TESTING POST-NEWTONIAN GRAVITY

WUGRAV

WASHINGTON UNIVERSITY GRAVITATION GROUP

- The Parametrized Post-Newtonian Formalism
    - GR and Scalar-Tensor Gravity
- Measurement of PPN parameters
    - Light Deflection and Time Delay
    - Mercury's Perihelion Advance
    - Tests of the Strong Equivalence Principle
    - Is G constant?
    - Is Momentum Conserved?
- Bounds on the PPN Parameters
    - The Rise and Fall (and Rise) of Scalar-Tensor Gravity
- The Search for Gravitomagnetism
    - Gravitomagnetism, Frame-Dragging and Mach's Principle
    - Gravity Probe-B
    - LAGEOS Satellite Tracking
- Future Tests

The Post-Newtonian Limit

$$\boxed{v^2 \sim U \sim \frac{p}{\rho} \ll 1}$$

$G = c = 1$

Geodesic equation from the action:

$$\delta \mathcal{L} = 0 = \delta \int_A^B d\tau = \delta \int_A^B \left(g_{00} + 2 g_{0i} v^i + g_{ij} v^i v^j \right)^{1/2} dt$$

$$\approx \delta \int_A^B dt \left\{ 1 - U + O(U^2) + O(V^i) v^i + \frac{1}{2} v^2 \left(1 + O(U) \right) \right\}$$

Need:

$$\boxed{g_{00} \text{ to } O(v^4) \, ; \; g_{0i} \text{ to } O(v^3) \, ; \; g_{ij} \text{ to } O(v^2)}$$

THE PARAMETRIZED POST-NEWTONIAN FORMALISM

A. PPN Parameters:

$$\gamma, \quad \beta, \quad \xi, \quad \alpha_1, \quad \alpha_2, \quad \alpha_3, \quad \zeta_1, \quad \zeta_2, \quad \zeta_3, \quad \zeta_4 \quad .$$

B. Metric:

$$\begin{aligned} g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi \Phi_W + (2\gamma+2+\alpha_3+\zeta_1-2\xi)\Phi_1 \\ & + 2(3\gamma-2\beta+1+\zeta_2+\xi)\Phi_2 + 2(1+\zeta_3)\Phi_3 + 2(3\gamma+3\zeta_4-2\xi)\Phi_4 \\ & - (\zeta_1-2\xi)A - (\alpha_1-\alpha_2-\alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3-\alpha_1)w^i V_i \end{aligned}$$

$$\begin{aligned} g_{0i} = & -\frac{1}{2}(4\gamma+3+\alpha_1-\alpha_2+\zeta_1-2\xi)V_i - \frac{1}{2}(1+\alpha_2-\zeta_1+2\xi)W_i \\ & - \frac{1}{2}(\alpha_1-2\alpha_2)w^i U - \alpha_2 w^j U_{ij} \end{aligned}$$

$$g_{ij} = (1+2\gamma U)\delta_{ij}$$

C. Metric Potentials:

$$U = \int \frac{\rho'}{|\mathbf{x}-\mathbf{x}'|} d^3x' \quad , \qquad U_{ij} = \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3} d^3x'$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|} \right) d^3x' d^3x''$$

$$A = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3} d^3x' \quad , \qquad \Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$

$$\Phi_2 = \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|} d^3x' \quad , \qquad \Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x}-\mathbf{x}'|} d^3x' \quad , \qquad \Phi_4 = \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$

$$V_i = \int \frac{\rho' v_i'}{|\mathbf{x}-\mathbf{x}'|} d^3x' \quad , \qquad W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x}-\mathbf{x}'|^3} d^3x'$$

Table 1. The PPN Parameters and Their Significance*

Parameter	What it measures relative to general relativity	Value in general relativity	Value in semi-conservative theories	Value in fully-conservative theories
γ	How much space-curvature is produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" is there in the superposition law for gravity?	1	β	β
ξ	Are there preferred location effects?	0	ξ	ξ
α_1	Are there preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
ζ_1	Is there violation of conservation of total momentum?	0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

---

*For a compendium of PPN parameter values in alternative theories together with derivations, see TEGP, Chapter 5.

C. Will, SSI 1998

METRIC THEORIES OF GRAVITY: PPN PARAMETER VALUES

Theory and its gravitational fields	Arbitrary functions or constants	Cosmological matching parameters	PPN parameters*					
			γ	β	ξ	α_1	α_2	(α_3, ζ_i)
(a) Purely dynamical theories								
(i) General relativity (**g**)	none	none	1	1	0	0	0	0
(ii) Scalar-tensor (**g**, ϕ)								
BWN	$\omega(\phi)$	ϕ_0	$\frac{1+\omega}{2+\omega}$	$1+\Lambda$	0	0	0	0
Bekenstein's VMT	$\omega(\phi), r, q$	ϕ_0	$\frac{1+\omega}{2+\omega}$	$1+\Lambda$	0	0	0	0
Brans–Dicke	ω	ϕ_0	$\frac{1+\omega}{2+\omega}$	1	0	0	0	0
(iii) Vector-tensor (**g**, **K**)								
General	$\omega, \eta, \varepsilon, \tau$	K	γ'	β'	0	α_1'	α_2'	0
Hellings–Nordtvedt	ω	K	γ'	β'	0	α_1'	α_2'	0
Will–Nordtvedt	none	K	1	1	0	0	$K^2/(1+\frac{1}{2}K^2)$	0
(b) Theories with prior geometry								
(iv) Bimetric theories								
Rosen (**g**, **η**)	none	c_0, c_1	1	1	0	0	$(c_0/c_1) - 1$	0
Rosen (**g**, γ)	none	a	1	1	0	0	0	0
Rastall (**g**, **η**, **K**)	none	K	1	1	0	0	α_2'	0
BSLL (**g**, **η**, **B**)	a, f, k	ω_0, ω_1	γ'	β'	0	α_1'	α_2'	0
(v) Stratified theories								
LLN (**g**, **η**, t, ϕ, **K**, **B**)	$f_1(\phi), f_2(\phi), f_3(\phi), e, \kappa_1, \kappa_2$	c_0, c_1, a, b, c, d	ac_0/c_1	β'	ξ'	α_1'	α_2'	0
Ni (**g**, **η**, t, ϕ, **K**, **B**)	$f_1(\phi), f_2(\phi), f_3(\phi), e$	c_0, c_1, a, b, c, d	ac_0/c_1	bc_0	0	α_1'	α_2'	0
Canuto (**g**, **η**, t)	$\beta(t)$	none	1	1	0	0	0	0

* Prime over a PPN parameter (e.g., γ') denotes a complicated function of arbitrary constants and cosmological matching parameters. See TEGP, Sec 5.2–5.6 for explicit formulae.

GENERAL RELATIVITY

Action

$$I = (16\pi G)^{-1}\int\sqrt{-g}\,R\,d^4x + I_m(\psi_m, g_{\mu\nu})$$

matter fields

universal coupling ⇒ metric theory

Field Equations

$$\frac{\delta I}{\delta g_{\mu\nu}} = 0 \quad \Rightarrow \quad G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\frac{\delta I}{\delta \psi_m} = 0 \quad \Rightarrow \quad \text{matter field equations}$$

$$\frac{\delta I}{\delta x^\mu} = 0 \quad \Rightarrow \quad T^{\mu\nu}{}_{;\nu} = 0$$

(general covariance)

SCALAR-TENSOR GRAVITY

Action

- Non-metric representation ("Einstein frame")

$$I = (16\pi G)^{-1}\int \left[R_* - 2 g_*^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - V(\varphi)\right]\sqrt{-g_*}\, d^4x$$

$$+ I_m\big(\psi_m,\ \underbrace{A^2(\varphi)\, g^*_{\mu\nu}}_{\text{universal coupling}}\big)$$

- Metric representation

$$g_{\mu\nu} = A^2(\varphi)\, g^*_{\mu\nu}$$

physical metric

$$I = (16\pi G)^{-1}\int \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \phi^2 V\right]\sqrt{-g}\, d^4x$$

$$+ I_m(\psi_m, g_{\mu\nu})$$

$$\phi = [A(\varphi)]^{-2}$$

$$3 + 2\omega(\phi) = [\alpha(\varphi)]^{-2}$$

$$\alpha(\varphi) = d[\ln A(\varphi)]/d\varphi$$

SCALAR-TENSOR GRAVITY

Field Equations

$$G_{\mu\nu} + \frac{1}{2}\phi V g_{\mu\nu} = \frac{8\pi G}{\phi}T_{\mu\nu} + \frac{\omega(\phi)}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}\right)$$
$$+ \phi^{-1}\left(\phi_{;\mu\nu} - g_{\mu\nu}\Box_g\phi\right)$$

$$\Box_g\phi + \frac{1}{2}\frac{\phi^2}{\sqrt{3+2\omega}}\left(\frac{dV}{d\varphi}\right) = \frac{1}{3+2\omega}\left(8\pi T - \frac{d\omega}{d\phi}\phi_{,\lambda}\phi^{,\lambda}\right)$$

Remarks

- $A^2(\varphi) = \phi^{-1}$ modifies G
- PN limit determined by ϕ_0 (cosmology) and

$$\omega = \omega(\phi_0)$$

$$\Lambda = \left[\frac{d\omega}{d\phi}\frac{1}{(3+2\omega)^2}\frac{1}{(4+2\omega)}\right]_{\phi_0}$$

- "Attractor" mechanism (Damour & Nordtvedt)
- Brans-Dicke : ω = const, ie

$$A(\varphi) = e^{\varphi/\sqrt{3+2\omega}}$$

Light Deflection and Time Delay :

Tests of γ

Deflection of Light

$$\Delta\theta = \left(\frac{1+\gamma}{2}\right)\frac{4M}{d}$$

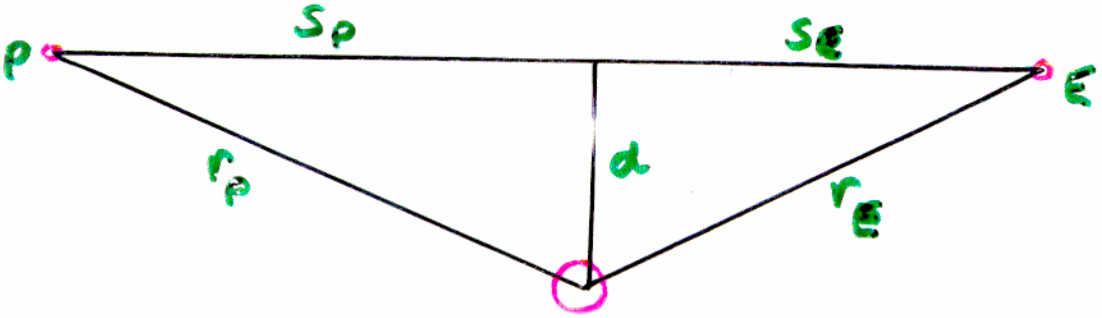
$$= \left(\frac{1+\gamma}{2}\right)\frac{1.''75}{(d/R_\odot)}$$

Shapiro Time Delay

$$\Delta t = \left(\frac{1+\gamma}{2}\right) 4M \ln\left|\frac{(r_E + S_E)(r_p + S_p)}{d^2}\right|$$

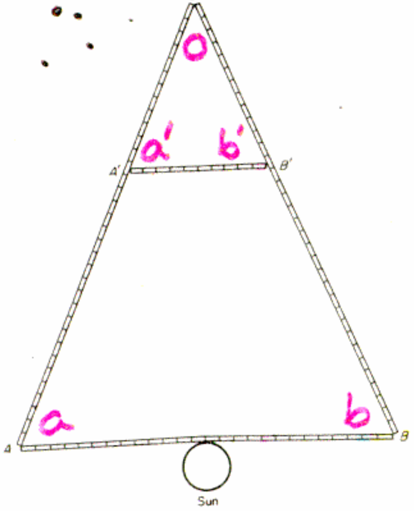
$$\simeq \left(\frac{1+\gamma}{2}\right) 250\left(1 - 0.16\ln(d/R_\odot)\right)\ \mu s$$

(Mars)



