2}\left[\kappa_1\left(1 + \frac{7}{2}e^2 + \frac{1}{2}e^4\right) - \kappa_2\left(\frac{1}{2}e^2 + \frac{1}{8}e^4\right)\right]$$

$$\kappa_1 = \mathcal{G}^2\left[12\left(1 - \frac{1}{2}\xi\right) + \xi\Gamma^2\right]$$

$$\kappa_2 = \mathcal{G}^2\left[11\left(1 - \frac{1}{2}\xi\right) + \frac{1}{2}\xi\left(\Gamma^2 - 5\Gamma\Gamma' - \frac{15}{2}\Gamma'^2\right)\right]$$

$$\Gamma = 1 - 2(m_1 s_2 + m_2 s_1)/m$$

$$\Gamma' = 1 - s_1 - s_2$$

$$\xi_D = \xi$$

$$G(e) = \left(1 + \frac{1}{2}e^2\right)(1-e^2)^{-5/2}$$

$$\mathcal{S} = s_1 - s_2$$

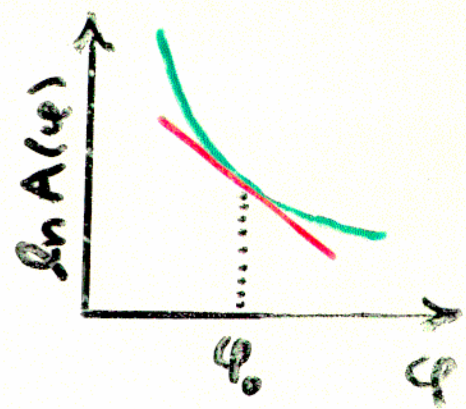
Values of s

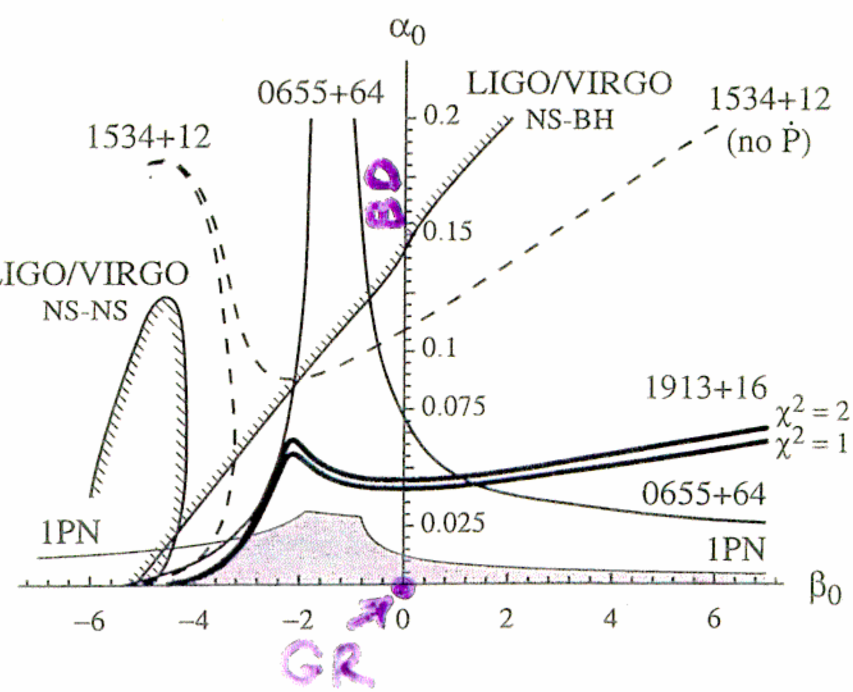
EOS	$1.38\,M_\odot$	$1.44\,M_\odot$
B	0.30	.....
O	0.143	0.145
M	0.124	0.125

