# TESTING THE FOUNDATIONS OF GENERAL RELATIVITY

WUGRAV

WASHINGTON UNIVERSITY GRAVITATION GROUP

- The Einstein Equivalence Principle
  - The Weak Equivalence Principle

    *The $TH\epsilon\mu$ framework*
  - Local Lorentz Invariance

    *The $c^2$ framework*
  - Local Position Invariance
- Gravitation Experiments and Searches for New Physics
  - The Rise and Fall of the Fifth Force
  - Future Experiments

## THE EINSTEIN EQUIVALENCE PRINCIPLE

- Test bodies fall with the same acceleration
  Weak Equivalence Principle (WEP)

- In a local freely falling frame, non-gravitational physics is independent of the frame's velocity
  Local Lorentz Invariance (LLI)

- In a local freely falling frame, non-gravitational physics is independent of the frame's location
  Local Position Invariance (LPI)

EEP $\longrightarrow$ Metric Theory of Gravity

- symmetric $g_{\mu\nu} \longleftrightarrow \eta_{\mu\nu}$ locally
- 'semicolon' $\longleftrightarrow$ 'comma'

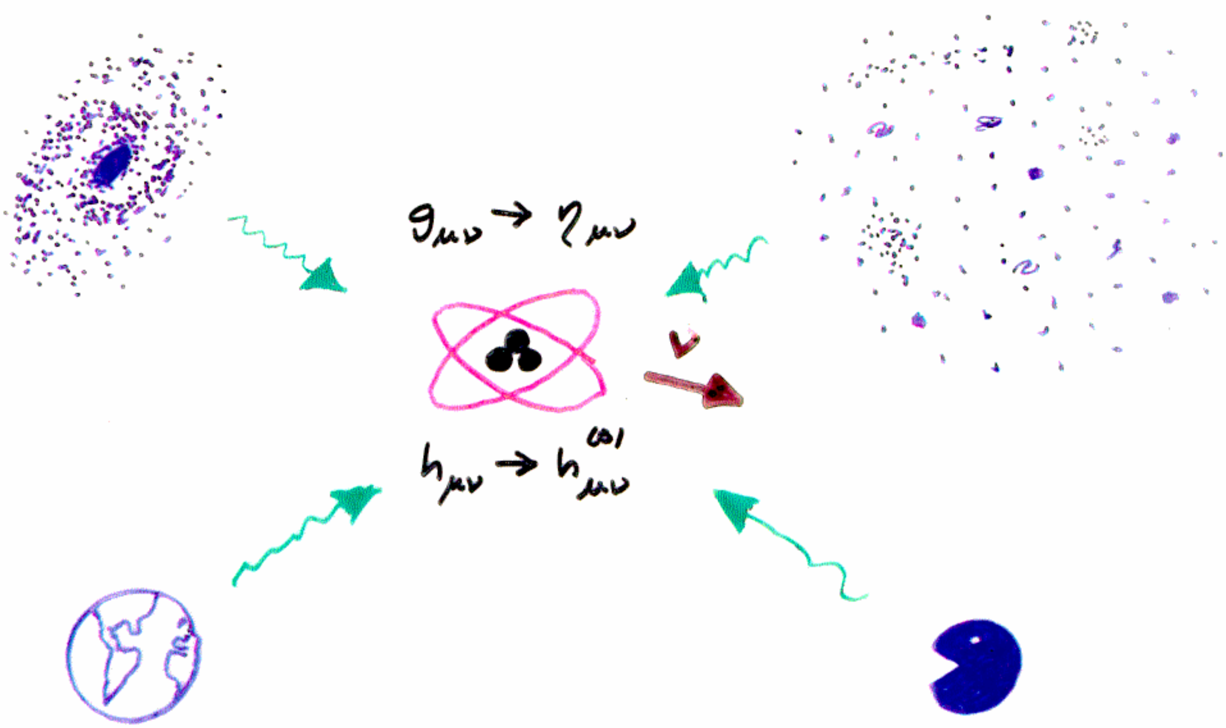
**Metric Theories:** *GR, Brans-Dicke, Rosen, ...*

**Nonmetric Theories:** *Moffat's NGT, String Theory ...*

# NON-METRIC FIELDS AND EEP

Matter action for charged particles:

$$
\begin{aligned}
I = &- \sum_a m_a \int (g_{\mu\nu} v_a^\mu v_a^\nu)^{1/2} dt \\
&- (16\pi)^{-1} \int \sqrt{-h}\, h^{\mu\alpha} h^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} d^4x \\
&+ \sum_a e_a \int A_\mu (x_a^\nu) v_a^\mu dt + I(g) + I(h)
\end{aligned}
$$



# WEP Phenomenology

$$m_I \underset{\sim}{a} = m_P \underset{\sim}{g} \quad ; \quad m_P = m_I + \sum_A \eta_A E_A$$

$$\boxed{\eta = \sum_A \eta_A \left[ \left(\frac{E_A}{m}\right)_2 - \left(\frac{E_A}{m}\right)_1 \right]}$$

## Estimate $E_A/m$

- Nuclear electrostatic (SEMF)

$$E_{ES} \approx \frac{1}{2} \sum_{ab} \frac{e_a e_b}{r_{ab}} \simeq 0.71 \frac{Z(Z-1)}{A^{1/3}} \quad \text{MeV}$$

- Nuclear Hyperfine

$$E_{HF} \simeq \frac{2\pi}{V} \mu_N^2 \left[ g_p^2 Z^2 + g_n^2 (A-Z)^2 \right] \qquad \text{(Haugan 1978)}$$

- Nuclear Magnetostatic

$$E_{MS} = \frac{1}{4} \sum_{ab} \frac{e_a e_b}{r_{ab}} \left[ v_a \cdot v_b + (v_a \cdot n_{ab})(v_b \cdot n_{ab}) \right]$$

(Haugan + Will 1977)

- Weak (P-conserving)

$$E_W = \frac{G}{2\sqrt{2} V} NZ \left[ (3\alpha^2 - 1) + 4 \sin^2\theta_W + \frac{1}{2} \frac{(N-Z)^2}{NZ} - \frac{Z}{N} \sin^2\theta_W (1 - 2\sin^2\theta_W) \right] \qquad \begin{pmatrix} \text{H + W 1976} \\ \text{Lobov 1990} \end{pmatrix}$$

# WEP Phenomenology (cont'd)

- Electronic Lamb Shift

$$E_L \simeq \frac{4\alpha}{3\pi}\frac{(Z\alpha)^4}{n^3} m_e \ln\left(\frac{1}{Z\alpha}\right) \qquad \ell = 0$$

(Alvarez & Mann 1995)