Light Deflection and Alternative Theories

$$\text{Deflection} = \left(\frac{1}{2} + \frac{\gamma}{2}\right) 1.''7505/d$$



$$o + a' + b' = 180^\circ$$

$$o + a + b = 180^\circ - \gamma\, 0.''875/d$$

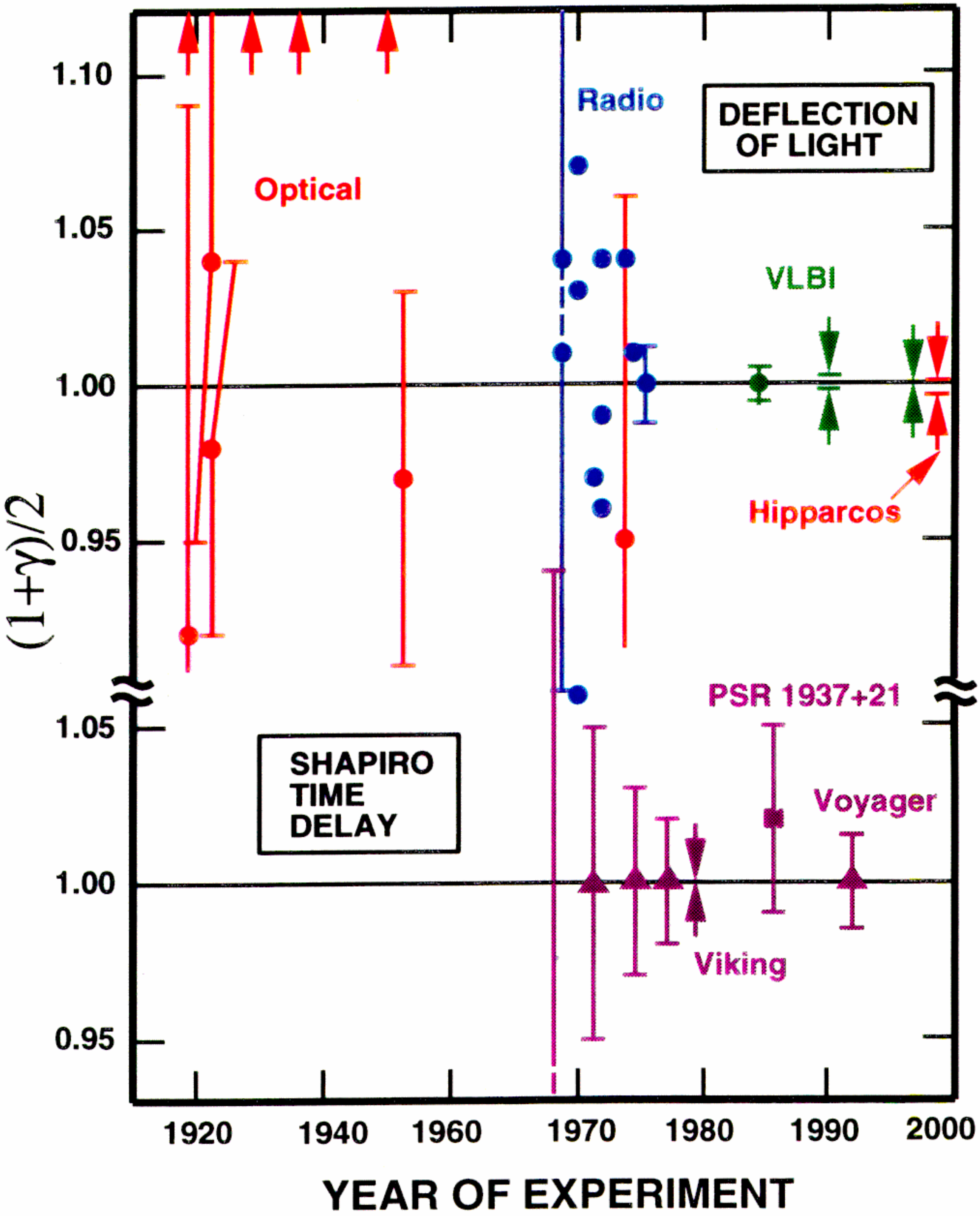
Cavendish	1784
von Soldner	1801
Einstein	1907

Space curvature

$$\boxed{\begin{array}{c}\text{GR} \\ \gamma = 1\end{array}}$$

C. Will, SSI 1998

THE PARAMETER $(1+\gamma)/2$



Mercury's Perihelion: Triumph or Trouble?

- In 1915: triumph
- 1966–80: trouble
- 1980 – : resolution

$$\omega = 42''.98 \, (\lambda_{REL} + \lambda_{QUAD})$$

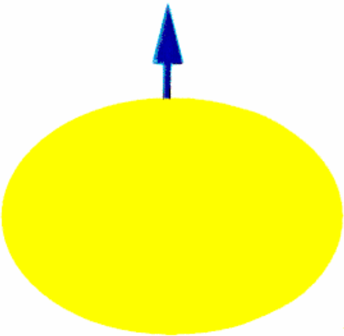
$$\lambda_{REL} = (2+2\gamma-\beta)/3 = 1 \text{ in GR}$$

- **Triumph**

$$\lambda_{OBS} = 1.000 \pm 0.001 \text{ (Mercury radar)}$$

- **Trouble and resolution**

$$\lambda_{QUAD} = 3 \times 10^{-4} \, (J_2 / 10^{-7})$$



Helioseismology ☛ $J_2 \cong 2 \times 10^{-7}$

THE STRONG EQUIVALENCE PRINCIPLE

- All bodies (laboratory-size, planets, stars) fall with the same acceleration

  Analogue of WEP

- In a local freely falling frame, non-gravitational as well as gravitational physics is independent of the frame's velocity

  Analogue of LLI

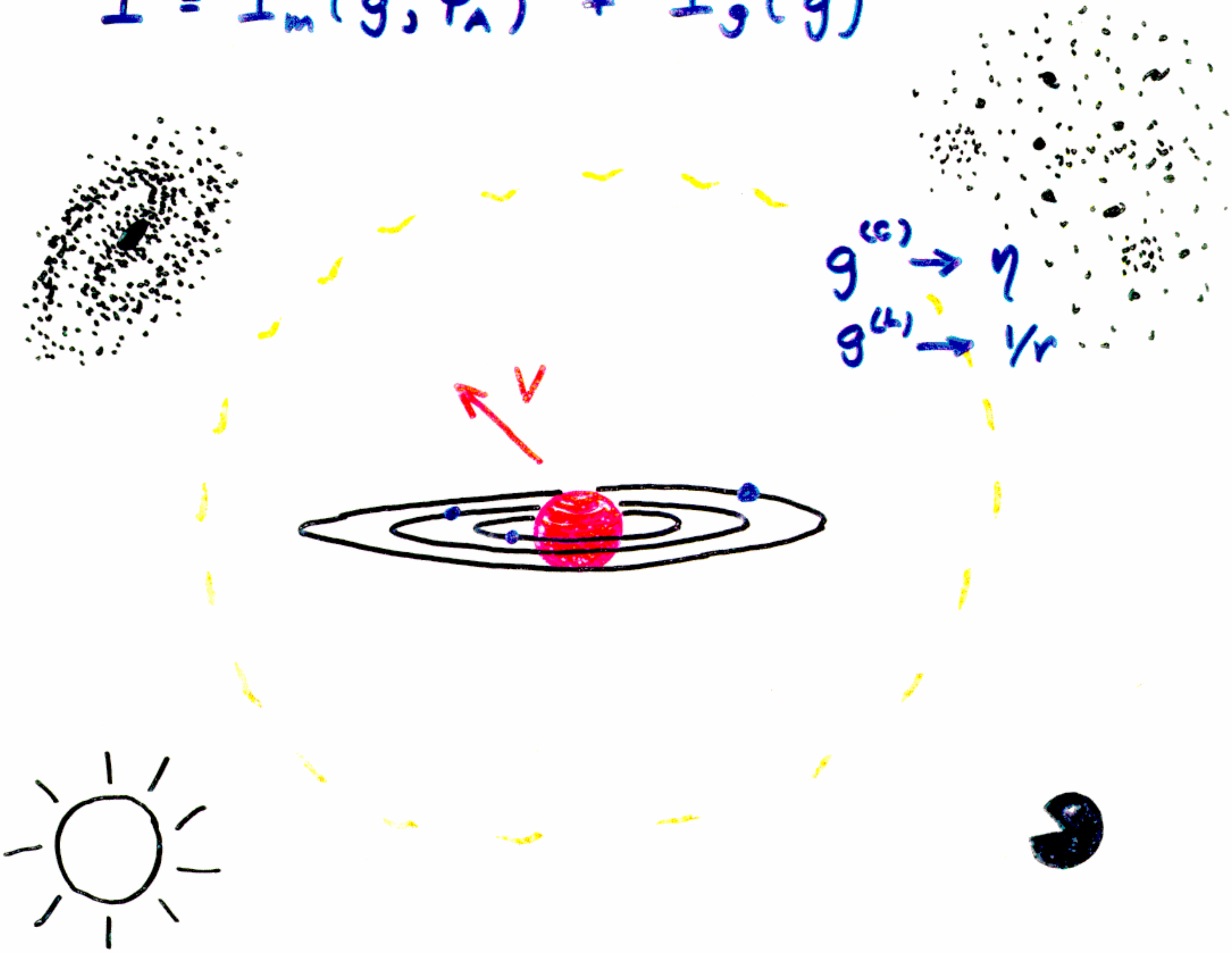
- In a local freely falling frame, non-gravitational as well as gravitational physics is independent of the frame's location