Tests of General Scalar-Tensor Gravity

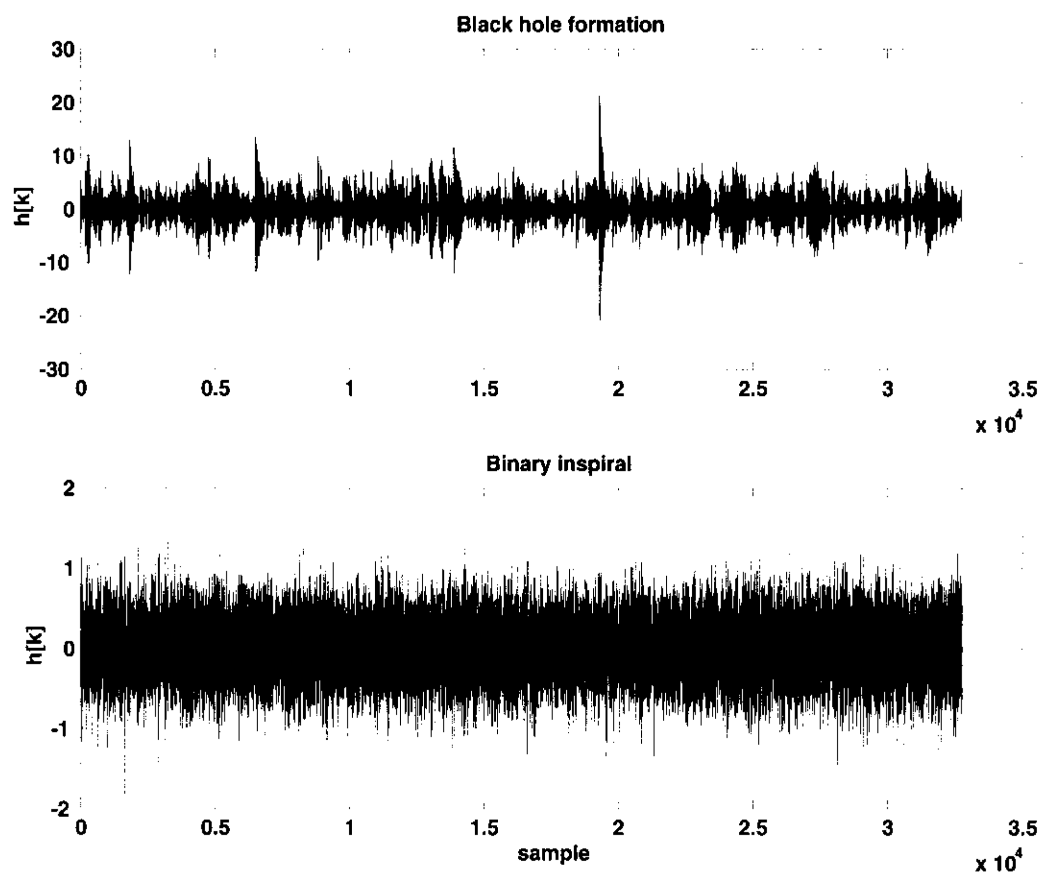
$$\gamma - 1 = -2\,\frac{\alpha_0^2}{1+\alpha_0^2}$$