- Weak (P-violating)

$$E_W \sim \frac{G_W}{V}(G_W m_\pi^2)$$

## The "5th Force" Viewpoint

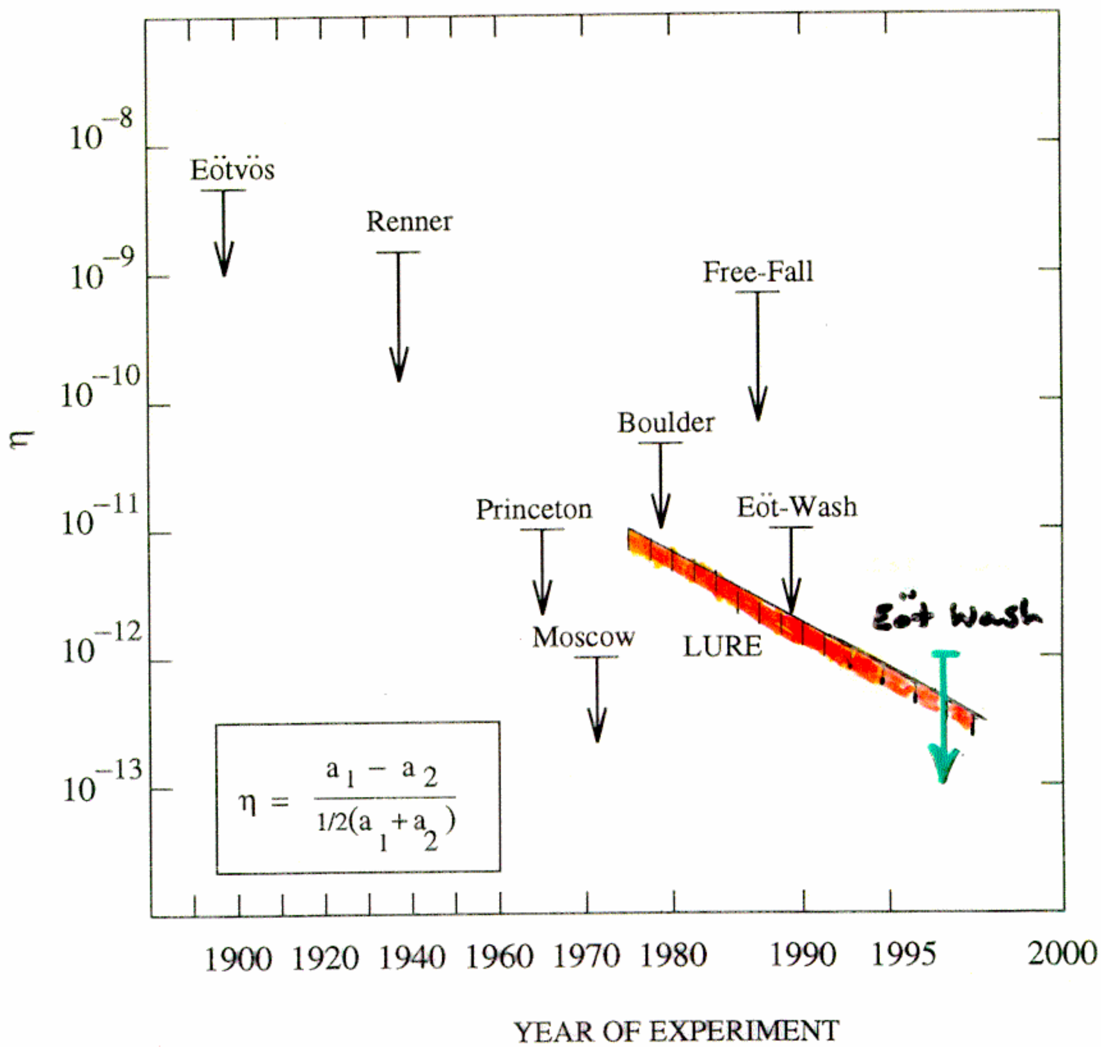
- exchange of scalar or vector bosons of mass $m_b$
- small masses
- weak coupling