  Analogue of LPI

GR satisfies SEP; all other metric theories (Brans-Dicke, etc.) don't

THE STRONG EQUIVALENCE PRINCIPLE

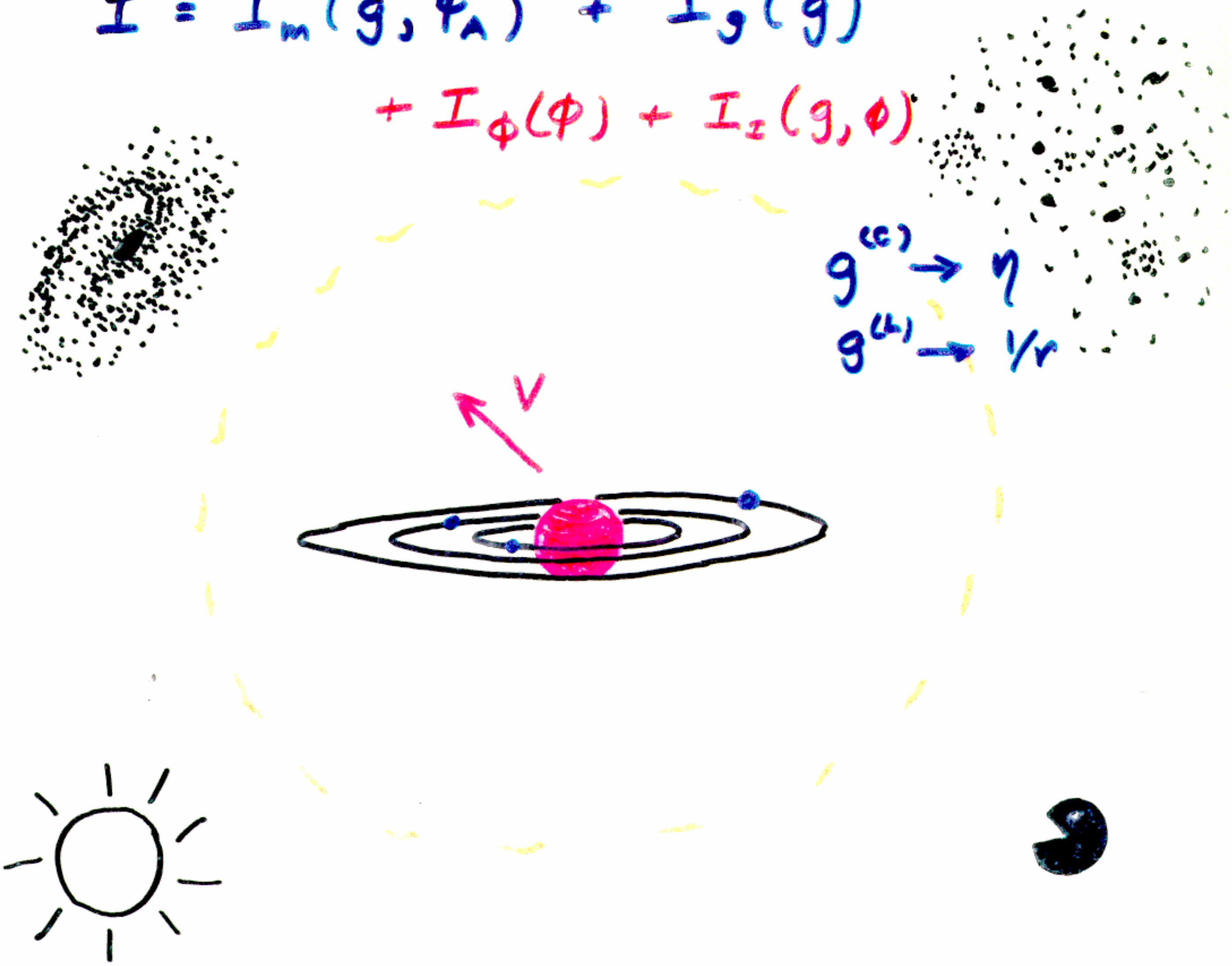
$$I = I_m(g, \psi_A) + I_g(g)$$



$$g_{\mu\nu} = g^{(C)}_{\mu\nu} + g^{(L)}_{\mu\nu}$$

THE STRONG EQUIVALENCE PRINCIPLE

$$I = I_m(g, \psi_A) + I_g(g)$$
$$+ I_\phi(\phi) + I_I(g, \phi)$$



$g^{(C)} \rightarrow \eta$

$g^{(L)} \rightarrow 1/r$

$$g_{\mu\nu} = g^{(C)}_{\mu\nu} + g^{(L)}_{\mu\nu}$$

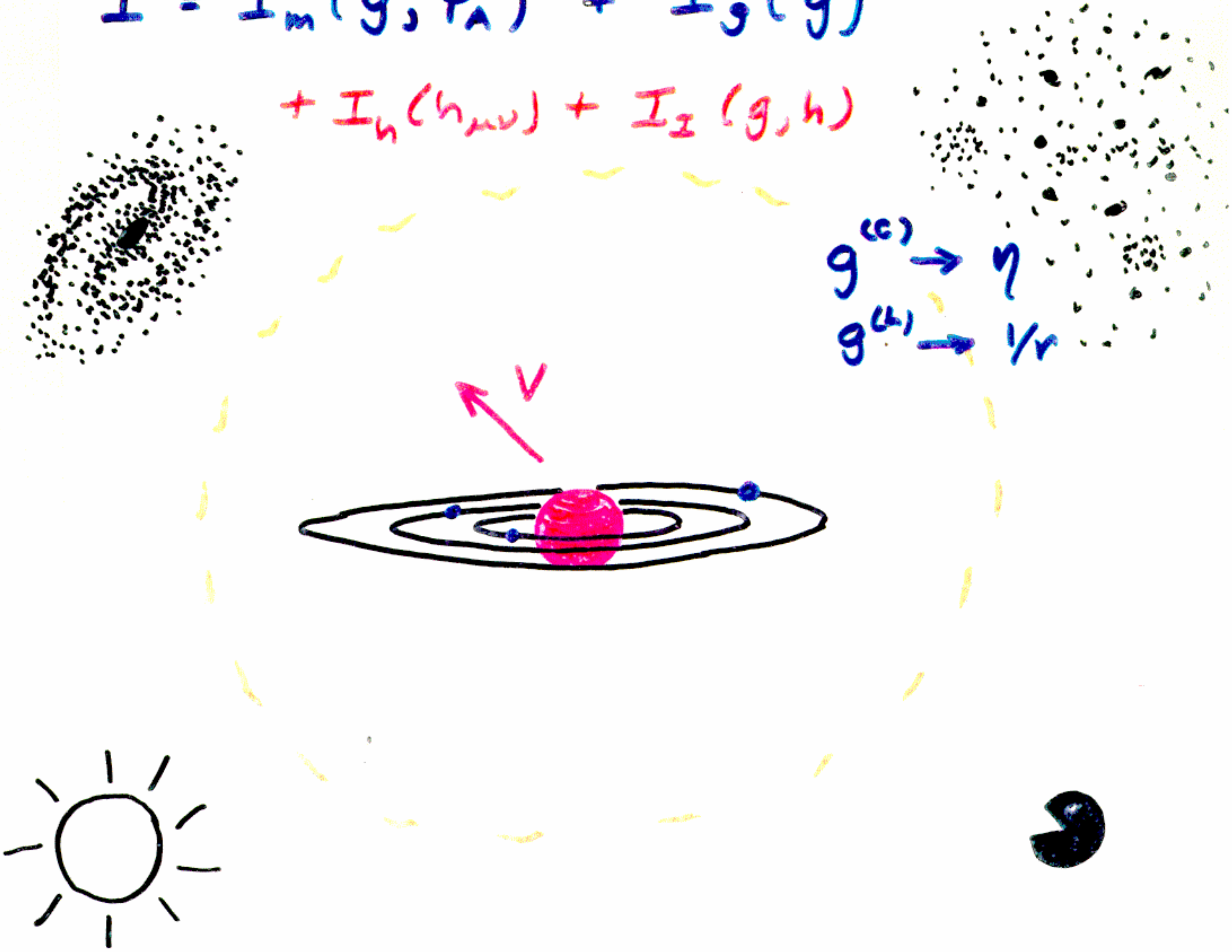
$$\phi = \phi^{(C)} + \phi^{(L)}$$

↑ can act back on $g^{(L)}$

THE STRONG EQUIVALENCE PRINCIPLE

$$I = I_m(g, \psi_A) + I_g(g)$$

$$+ I_h(h_{\mu\nu}) + I_I(g, h)$$



$g^{(c)} \to \eta$

$g^{(L)} \to 1/r$

$$g_{\mu\nu} = g^{(C)}_{\mu\nu} + g^{(L)}_{\mu\nu}$$

$$h_{\mu\nu} = h^{(C)}_{\mu\nu} + h^{(L)}_{\mu\nu}$$

↑ can act back on $g_{\mu\nu}$

can depend on $\underset{\sim}{V}$

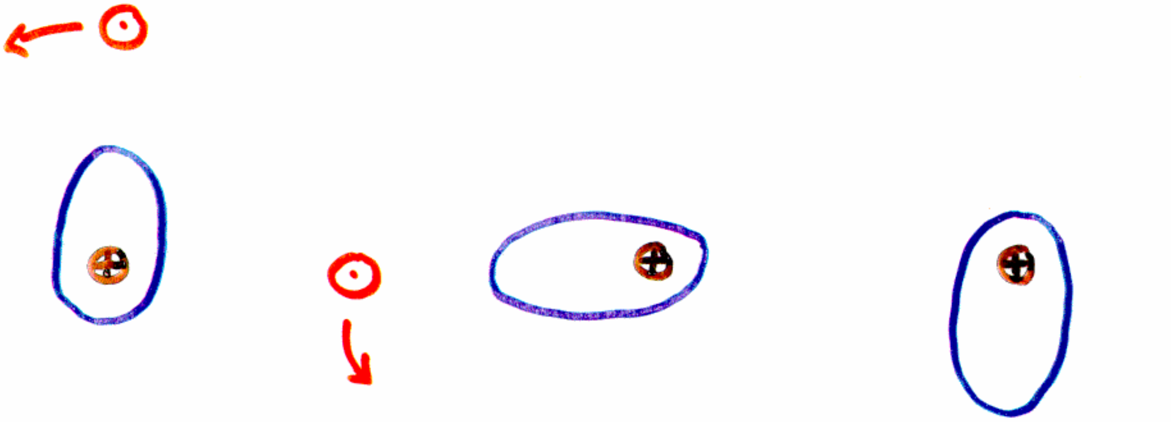
The Moon and the Strong Equivalence Principle

Strong Equivalence Principle

(i) all bodies (even with self-gravity) fall with the same acceleration

(ii) in "local" freely falling frame, laws of gravitation are independent of location and velocity of frame

$\Rightarrow$ general relativity (?)



$\Delta r = \eta \; 13.1 \text{ m}$

$$\eta = 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2$$

[Nordtvedt effect]

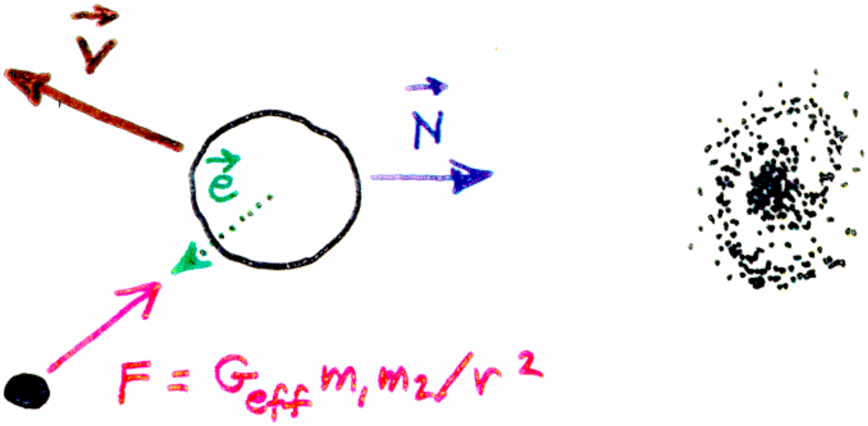
LURE (1969–) : $\Delta r < 1$ cm

$$\boxed{\begin{gathered} |\eta| < 1 \times 10^{-3} \\ |\Delta a/a| < 7 \times 10^{-13} \end{gathered}}$$

BIG "G" AND SEP

Effective Gravitational Constant:

$$
\begin{aligned}
G_{\mathrm{eff}} = & 1 - (4\beta - \gamma - 3 - 3\xi)U_{\mathrm{ext}} \\
& - \frac{1}{2}(\alpha_1 - \alpha_2)v^2 - \frac{1}{2}\alpha_2(\mathbf{v}\cdot\mathbf{e})^2 \\
& + \xi U_{\mathrm{ext}}(\mathbf{N}\cdot\mathbf{e})^2
\end{aligned}
$$



Is Newton's Constant Constant?

A History of G

1687	No G in the Principia	
1730	Bouguer - density of Earth	
1797	Cavendish - " "	
1800-20	Laplace : Mécanique Céleste Poisson : Traité de Mécanique	} G appears
1890	Poynting, Boys - metrology	
1929	Expansion of universe	
1930's	Dirac	$\dot G$ appears
1960's	Brans-Dicke theory	

$$\boxed{\frac{\dot G}{G} \equiv \sigma H_0 = \left(5\times 10^{-11}\,yr^{-1}\right)\sigma\left(\frac{H_0}{55}\right)}$$

Table 6. Constancy of the gravitational constant

Method	$\dot{G}/G(10^{-12}\text{yr}^{-1})$	Reference
Lunar laser ranging	0 ± 30	91
	0 ± 11	84
Viking radar	2 ± 4	92
	-2 ± 10	69
Binary pulsar[1]	11 ± 11	93–95
Pulsar PSR 0655 + 64[1]	<55	96

[1] Bounds dependent upon theory of gravity in strong-field regime and on neutron star equation of state.