$$\beta - 1 = \frac{1}{2}\,\frac{\beta_0\alpha_0^2}{(1+\alpha_0^2)^2}$$

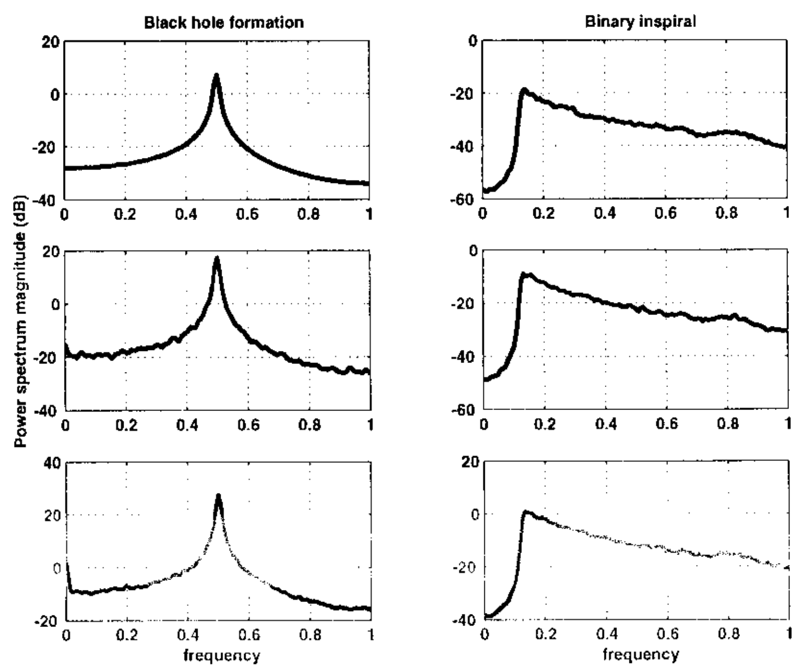




Same distribution, different signals

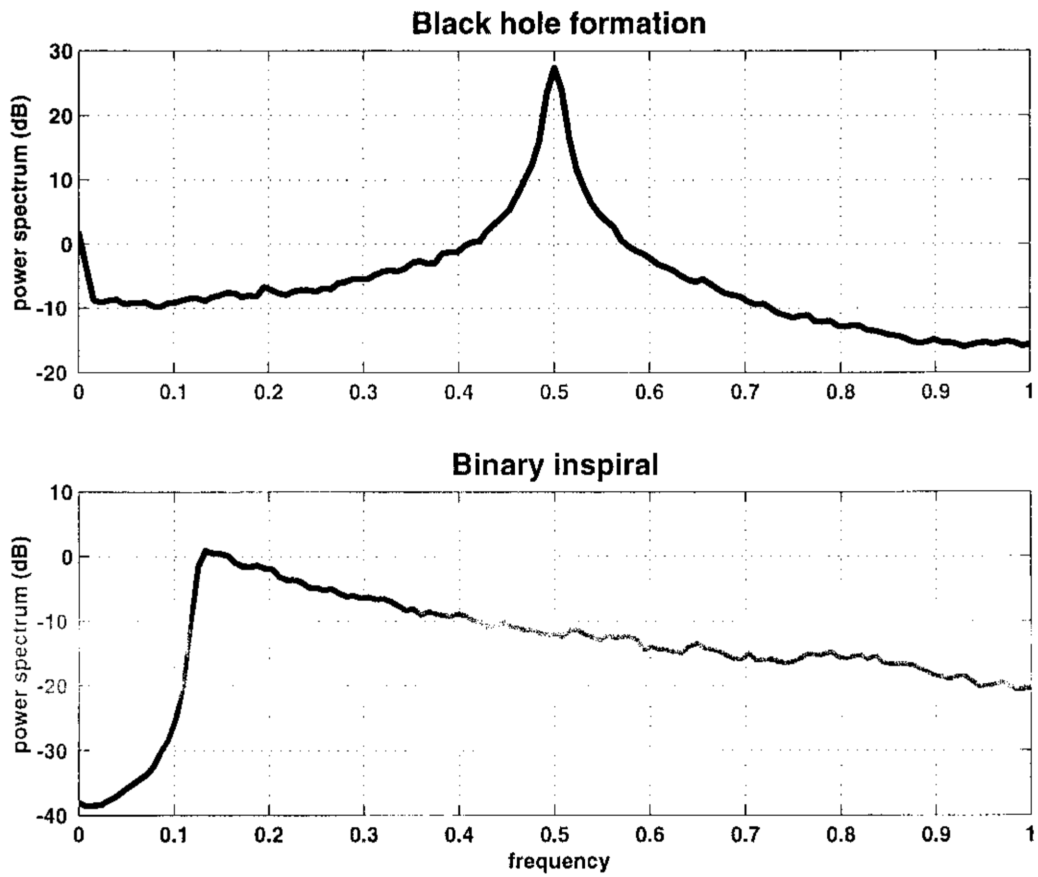


Power is proportional to rate



- $\dot{n}$
- $10\dot{n}$
- $100\dot{n}$

Correlations distinguish stochastic signals

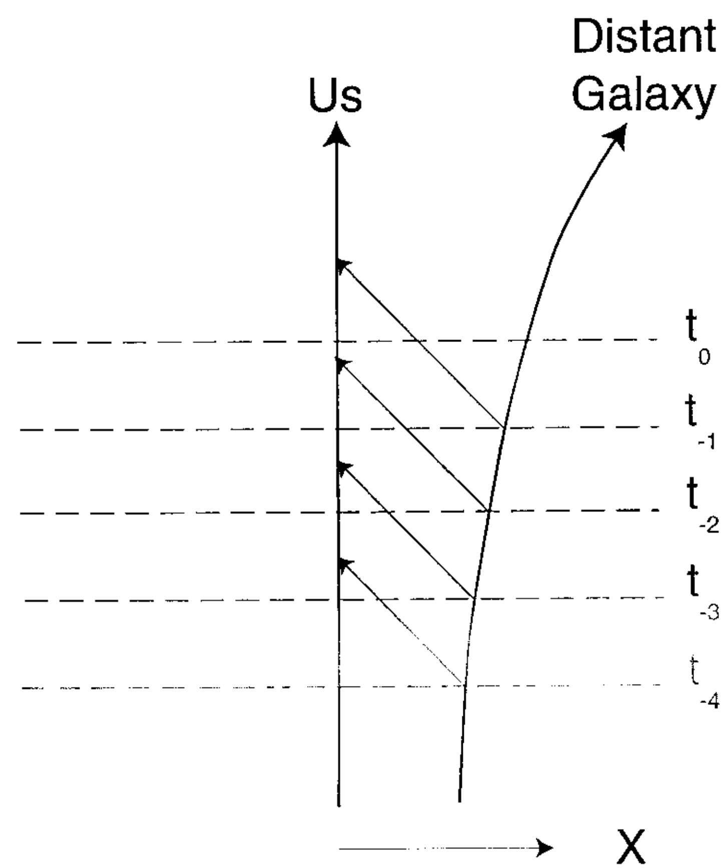


Murmuring or Deafening?

- Signals come from sources near and far