$$V_{AB}(r) = \mp \frac{g^2}{4\pi}(q_5)_A (q_5)_B \frac{e^{-r/\lambda}}{r}$$

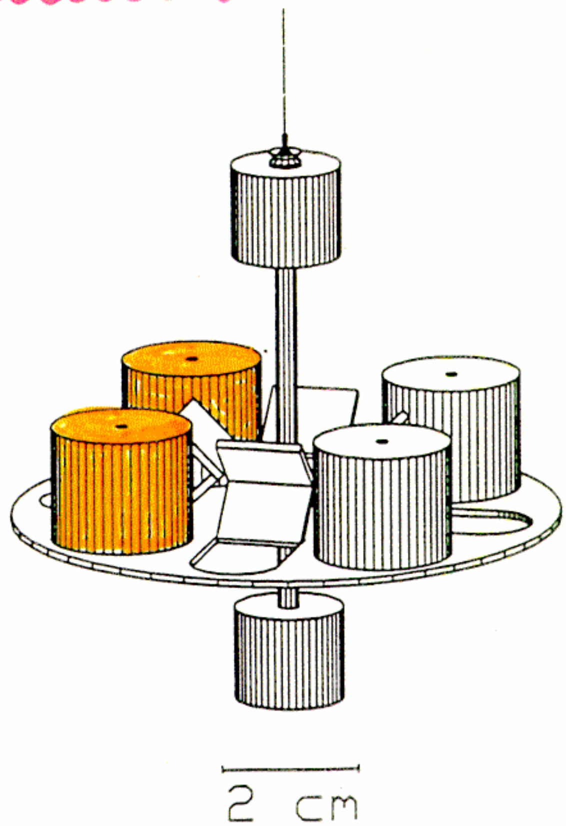
$$\lambda = \hbar / m_b c$$

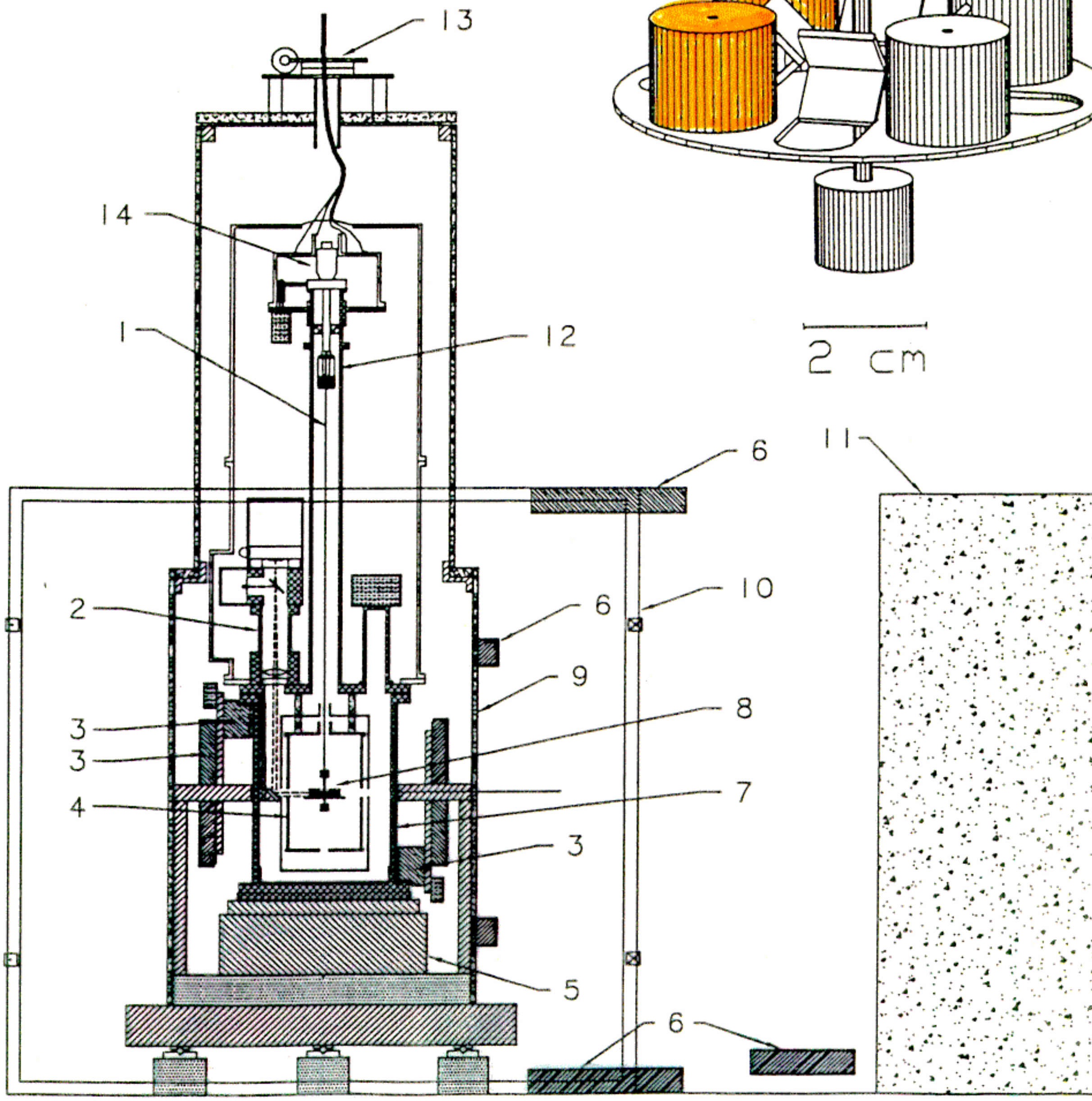
$$q_5 = B\cos\theta_5 + L\sin\theta_5$$

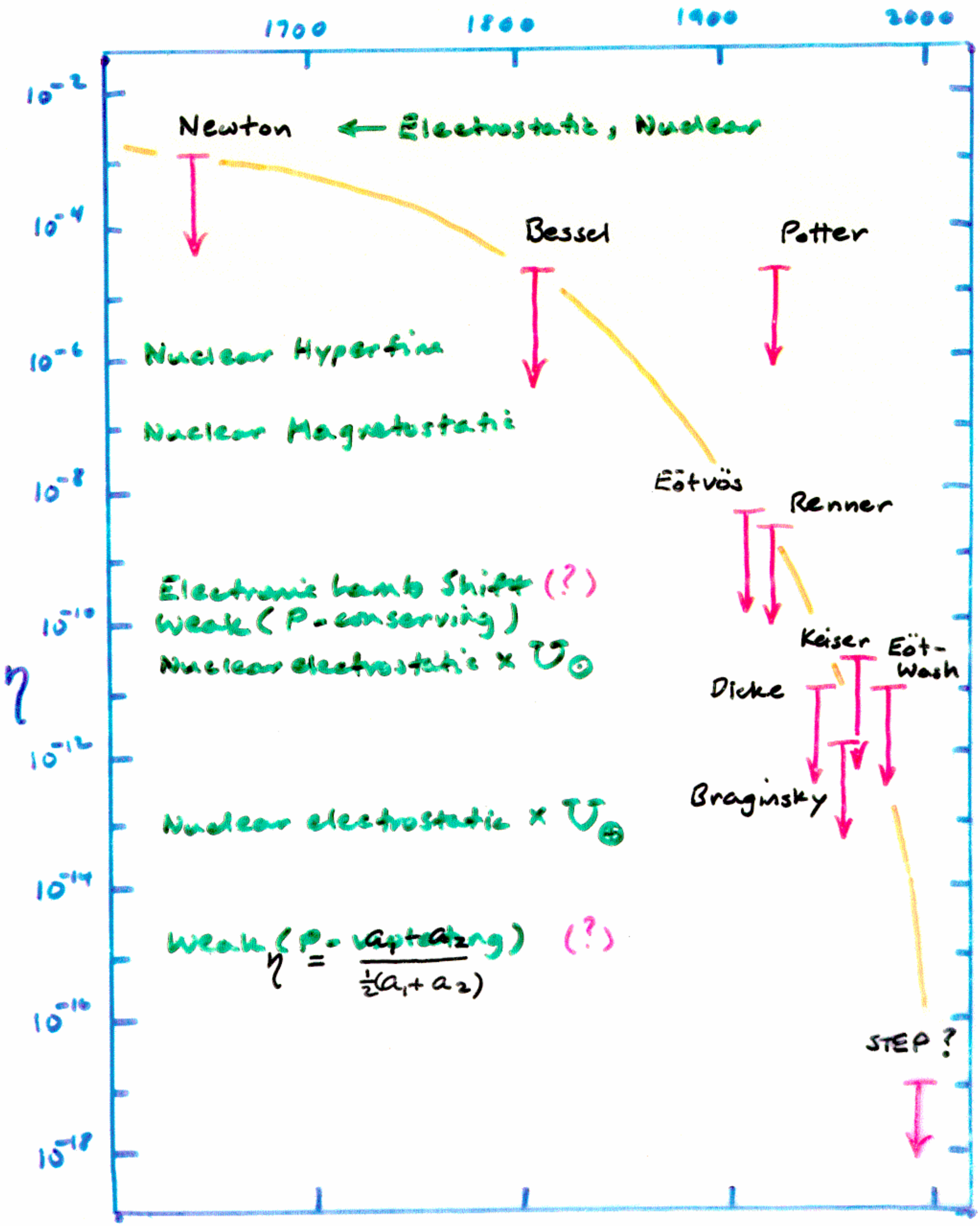
# TESTS OF THE WEAK EQUIVALENCE PRINCIPLE



# The Eöt-Wash Experiments




1700
1800
1900
2000
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^{-12}$
$10^{-14}$
$10^{-16}$
$10^{-18}$
$\eta$
Newton
← Electrostatic, Nuclear
Bessel
Potter
Nuclear Hyperfine
Nuclear Magnetostatic
Eötvös
Renner
Electronic Lamb Shift (?)
Weak (P-conserving)
Nuclear electrostatic × $v_{\odot}$
Keiser
Eöt-Wash
Dicke
Braginsky
Nuclear electrostatic × $v_{\oplus}$
Weak (P-violating) (?)
$\eta = \frac{a_1 - a_2}{\frac{1}{2}(a_1 + a_2)}$
STEP ?