ELECTROMAGNETISM AND GRAVITOMAGNETISM

ELECTROMAGNETISM:

1. Gauss' Law: $\nabla \cdot \vec{E} = 4\pi\rho$

2. Ampere-Maxwell Law: $\nabla \times \vec{B} - \frac{\partial}{\partial t}\vec{E} = 4\pi\vec{J}$

3. Gauss' Law for Magnetism: $\nabla \cdot \vec{B} = 0$

4. Faraday's Law: $\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0$

5. Equation of motion: $m d\vec{v}/dt = e(\vec{E} + \vec{v} \times \vec{B})$

6. Potentials: $\vec{E} = -\nabla\phi - \partial\vec{A}/\partial t, \quad \vec{B} = \nabla \times \vec{A}$

ELECTROMAGNETISM AND

GRAVITOMAGNETISM

Gravito~~ELECTRO~~MAGNETISM:

1. Gauss' Law: $\nabla \cdot \vec{E}_g \simeq -4\pi\rho$

2. Ampere-Maxwell Law: $\nabla \times \vec{B}_g - \frac{\partial}{\partial t}\vec{E}_g \simeq -\overset{16}{\cancel{4}}\pi \vec{J}$

3. Gauss' Law for Magnetism: $\nabla \cdot \vec{B}_g \simeq 0$

4. Faraday's Law: $\nabla \times \vec{E}_g + \frac{\partial}{\partial t}\vec{B}_g \simeq 0$

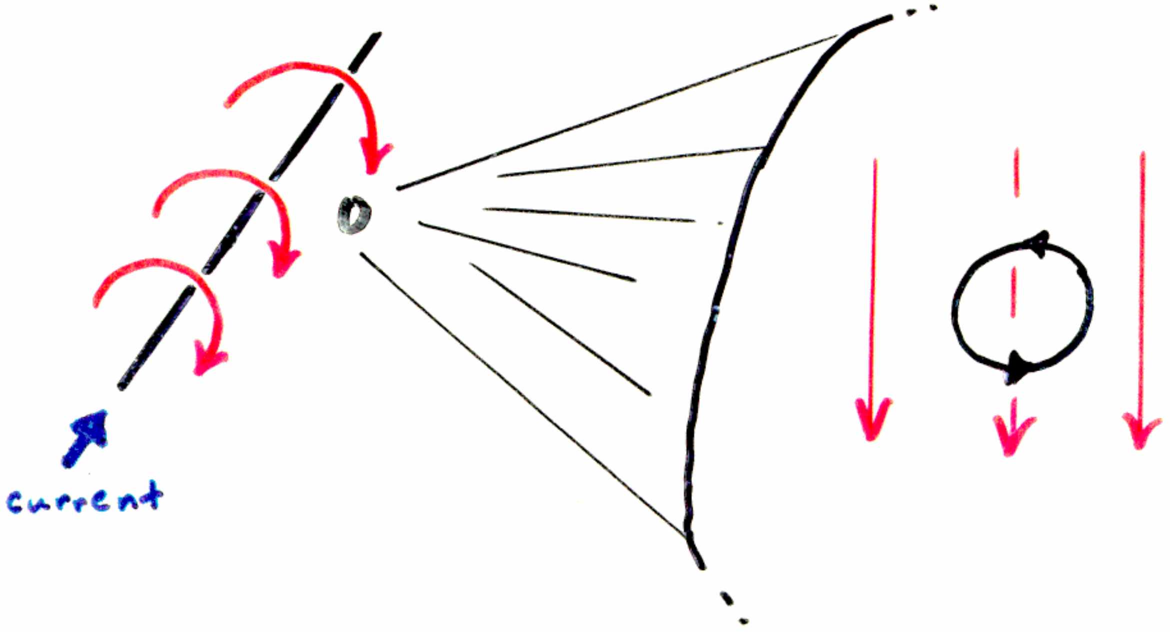
5. Equation of motion: $m d\vec{v}/dt \simeq \overset{m}{\cancel{q}}(\vec{E}_g + \vec{v} \times \vec{B}_g)$

6. Potentials: $\vec{E}_g = -\nabla\phi_g - \partial\vec{A}_g/\partial t, \quad \vec{B}_g = \nabla \times \vec{A}_g$

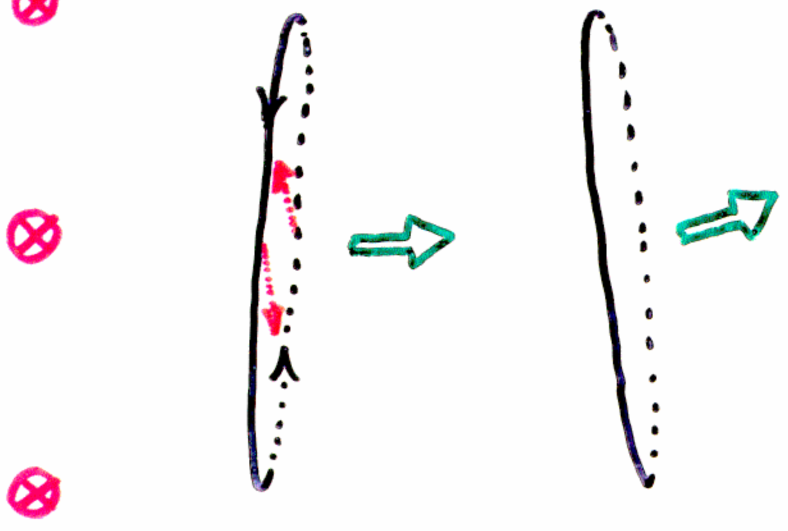
$$\phi_g \simeq -\frac{1}{2}(1+g_{00}) \qquad A_i \simeq g_{0i}$$