  - nearer: stronger, rarer
  - farther: weaker, more frequent
- Rate of signals arising fron $(r,\ r+dr)$
  - $\propto 4\pi r^2 dr$
- Mean source energy
  - $1/r^2$
- Power from $(r,\ r+dr)$
  - $\propto 4\pi r^2 dr\,/\,r^2 = 4\pi\, dr$
- Total power incident on detector
  - $\propto \int_0^\infty 4\pi\, dr = \infty$?!

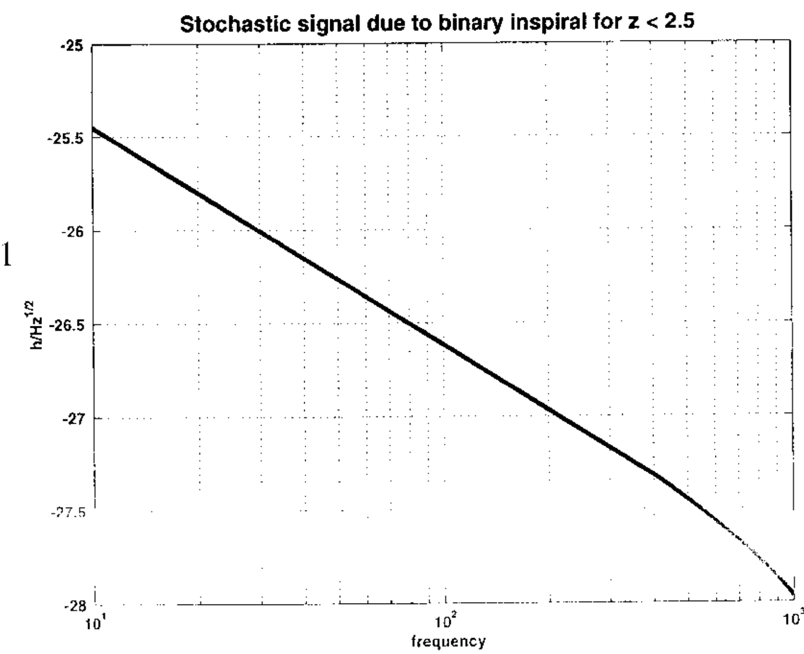
Why is the night sky dark?