# THE $TH\epsilon\mu$ FORMALISM

## Assumptions

(1) Charged point masses

(2) Static, spherically symmetric gravity field

(3) Action principle

$$\boxed{I = -\sum_a m_a \int (T - H v_a^2)^{1/2}\, dt + \sum_a e_a \int A_\mu(x_a)\, v_a^\mu\, dt + \frac{1}{8\pi}\int (\epsilon E^2 - \mu^{-1} B^2)\, d^3x\, dt}$$

$$v_a^\mu = dx_a^\mu / dt$$

$$\mathbf{E} = \nabla A_0 - \dot{\mathbf{A}}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$T = T(r), \text{ etc}$$

(Lightman & Lee 1973)

# THE$\epsilon\mu$ with Minimally Coupled Metric

$$I = -\sum_a m_a \int (g_{\mu\nu} v_a^\mu v_a^\nu)^{1/2} dt + \sum_a e_a \int A_\mu(x_a) v_a^\mu dt$$

$$- \frac{1}{16\pi} \int \sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}\, d^3x\, dt$$

$$g_{\mu\nu} = \begin{pmatrix} -T(r) & & & \\ & H(r) & & \\ & & H(r) & \\ & & & H(r) \end{pmatrix}$$

$$I = -\sum_a m_a \int (T - H v_a^2)^{1/2} dt \quad + \quad \cdots$$

$$+ \frac{1}{8\pi} \int d^3x\, dt \left( \epsilon E^2 - \mu^{-1} B^2 \qquad \right)$$

$$\epsilon = \left(\frac{H}{T}\right)^{1/2}$$

$$\mu = \left(\frac{H}{T}\right)^{1/2}$$

# THεμ Dynamics

## Lorentz and Maxwell Equations

$$(d/dt)(HW^{-1}\mathbf{v}_a) + \frac{1}{2}W^{-1}\nabla(T - Hv_a^2) = \mathbf{a}_L(\mathbf{x}_a) ,$$

$$\mathbf{a}_L(\mathbf{x}_a) = (e_a/m_{0a})\{\nabla A_0(\mathbf{x}_a) + \nabla[\mathbf{v}_a \cdot \mathbf{A}(\mathbf{x}_a)] - d\mathbf{A}(\mathbf{x}_a)/dt\} ,$$

$$\nabla \cdot (\varepsilon \mathbf{E}) = 4\pi\rho ,$$

$$\nabla \times (\mu^{-1}\mathbf{B}) = 4\pi\mathbf{J} + \partial(\varepsilon\mathbf{E})/\partial t ,$$

## Approximations

$$v^2 \sim e^2/m_0 r \ll 1 , \qquad T(\Phi) = T_0 + T_0' \, \mathbf{g}_0 \cdot \mathbf{x} + O(\mathbf{g}_0 \cdot \mathbf{x})^2 ,$$

$\Phi = M/r$ , $\nabla\Phi = \mathbf{g}_0$

## Equation of Motion

$$d\mathbf{v}_a/dt = -\frac{1}{2}T_0' H_0^{-1}\mathbf{g}_0 + \frac{1}{2}H_0' H_0^{-1}\mathbf{g}_0 v_a^2$$

$$+ (T_0' T_0^{-1} - H_0' H_0^{-1})\mathbf{g}_0 \cdot \mathbf{v}_a \mathbf{v}_a + T_0^{1/2} H_0^{-1} \mathbf{a}_L(\mathbf{x}_a) .$$

## Vector Potential

$$A_0 = -\phi + \frac{1}{2}\varepsilon_0\mu_0\chi_{,00} + \varepsilon_0'\varepsilon_0^{-1}(\mathbf{g}_0 \cdot \mathbf{x}\phi + \frac{1}{2}\mathbf{g}_0 \cdot \nabla\chi) ,$$

$$\mathbf{A} = \mathbf{A}^{(0)} + O(g_0) ,$$

where

$$\phi \equiv \varepsilon_0^{-1}\sum_a e_a |\mathbf{x} - \mathbf{x}_a|^{-1} , \qquad \chi \equiv -\varepsilon_0^{-1}\sum_a e_a |\mathbf{x} - \mathbf{x}_a| ,$$

$$\mathbf{A}^{(0)} \equiv \mu_0 \sum_a e_a \mathbf{v}_a |\mathbf{x} - \mathbf{x}_a|^{-1} .$$

# The Weak Equivalence Principle

## Center of Mass

$$\underset{\sim}{X} \equiv \frac{1}{m} \sum_a m_a \underset{\sim}{x}_a \qquad m \equiv \sum_a m_a$$

## Virial Relations

$$\left\langle \sum_a m_a v_a^i v_a^j + \frac{1}{4} T_o^{1/2} H_o^{-1} \epsilon_o^{-1} \sum_{ab} e_a e_b \frac{x_{ab}^i x_{ab}^j}{r_{ab}^3} \right\rangle = 0$$

## CM Acceleration

$$\boxed{\ddot{\underset{\sim}{X}} = \underset{\sim}{g}\left[1 - \frac{E_e}{mc^2}\left(\Gamma_o - \frac{4}{3}\Upsilon_o\right)\right]}$$

$$E_e = \frac{1}{2} \sum_{ab} e_a e_b / r_{ab}$$

$$c \equiv (T/H)_o^{1/2}$$

$$\Gamma_o \equiv -c^2 \frac{\partial}{\partial \Phi} \ln\left[\epsilon\left(\frac{T}{H}\right)^{1/2}\right]_o$$

$$\Upsilon_o \equiv 1 - \left(\frac{T}{H}\epsilon\mu\right)_o$$

# The $c^2$-Formalism

Haugan + Will 1987
Gabriel and Haugan 1990

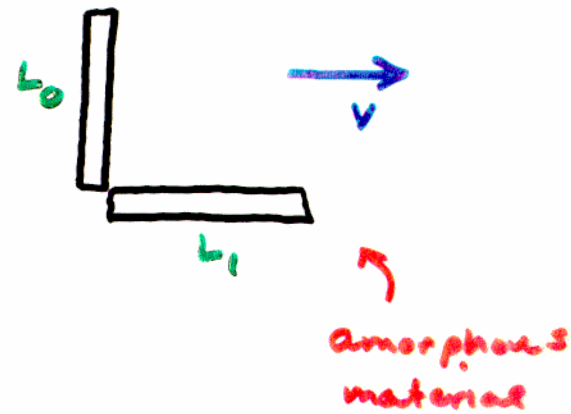
$$\boxed{I = \sum_i \int \left[ -m_i (1-v_i^2)^{1/2} + e_i A_\mu v_i^\mu \right] dt + \frac{1}{8\pi} \int \left[ E^2 - c^2 B^2 \right] d^3x \, dt}$$

- version of $TH\epsilon\mu$ $\quad c^2 = (H/T\epsilon\mu)$
- valid in universal rest frame (3K)
- in universe frame, speed of light is $c$ (isotropic)
- $c = 1 \iff$ LLI
- in $g$-$h$ model, $c \leftrightarrow h_{00}^{(0)}/h_{ii}^{(0)}$

# Features of the $c^2$-Formalism

- **Length Contraction**

$$L_1 \simeq L_0\left(1-\tfrac{1}{2}V^2\right)$$




**Consequence:**

Michelson-Morley expt:

$$\Delta t_{\parallel} - \Delta t_{\perp} = \frac{L_0 V^2}{c}\left(\frac{1}{c^2}-1\right)$$

## Non-Universality of Energy & Momentum

For a electromagnetically bound body moving at $\underline{V}$:

$$E = M_R c^2 + \frac{1}{2} M_R V^2 + \frac{1}{2} \delta M_I^{ab} V^a V^b + \ldots$$

$$P^a = M_R V^a + \delta M_I^{ab} V^b$$

$$\delta M_I^{ab} = \left(\frac{1}{c^2} - 1\right)\left(\frac{1}{2}\Omega^{ab} + \frac{2}{3}\Omega\,\delta^{ab}\right)$$

$$\Omega^{ab} = \left\langle \frac{1}{2} \sum_{ij} \frac{e_i e_j}{r_{ij}} \left(\hat{n}_{ij}^a n_{ij}^b - \frac{1}{3}\delta^{ab}\right)\right\rangle$$

$$\Omega = \left\langle \frac{1}{2} \sum_{ij} \frac{e_i e_j}{r_{ij}} \right\rangle = 2E_B$$