THE SEARCH FOR MAGNETIC GRAVITY

Electric Currents and Magnetic Fields



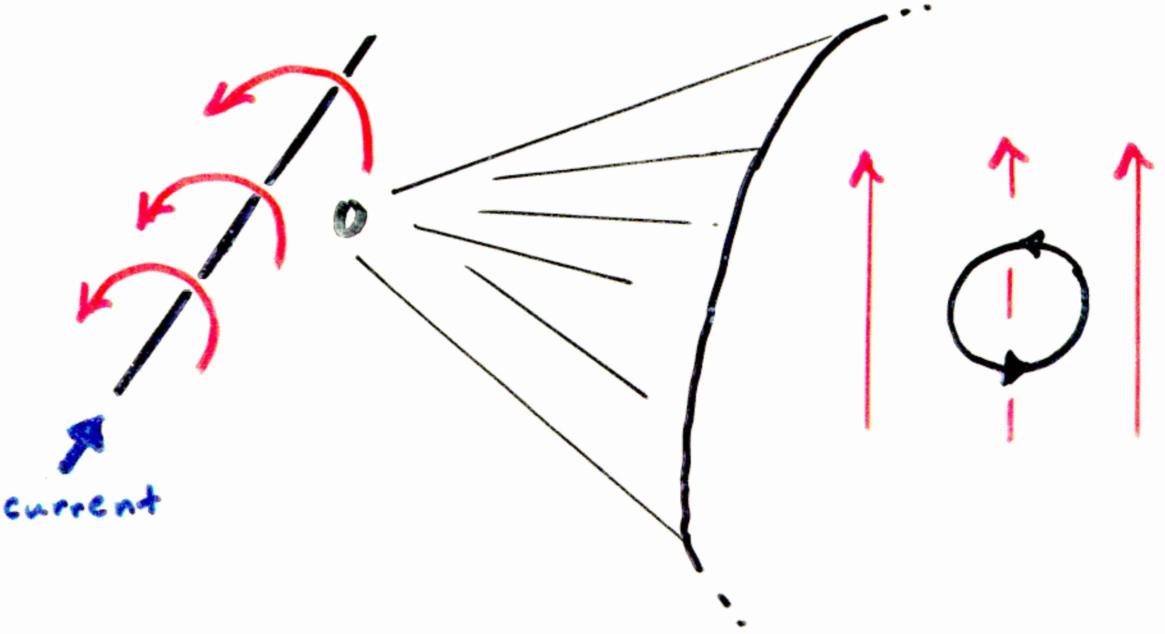
TOP VIEW



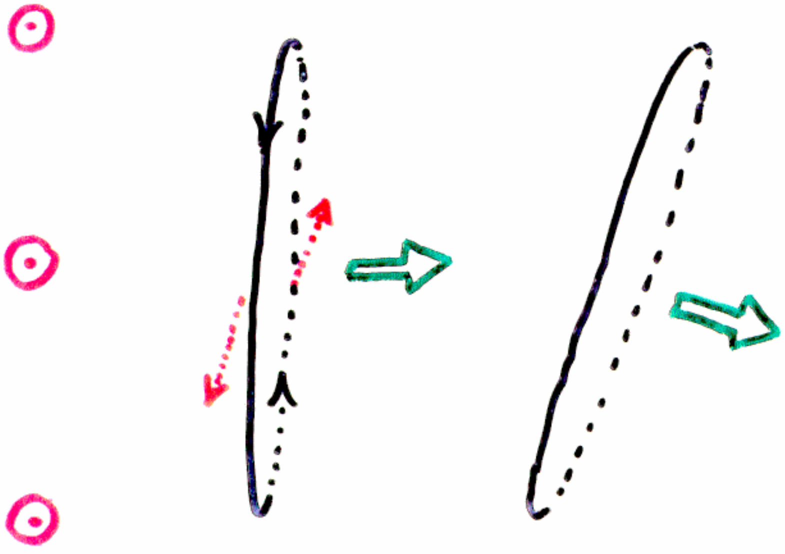
PRECESSION OF SPIN IN MAGNETIC FIELD

THE SEARCH FOR MAGNETIC GRAVITY

Mass Currents and Gravitomagnetic Fields



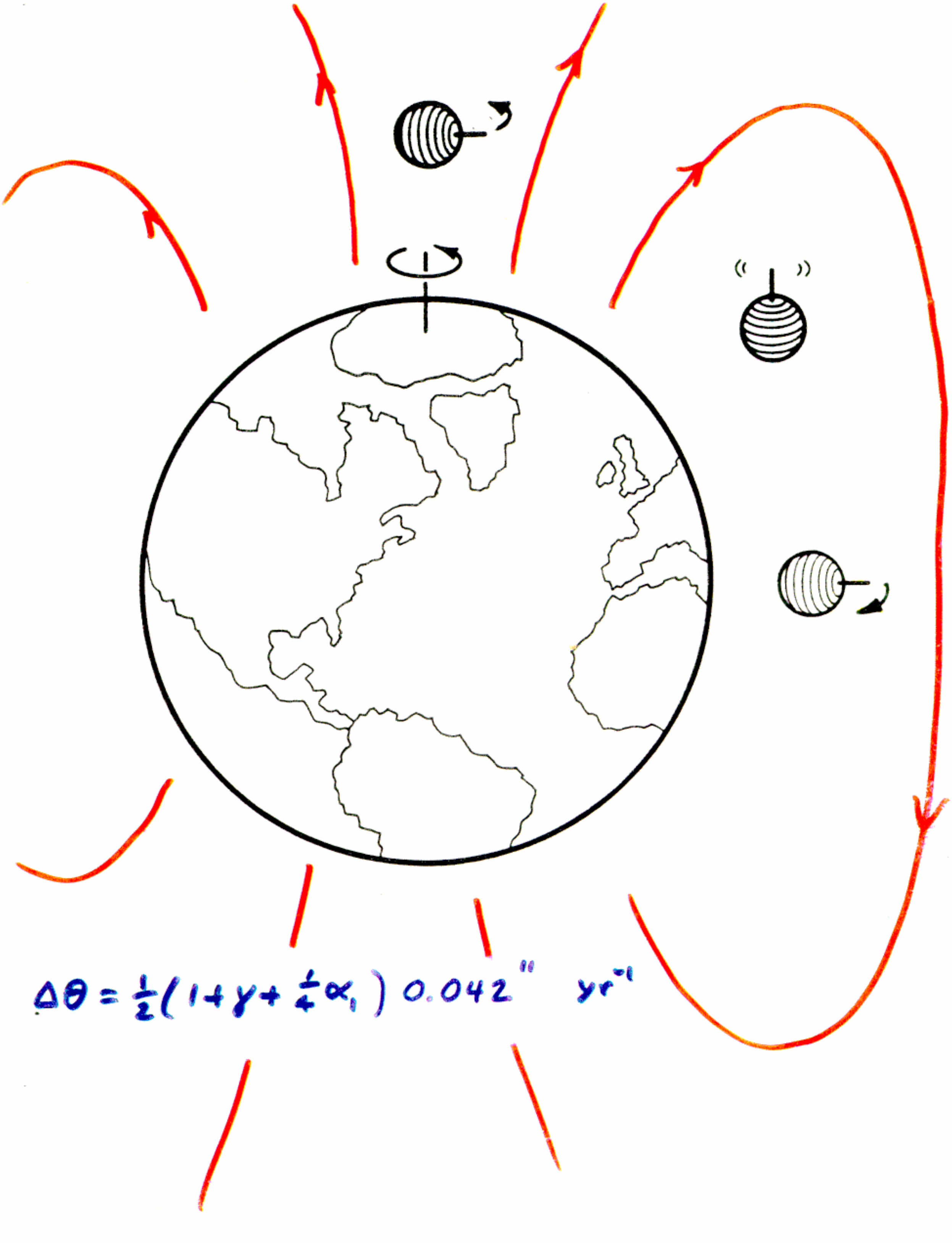
TOP VIEW



PRECESSION OF
SPIN
IN GRAVITOMAGNETIC
FIELD

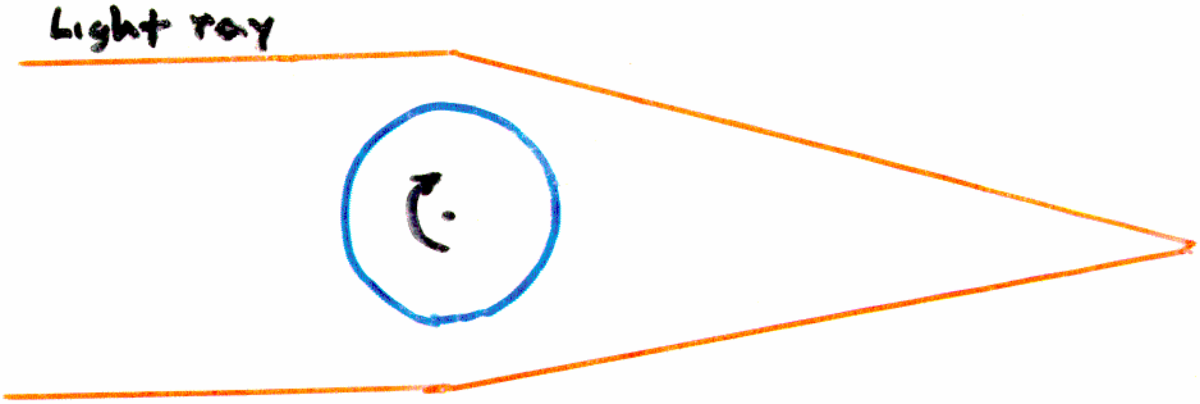
SEARCH FOR GRAVITOMAGNETISM

Precession in the Earth's Gravitomagnetic Field



$$\Delta\theta = \frac{1}{2}\left(1+\gamma+\frac{1}{4}\alpha_1\right) 0.042'' \ \mathrm{yr}^{-1}$$

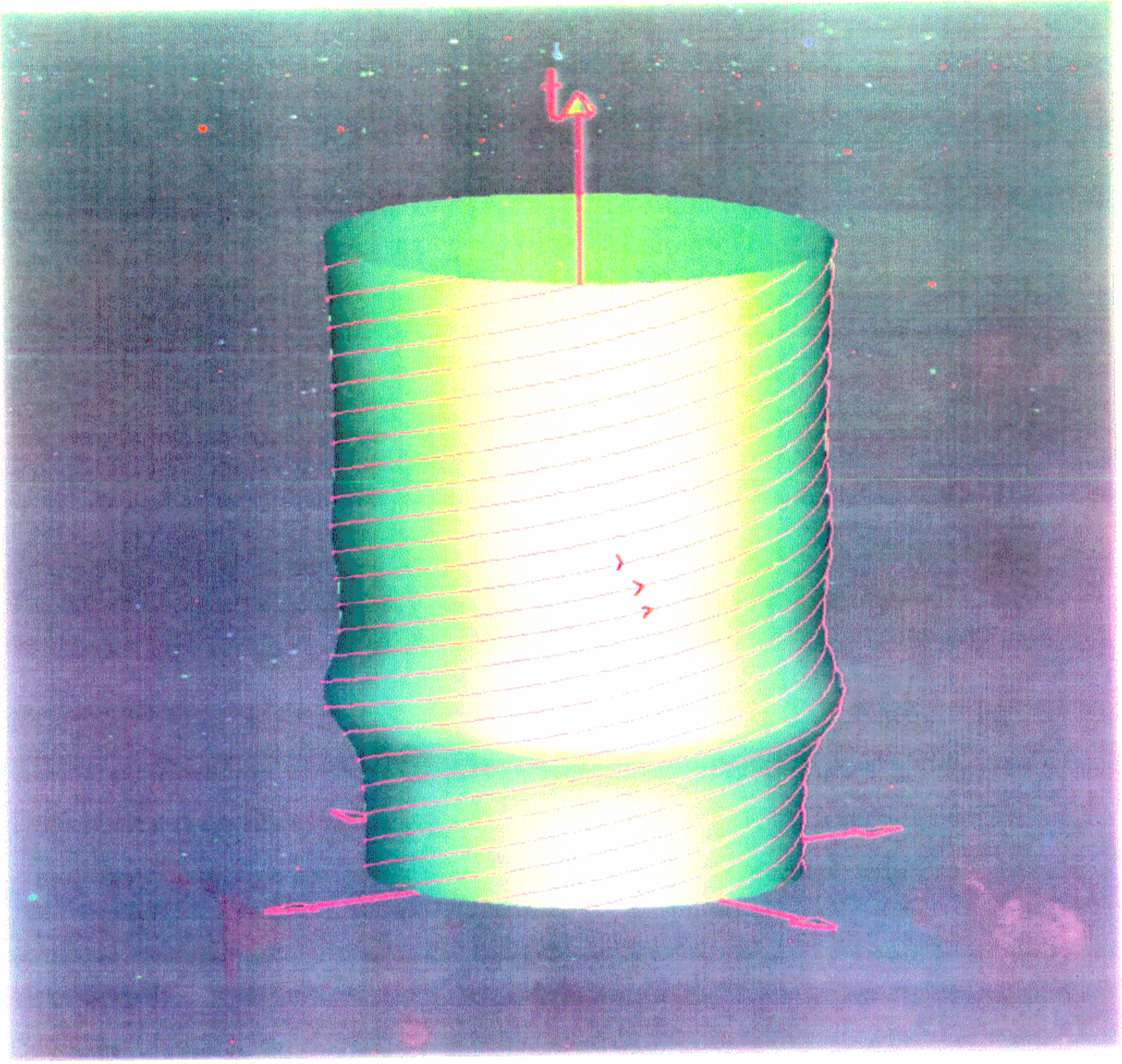
Frame-Dragging



$$\Omega_{FD} \sim \omega \left(\frac{m}{r}\right)\left(\frac{R}{r}\right)^2$$

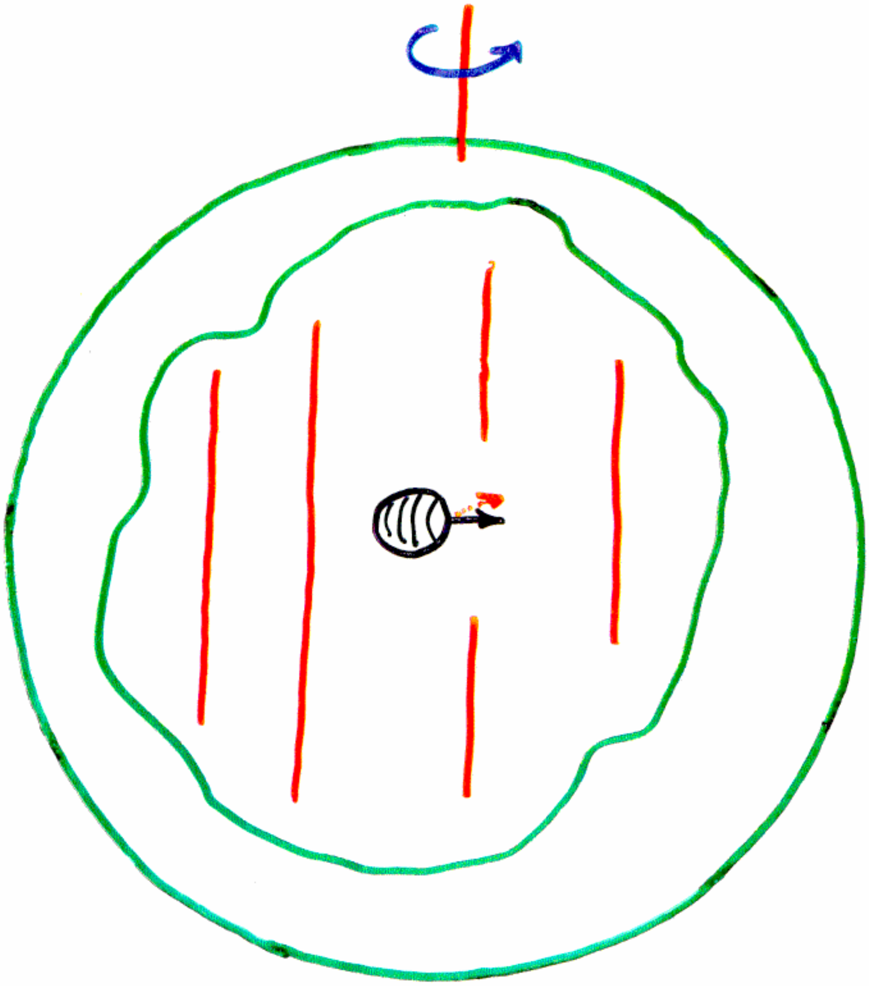
$$\ll \omega$$

Barber Pole



Frame Dragging and Mach's Principle

(Brill & Cohen 1966)

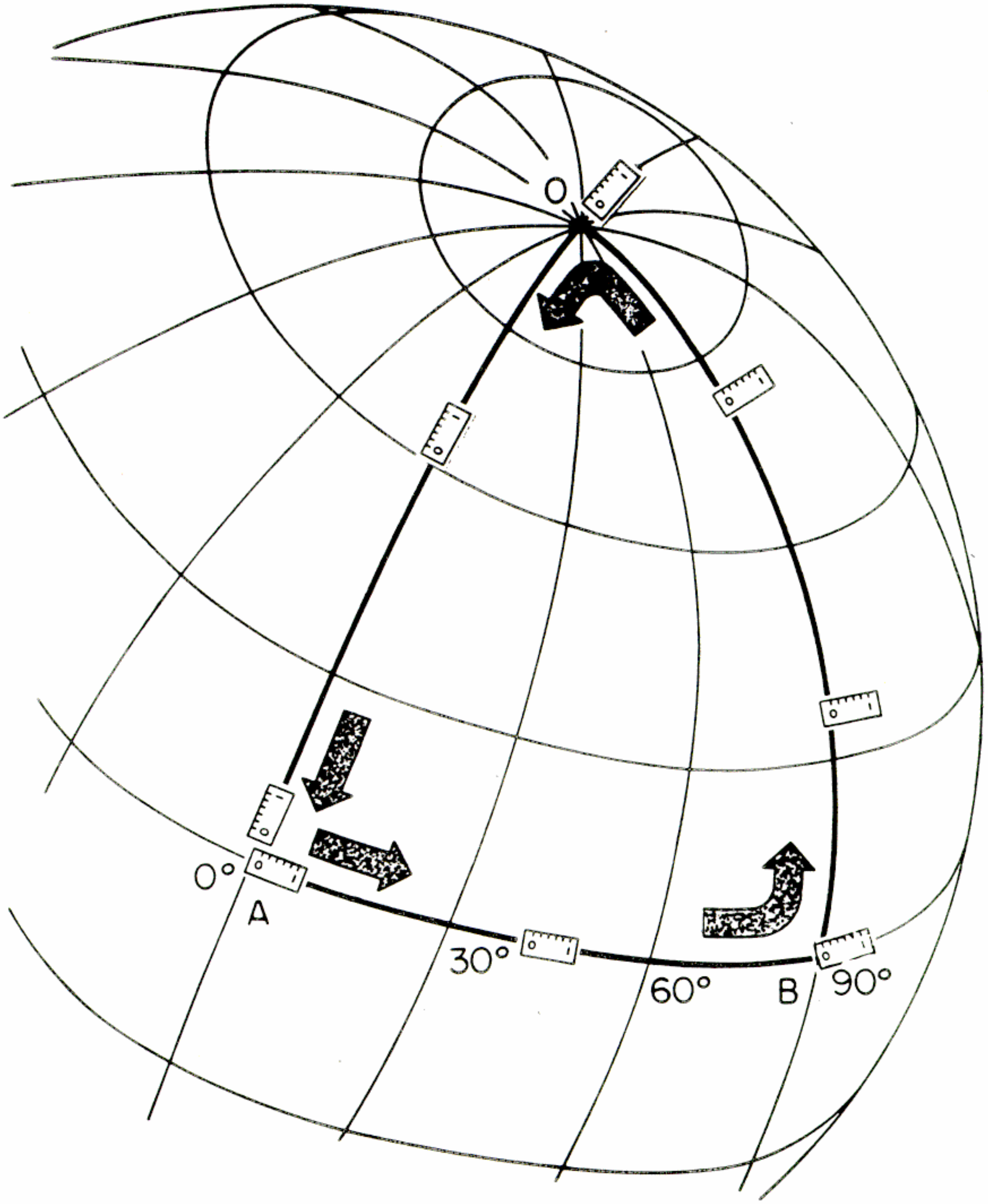


$$\Omega_{Gyro} = \omega_s \left[\frac{1}{1 + \frac{3(R-R_s)^2}{8R_s(R-R_s/2)}} \right]$$