- Because the Universe is expanding:
  - expansion decreases apparent rate of distant sources
  - expansion decreases apparent energy of distant sources
  - Universe had a beginning: no sources before "date certain"



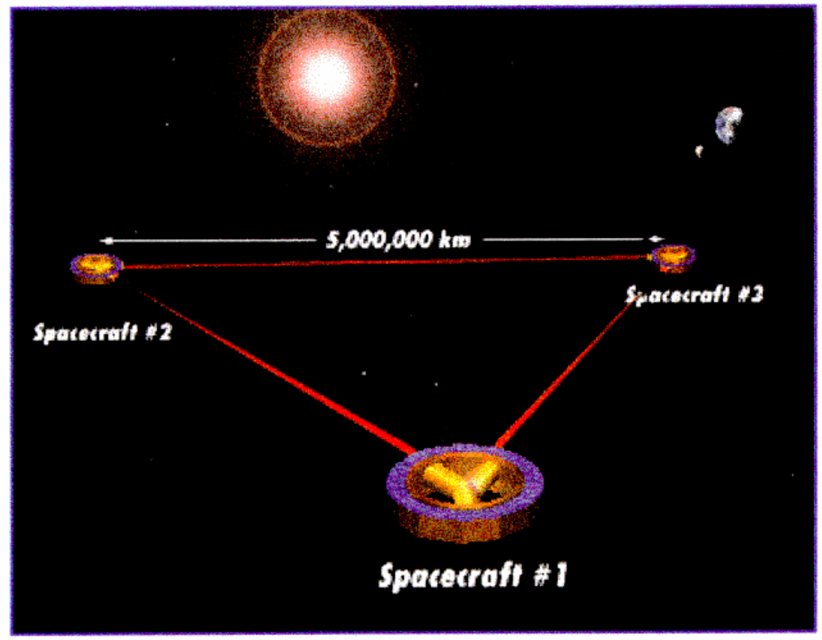
Binary inspiral stochastic signal

- Assumptions
  - Uniform rate density
    - » better to track star formation
    - » $\dot{n}_0 = 8 \times 10^{-8}\,\mathrm{Mpc}^{-3}\mathrm{yr}^{-1}$
  - Cut-off at quasar distribution peak
  - $\Omega=1$, $\Lambda=0$ (H_0 cancels)



LISA

- Laser Interferometer Space Antenna
  - $10^{-4} < f\,/\,\mathrm{Hz} < 10^{-1}$
  - ESA, maybe NASA, project
- Galactic binaries
  - Confusion limited



CWDB Census

- CWDBs
  - follow common envelope binary evolution phase
  - evolve by gravitational radiation reaction
  - confusion limited source: dn/df observed
- Continuity equation
  - $$\frac{dn_+}{df} - \frac{dn_-}{df} = \frac{d}{df}\left[\frac{dn}{df}\frac{df}{dt}\right]$$
  - observing spectrum tells us birth rate dn_+/df

Summary

- Periodic sources
  - all involve rapidly rotating neutron stars
  - require mechanism to generate, maintain asymmetry
- Unknowns
  - Mechanisms for producing asymmetries
  - QPOs promising; R-modes less so
- Detection challenges
  - frequency modulated signal
  - weak signal requires long (month to year) integration times