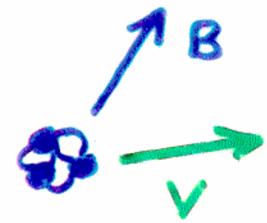
$$M_R = M_0 - E_B / c^2$$

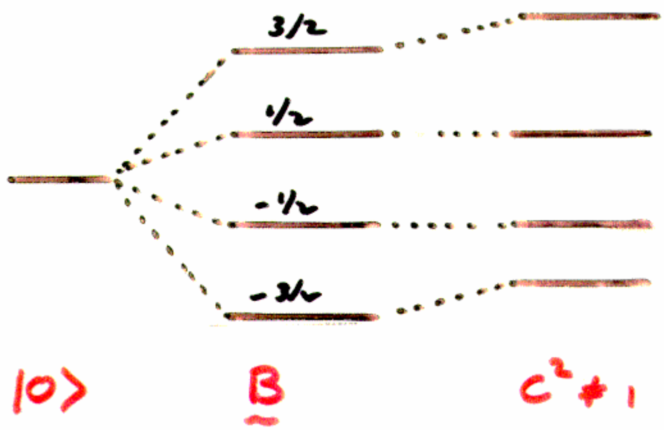
**Consequences:**

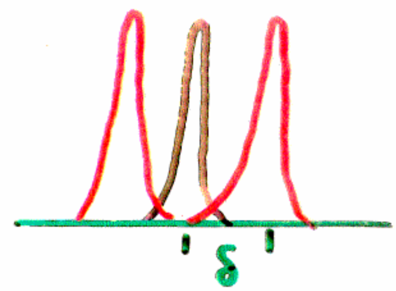
- time dilation ($h\nu = \Delta E$) depends on nature of clock
- energy level shifts depending on orientation

# Energy Anisotropy and the $c^2$-Formalism

$Li^7$ : $J = 3/2$ , valence proton



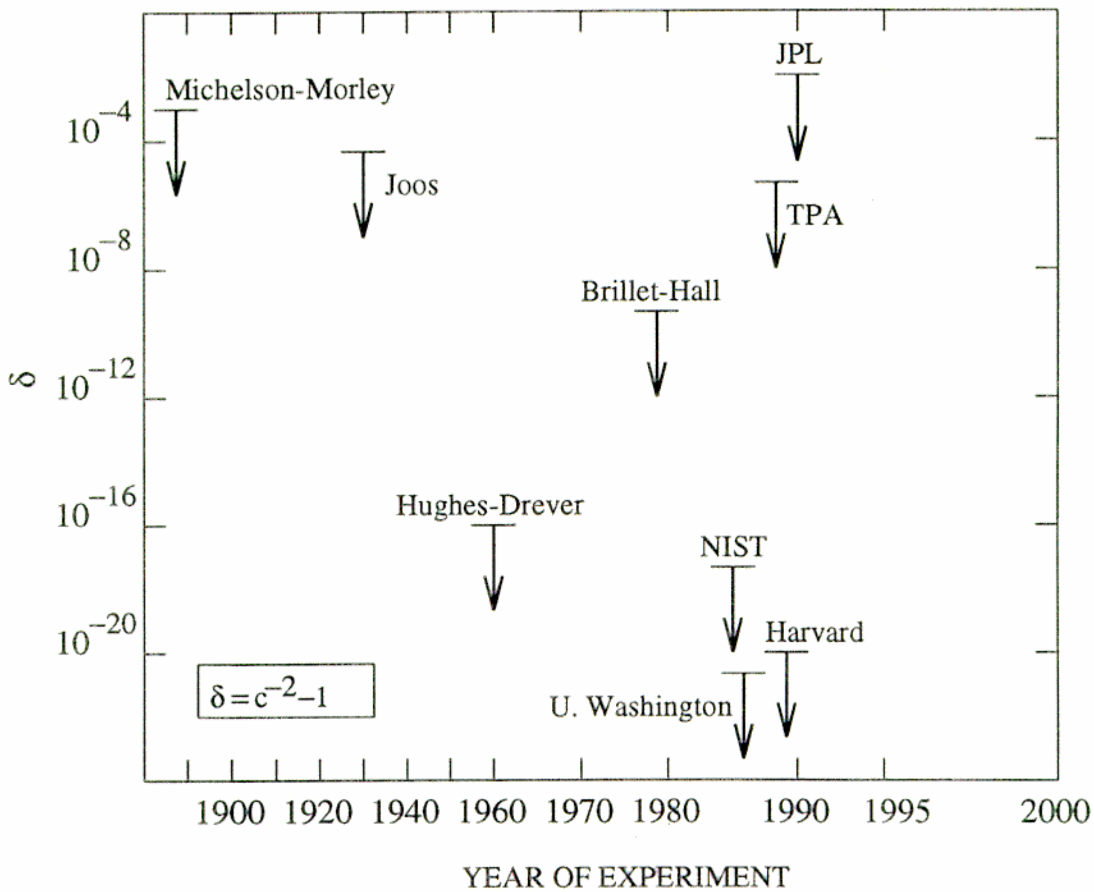




$$\delta = \frac{2}{15}\left(\frac{E_B}{c^2}\right)\left(\frac{1}{c^2} - 1\right)\left(2v_{\parallel}^2 - v_{\perp}^2\right)$$

Hughes-Drever Expt.

# TESTS OF

# LOCAL LORENTZ INVARIANCE



# Local Position Invariance (LPI)

Implication : local non-gravitational constants are constant in space and time

## Gravitational Redshift Experiments

In local freely falling frame, relative to some standard clock, suppose

$$\tau = \tau_0 ( 1 + \alpha U )$$

$\alpha = 0 \Rightarrow$ LPI valid

At emission :

$$\tau = \tau_0 (1 + \alpha U_e)$$

At reception

$$\tau = \tau_0 (1 + \alpha U_R)$$

But two frames have relative velocity

$$v = g \Delta t = U_e - U_R$$

Thus

$$\boxed{z = \frac{\nu_r - \nu_e}{\nu_r} = (1+\alpha) \Delta U}$$

From the THεμ framework:

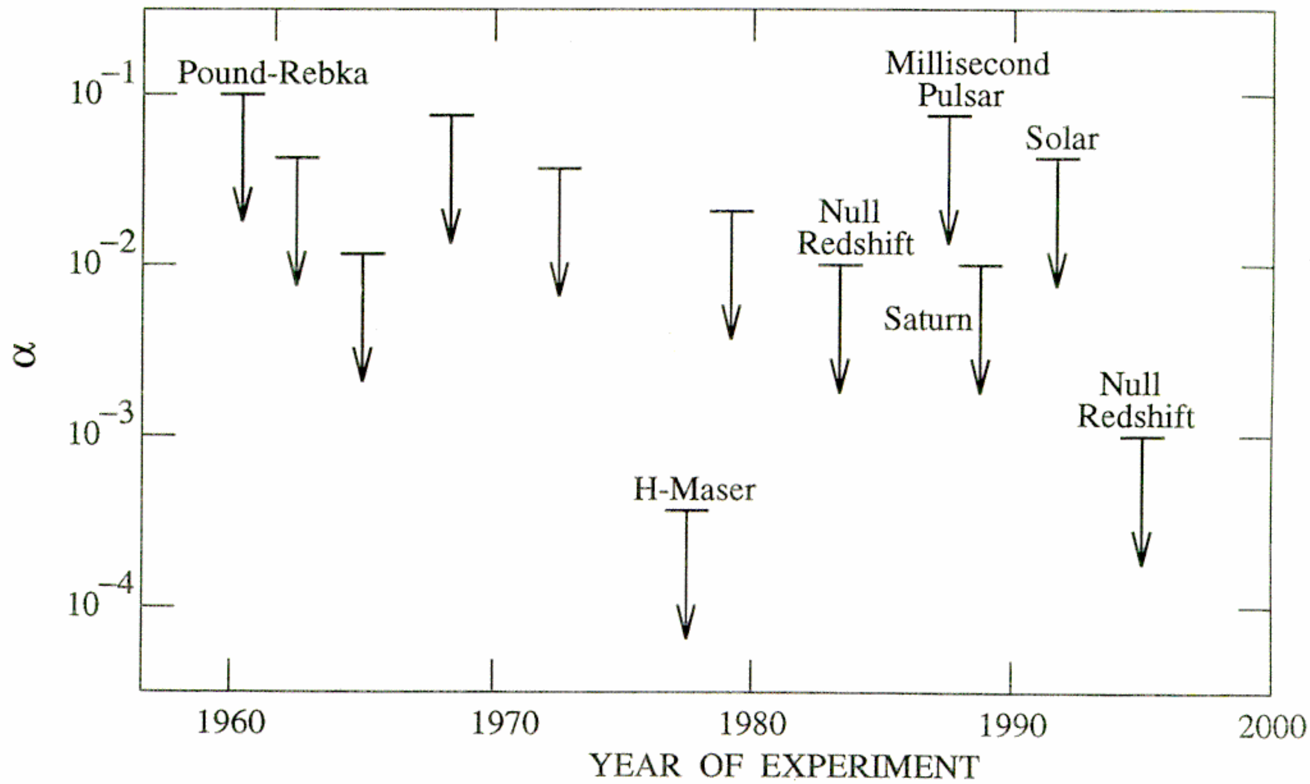
$$\alpha^{ES} = -2\Gamma_0$$

$$\alpha^{HF} = -3\Gamma_0 + \Lambda_0$$

$$\alpha^{SCSO} = -\frac{3}{2}\Gamma_0 - \frac{1}{2}\Lambda_0$$

# TESTS OF

# LOCAL POSITION INVARIANCE



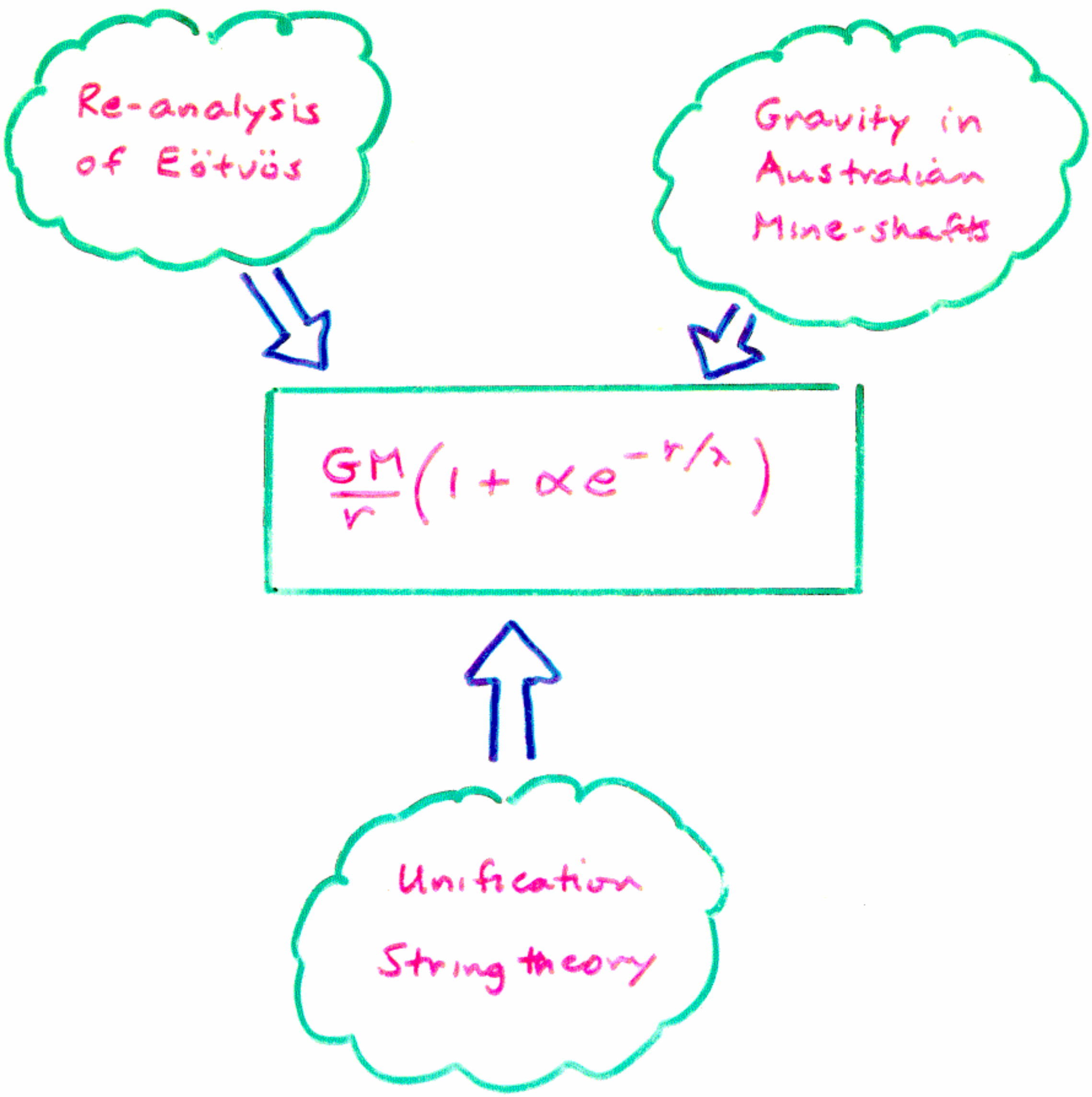

$$\boxed{\frac{\Delta \nu}{\nu} = (1+\alpha)\,\Delta U/c^2}$$

## COSMOLOGICAL VARIATION OF FUNDAMENTAL CONSTANTS

| Constant $k$ | Limit on $\dot{k}/k$ per Hubble time $(2 \times 10^{10}\ \mathrm{yr})$[1] | Method | Reference |
|---|---|---|---|
| *Fine Structure Constant:* | | | |
| $\alpha = e^2/\hbar c$ | $7 \times 10^{-4}$ | H-maser vs Hg ion clock | Prestage *et al.* 1995 |
| | $10^{-6}$ | Oklo Natural Reactor | Damour & Dyson 1996 |
| | $10^{-5}$ | 21-cm vs molecular absorption at $Z = 0.7$ | Drinkwater *et al.* 1997 |
| *Weak Interaction Constant:* | | | |
| $\beta = G_f m_p^2 c/\hbar^3$ | 2 | $^{187}$Re, $^{40}$K decay rates | Dyson (1972) |
| | 0.2 | Oklo Natural Reactor | Damour & Dyson 1996 |
| | 0.06 | Big Bang Nucleosynthesis | Malaney *et al.* 1993 |
| *e-p mass ratio* | | | |
| $m_e/m_p$ | 1 | Mass shift in quasar spectra at $Z \sim 2$ | Pagel 1977 |
| *Proton g-factor* | | | |
| $g_p$ | $2 \times 10^{-5}$ | 21-cm vs molecular absorption at $Z = 0.7$ | Drinkwater *et al.* 1997 |