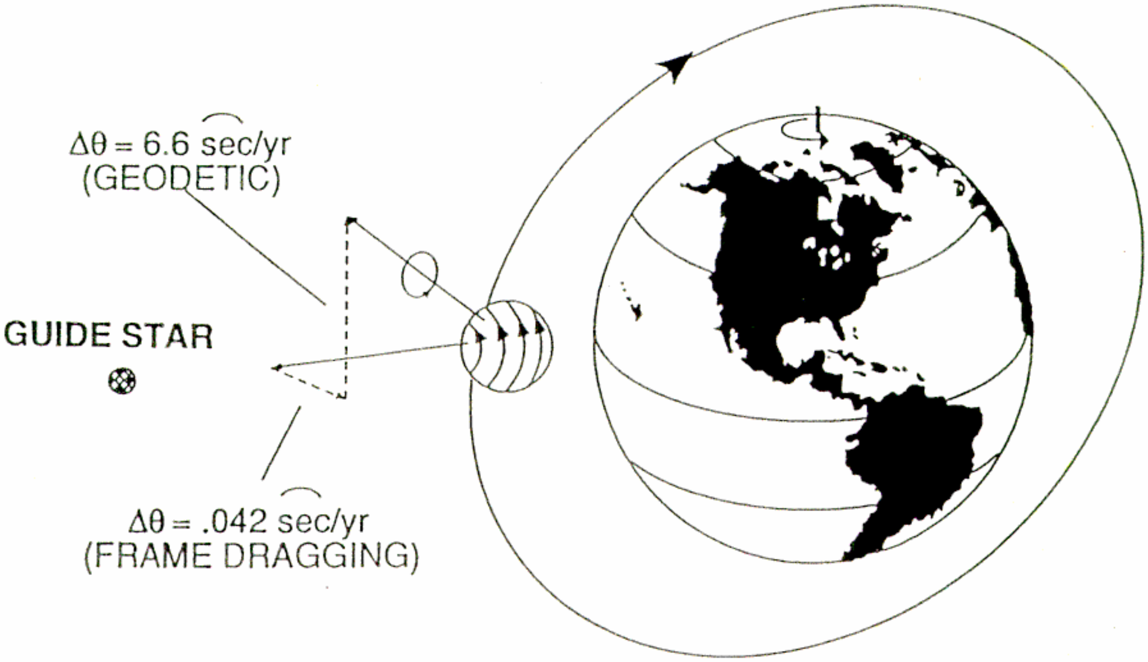
$\rightarrow \omega_s \left(\frac{8R_s}{3R}\right)$ $R \gg R_s$

$\rightarrow \omega_s$ $R \rightarrow R_s$

C. Will, SSI 1998

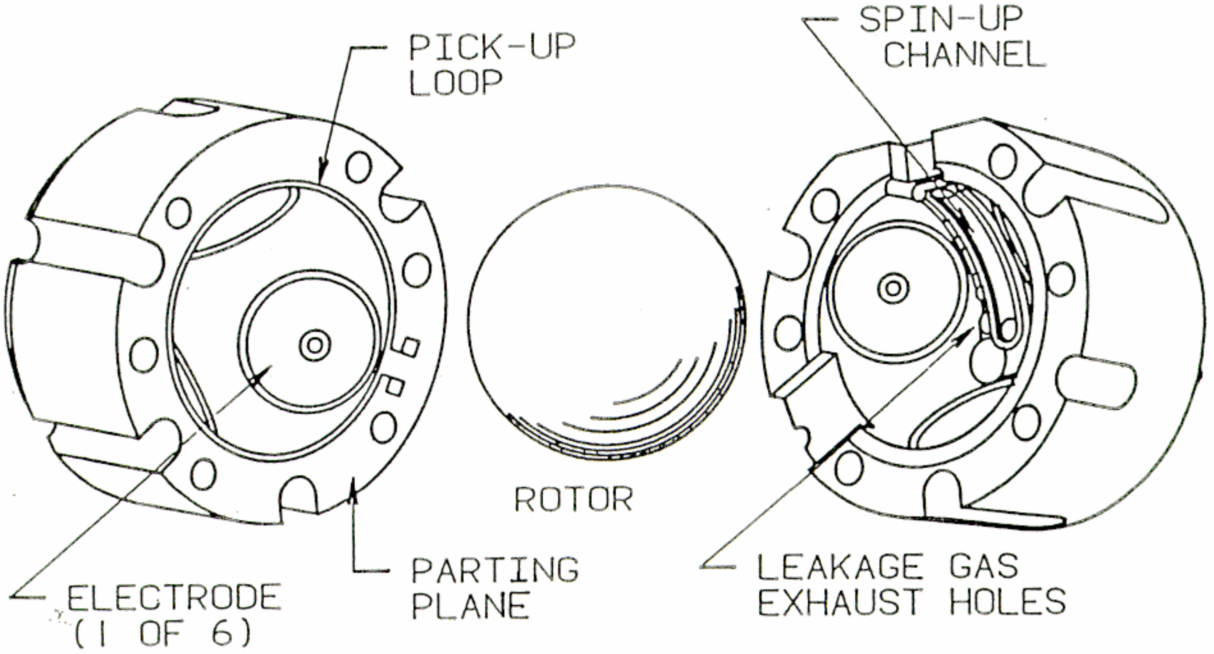


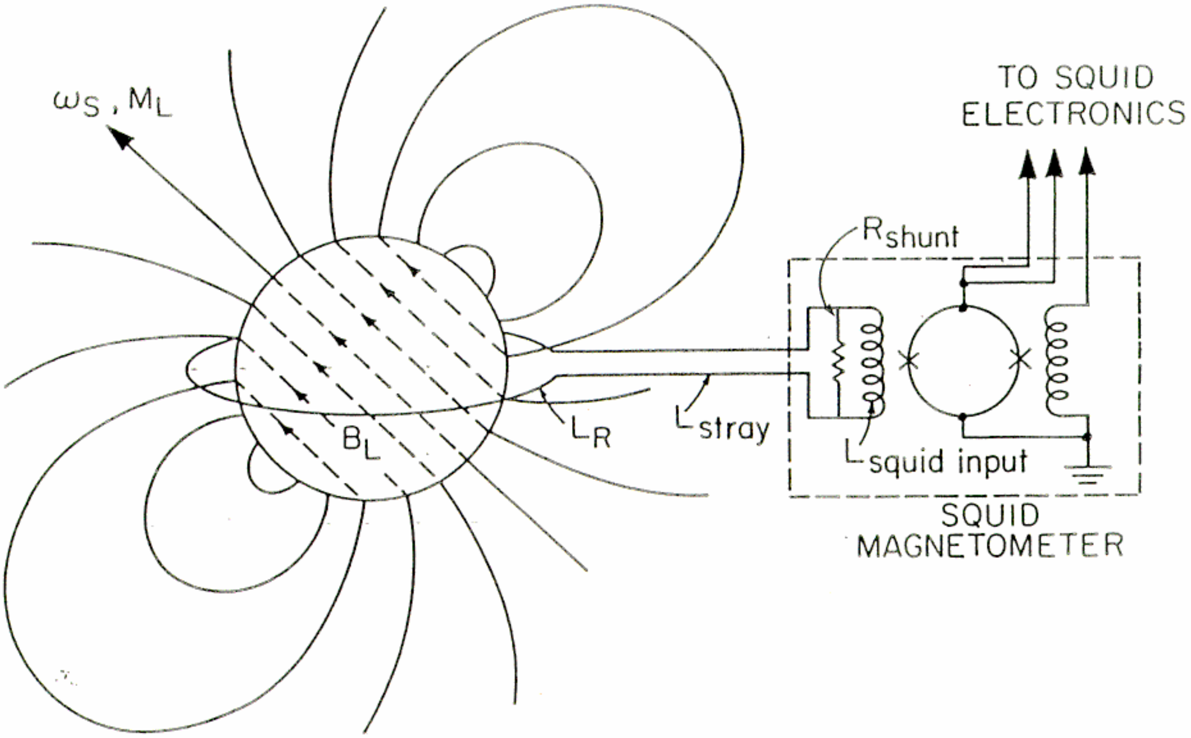
GRAVITY PROBE - B

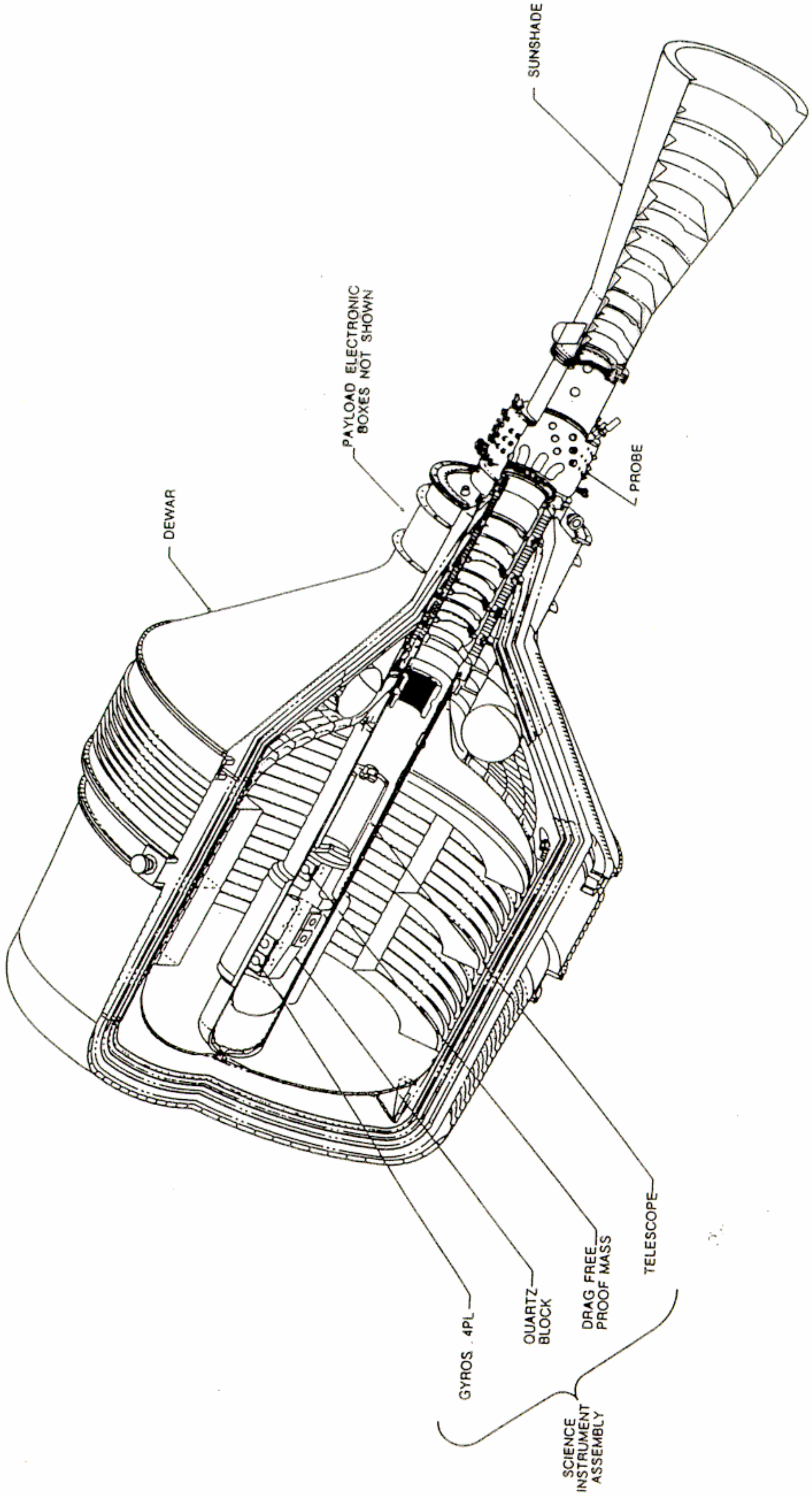


Goal : ± 0.0004 ″/yr

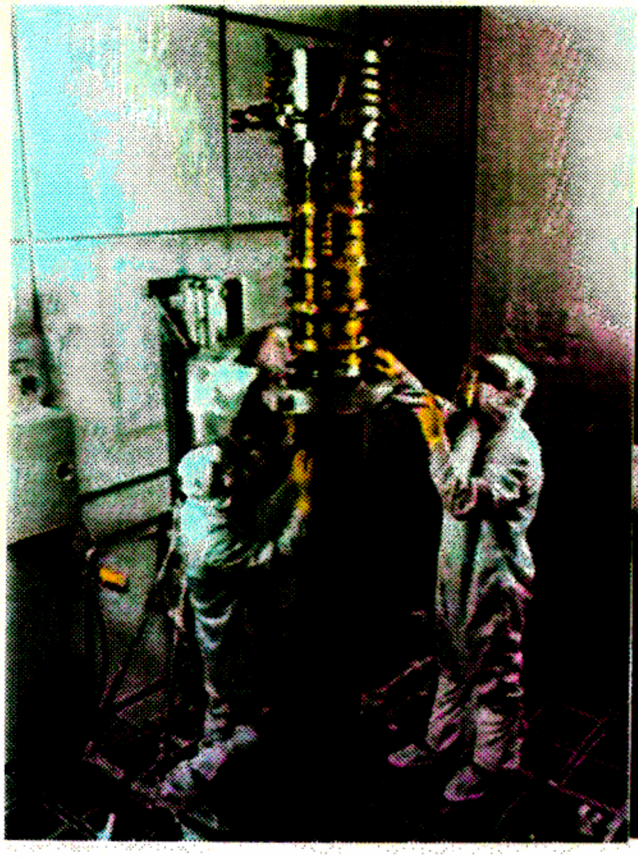
Launch : March 2000

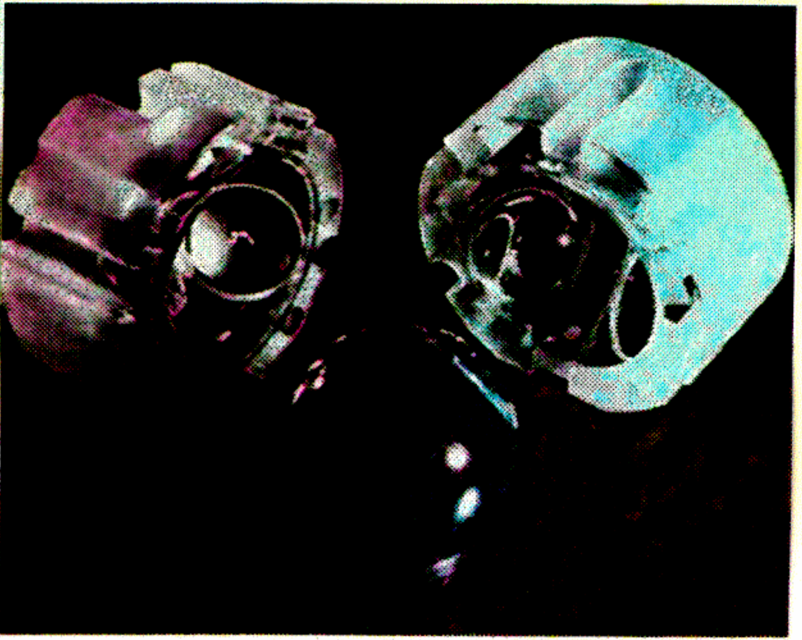

PICK-UP LOOP
SPIN-UP CHANNEL
ROTOR
ELECTRODE (1 OF 6)
PARTING PLANE
LEAKAGE GAS EXHAUST HOLES

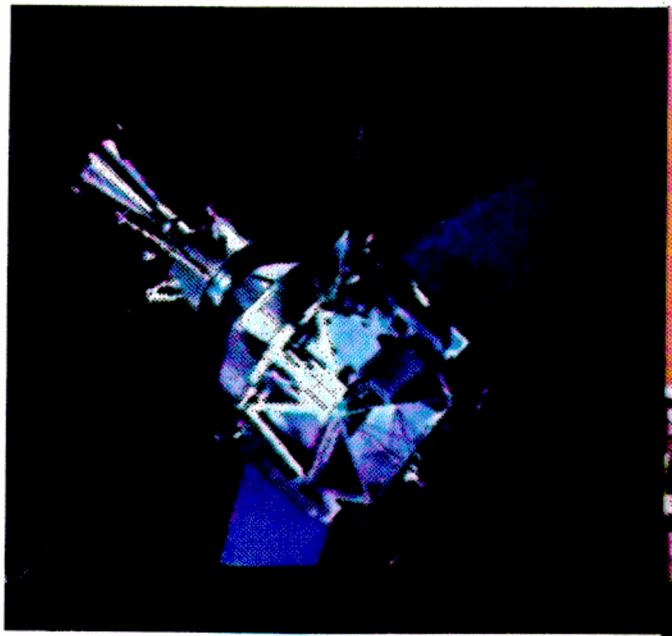

ω_S, M_L
TO SQUID ELECTRONICS
R_{shunt}
B_L
L_R
L_{stray}
$L_{squid\ input}$
SQUID MAGNETOMETER

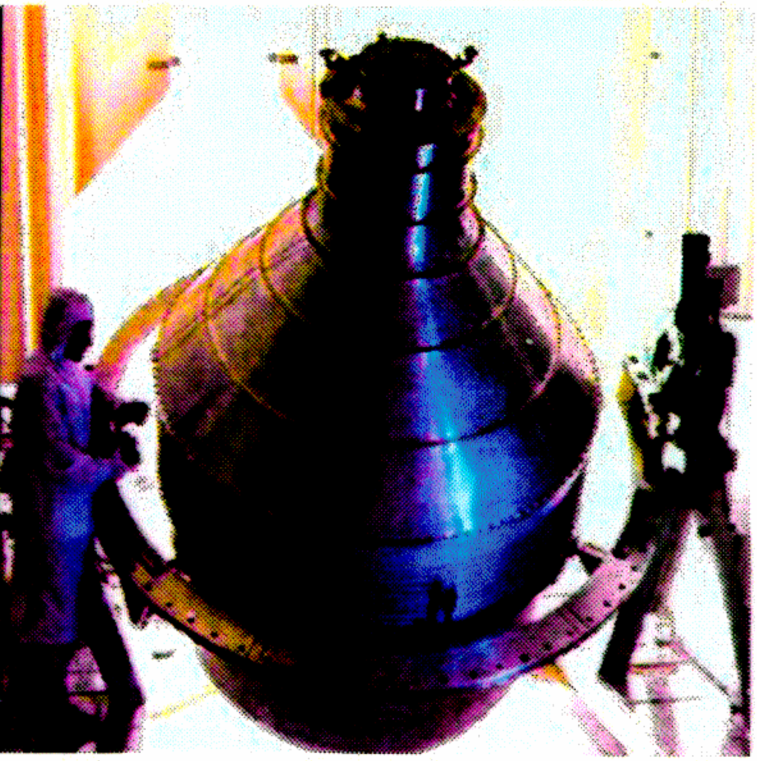

SUNSHADE
PAYLOAD ELECTRONIC BOXES NOT SHOWN
DEWAR
PROBE
TELESCOPE
DRAG FREE PROOF MASS
QUARTZ BLOCK
GYROS, 4PL
SCIENCE INSTRUMENT ASSEMBLY

GRAVITY
PROBE-B





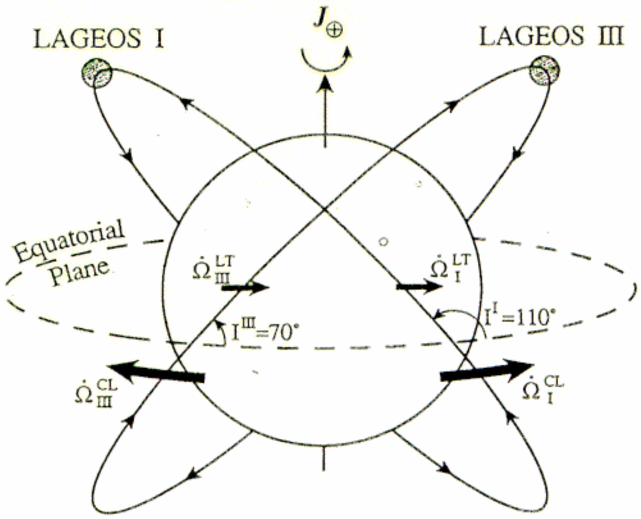




NASA
National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

LAGEOS Test of Gravitomagnetism



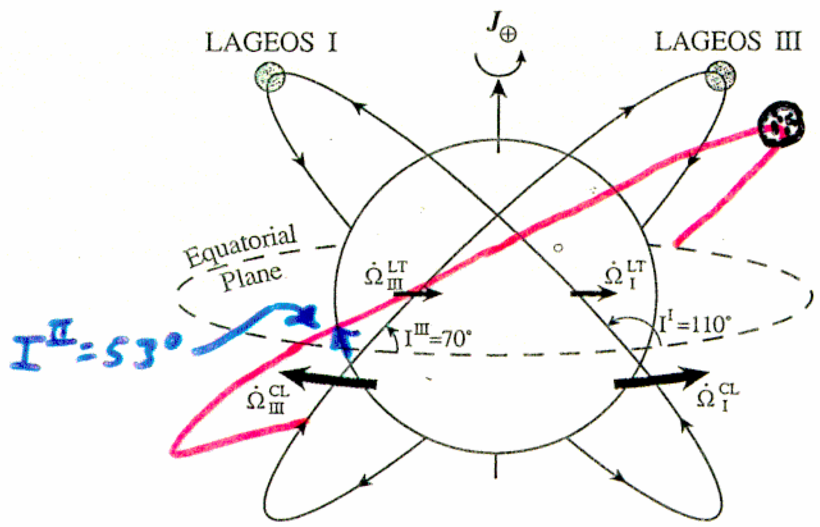
$\Omega_{LT} \sim 31$ mas/yr

$\Omega_{CL} \sim 126°/yr$

Ideal : I & III

- effects of $J_{2\ell}$ cancel
- could reach 10 %

LAGEOS Test of Gravitomagnetism



$\Omega_{LT} \sim 31$ mas/yr

$\Omega_{CL} \sim 126^\circ$/yr

Ideal: I & III

- effects of $J_{2\ell}$ cancel
- could reach 10%

Actual: I & II

- 3 unknowns: α_{LT}, δJ_2, δJ_4
- 3 measurables: Ω_{I}, Ω_{II}, $\dot\omega_{II}$
- claim: 20-25%

small e

(Ciufolini et al)