[1] $H_0 = 55\mathrm{km\,s^{-1}\,Mpc^{-1}}$

# IS IT TWILIGHT TIME FOR THE FIFTH FORCE?




(Will, Sky & Telescope Nov 1990)

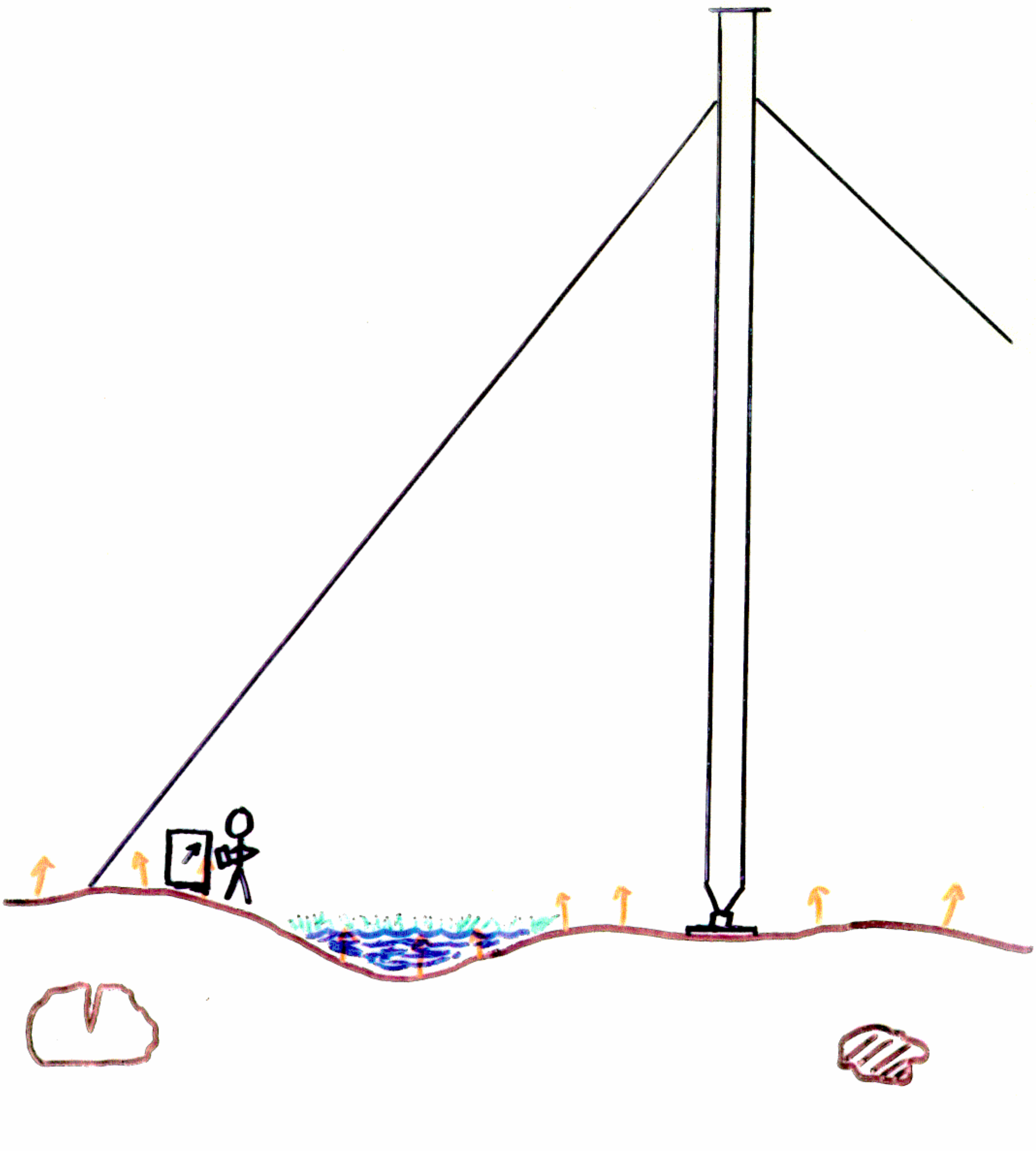
# COMPOSITION-DEPENDENT TESTS OF THE FIFTH FORCE

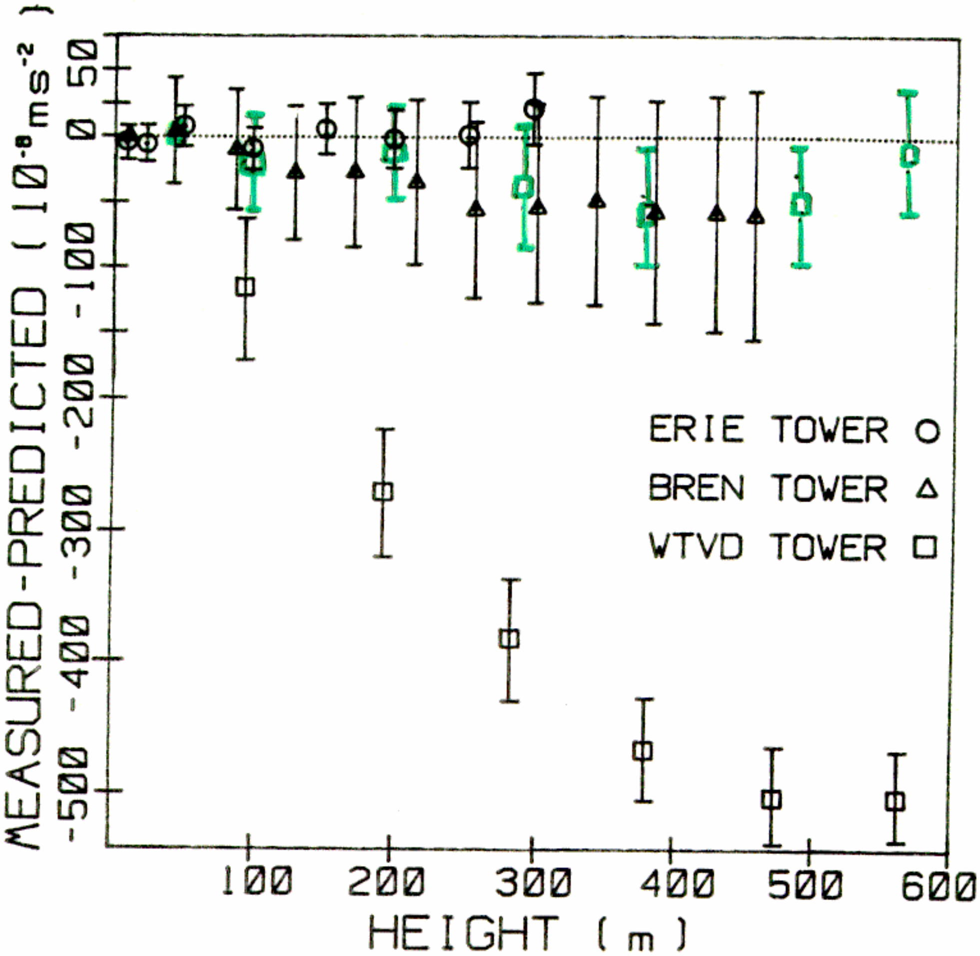
| Experiment Name or Place | Year | Method | Substance Compared | Source of Force |
|---|---|---|---|---|
| Palisades, NY | 1986 | Flotation | $Cu/H_2O$ | Cliff |
| Eöt-Wash | 1986 | Torsion balance | Cu/Be | Hillside |
| Boulder, CO | 1987 | Free fall | Cu/U | Earth |
| Eöt-Wash | 1987 | Torsion balance | Be/Al | Hillside |
| Index, WA | 1987 | Torsion balance | Be/Al | Cliff |
| Montana | 1988 | Torsion balance | $Cu/CH_2$ | Hillside |
| Paris | 1988 | Beam balance | Cu/Pb, C/Pb | Lead, brass masses |
| Bombay | 1988 | Torsion balance | Cu/Pb | Lead masses |
| Snake River | 1988 | Torsion balance | C/Pb | Water in lock |
| Eöt-Wash II | 1988 | Torsion balance | Be/Al | Lead masses |
| Japan | 1988 | Free fall | Al/Cu, Al/C | Earth |
| Florence | 1988 | Flotation | $Plastic/H_2O$ | Mountain |
| Bombay II | 1989 | Torsion balance | Cu/Pb | Lead masses |
| Irvine, CA | 1989 | Torsion balance | Cu/Pb | Lead masses |
| Eöt-Wash III | 1989 | Torsion balance | Cu/Be, Al/Be | Hillside |
| Index, WA II | 1989 | Torsion balance | $Cu/CH_2$ | Cliff |
| Florence II | 1989 | Flotation | $Plastic/H_2O$ | Mountain |