Energy-Momentum Conservation: Three viewpoints

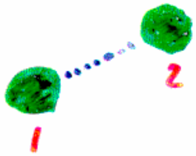
I. GENERAL

$$\begin{array}{c} \text{Conservation} \\ \text{of } E + P \end{array} \xLeftarrow[\text{Noether's Theorem}]{} \begin{array}{c} \text{Invariant Action} \\ I = \int \mathcal{L}\, d^4x \end{array}$$

II. NEWTONIAN

$$\underset{\sim}{a} = \underset{\sim}{F}/m_i \qquad \underset{\sim}{F} = m_p \underset{\sim}{g} \qquad \underset{\sim}{g} = \underset{\sim}{\nabla} \frac{Gm_a}{r}$$

$$\frac{d\underline{\underline{P}}}{dt} = F_1 + F_2 = 0$$



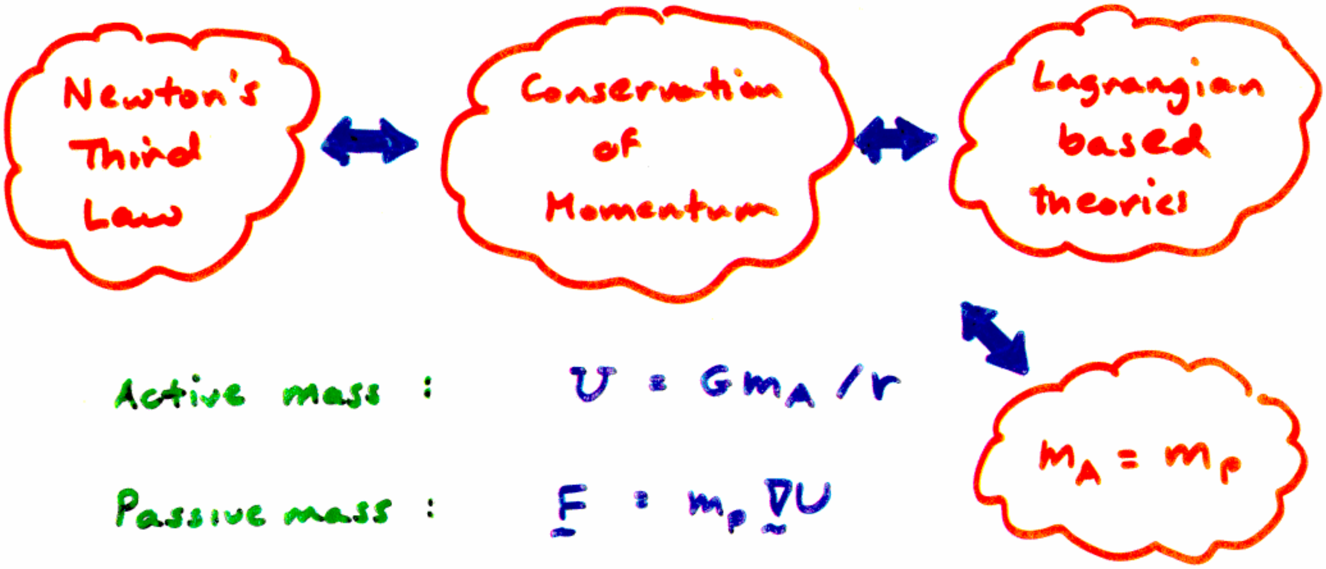
$\uparrow$ Action = Reaction

$$\boxed{m_a = m_p}$$

III. POST-NEWTONIAN

$$\boxed{\begin{array}{c} T^{\mu\nu}{}_{;\nu} = 0 \\ \Downarrow \\ \tau^{\mu\nu}{}_{,\nu} = 0 \\ \Downarrow \\ P^\mu = \int \tau^{\mu 0} d^3x = \text{const} \end{array}} \Longleftrightarrow \boxed{\begin{array}{c} \text{PPN Parameters} \\ \alpha_3 = \zeta_1 = \zeta_2 = \\ \zeta_3 = \zeta_4 = 0 \\ \text{(Will 1971)} \end{array}}$$

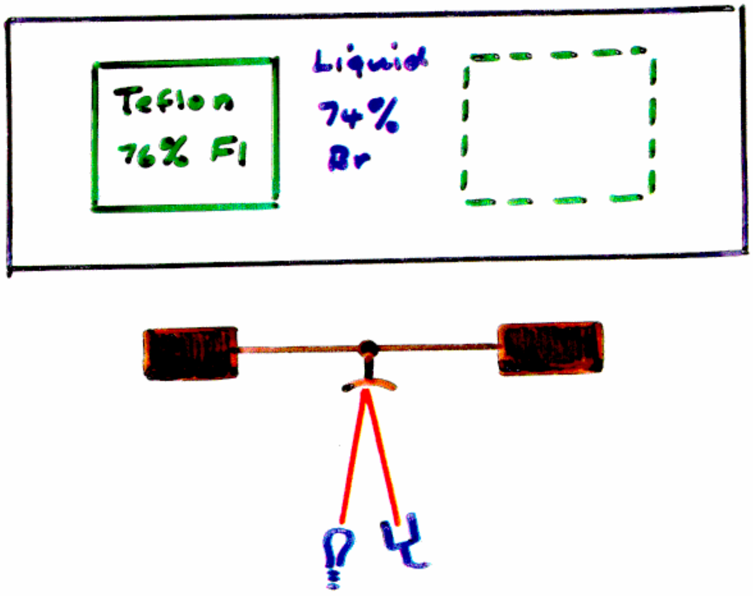
Does Action Equal Reaction?



Active mass : $U = G m_A / r$

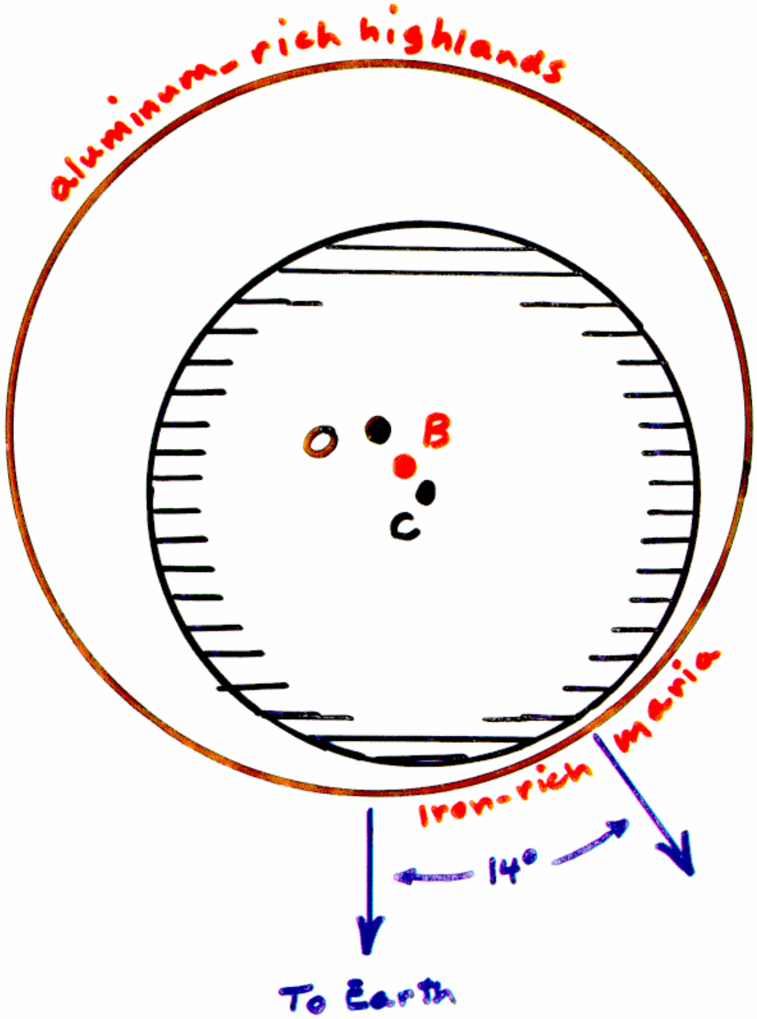
Passive mass : $\underline{F} = m_P \underline{\nabla} U$

Kreuzer's Experiment (1968)



$$\left| \frac{(m_A/m_P)_{Fl} - (m_A/m_P)_{Br}}{(m_A/m_P)_{Br}} \right| < 5 \times 10^{-5}$$

Action, Reaction and the Moon (1986) (Bartlett & van Buren)

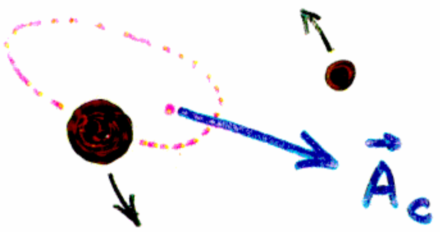


O = center of figure
B = center of mass
OB ≃ 2 km

$$\left|\frac{(m_A/m_p)_{Fe} - (m_A/m_p)_{Al}}{(m_A/m_p)_{Al}}\right| < 4\times 10^{-12}$$

$$\boxed{|\zeta_3| < 10^{-8}}$$

Is Momentum Conserved ?



$$\boxed{\begin{array}{c}\text{Conserved } E, \vec{p} \\ \Updownarrow \\ \text{Invariant Action}\end{array}}$$

$$\boldsymbol{A}_c = \zeta_2 \left\langle A_N \left(\frac{Gm}{rc^2} \right) \right\rangle \frac{\mu}{m} \frac{m_1 - m_2}{m} \, e \, \hat{\boldsymbol{n}}_p$$

$\uparrow$ 0 in GR, any action-based theory

Test using the binary pulsar

$$\dot{P}/P \simeq \vec{A}_c \cdot \vec{N}$$

$$\ddot{P}/P \simeq \dot{\vec{A}}_c \cdot \vec{N}$$

For the pulsar:

$\ddot{P}_p \simeq 10^{-25} \zeta_2 \; s^{-1}$ (use GR values for m_1, m_2)

($\delta m/m = 3.8 \pm 0.1\%$)

$|\ddot{P}_p|_{OBS} < 4 \times 10^{-30} \; s^{-1}$

$$\boxed{|\zeta_2| < 4 \times 10^{-5}}$$

(C.W. 1992)

A Complete Bound on Post-Newtonian Gravity

A (weak) Theoretical Assumption

$$\left(\begin{array}{l}\text{perfect fluid PPN}\\ \text{metric}\end{array}\right) \Longleftrightarrow \left(\begin{array}{l}\text{average of point-mass}\\ \text{PPN metric + point charges}\end{array}\right)$$

$$\underset{\substack{\uparrow\\ \text{energy}\\ \text{density}}}{\zeta_3} + 2\underset{\substack{\uparrow\\ \text{pressure}}}{\zeta_4} \underbrace{- \alpha_3 - \tfrac{2}{3}\zeta_1}_{\rho v^2} = 0$$

$$\boxed{\text{10 PPN parameters} = \text{9 constraints} + \text{1 equation}}$$

A. LURE

$$|\eta| = |4\beta - \gamma - 3 - \tfrac{10}{3}\xi - \alpha_1 + \tfrac{2}{3}\alpha_2 - \tfrac{2}{3}\zeta_1 - \tfrac{1}{3}\zeta_2| < .0015$$

$$\Rightarrow \boxed{|\zeta_1| < 3 \times 10^{-2}}$$

B. Equation from assumption

$$\boxed{|\zeta_4| < 10^{-2}}$$

C. Will, SSI 1998

CURRENT LIMITS ON THE PPN PARAMETERS

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2×10^{-3}	Viking ranging
	light deflection	3×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	6×10^{-4}	$\eta = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging PSR J2317+1439
α_2	solar spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	2×10^{-20}	pulsar $\dot{P}$ statistics
η	Nordtvedt effect[1]	10^{-3}	Lunar laser ranging
ζ_1	–	2×10^{-2}	combined PPN bounds
ζ_2	binary self-acceleration	4×10^{-5}	$\ddot{P}$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	Lunar acceleration
ζ_4	–	–	not independent

---

[1] Here $\eta = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$