TOWER AND BORE-HOLE TESTS OF THE FIFTH FORCE

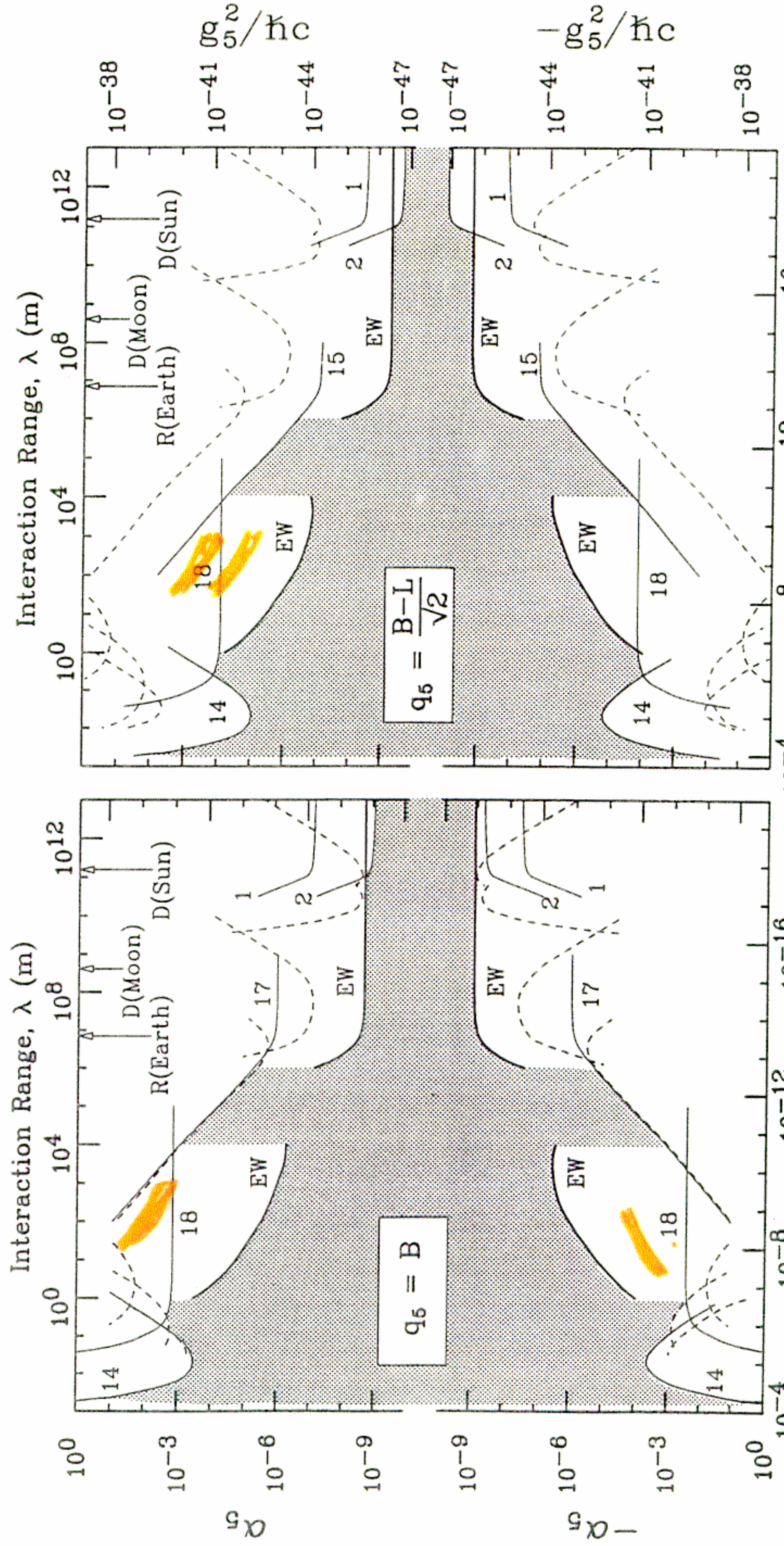
| Experiment Name or Place | Year | Method |
|---|---|---|
| Australia | 1981- | Mine Shaft |
| Michigan | 1987 | Mine Shaft |
| North Carolina | 1988 | TV Tower |
| Greenland | 1989 | Borehole |
| Nevada Test Site | 1989 | Tower |
| Colorado | 1989 | Tower |
| North Carolina | 1990 | TV Tower |
| Nevada Test Site | 1990 | Borehole |
| East Pacific Ocean | 1991 | Submersibles |

# Tower Gravity and Upward Continuation





MEASURED-PREDICTED ( $10^{-8} ms^{-2}$ )
50
0
-100
-200
-300
-400
-500
100
200
300
400
500
600
HEIGHT ( m )
ERIE TOWER ○
BREN TOWER △
WTVD TOWER □


Interaction Range, λ (m)
$m_b c^2$ (eV)
$\alpha_5$
$-\alpha_5$
$g_5^2/\hbar c$
$-g_5^2/\hbar c$
$q_5 = B$
$q_5 = \frac{B-L}{\sqrt{2}}$
R(Earth)
D(Moon)
D(Sun)
EW
14
18
17
15
1
2