Detection of Gravitational Waves II. Strategies

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Overview of 3 Lectures

I. Principles (Fri 7 Aug)
- nature of gravitational waves
- experimental principles

II. Strategies (Today, Mon 10 Aug)
- **resonant-mass detector systems**
- **interferometer systems**

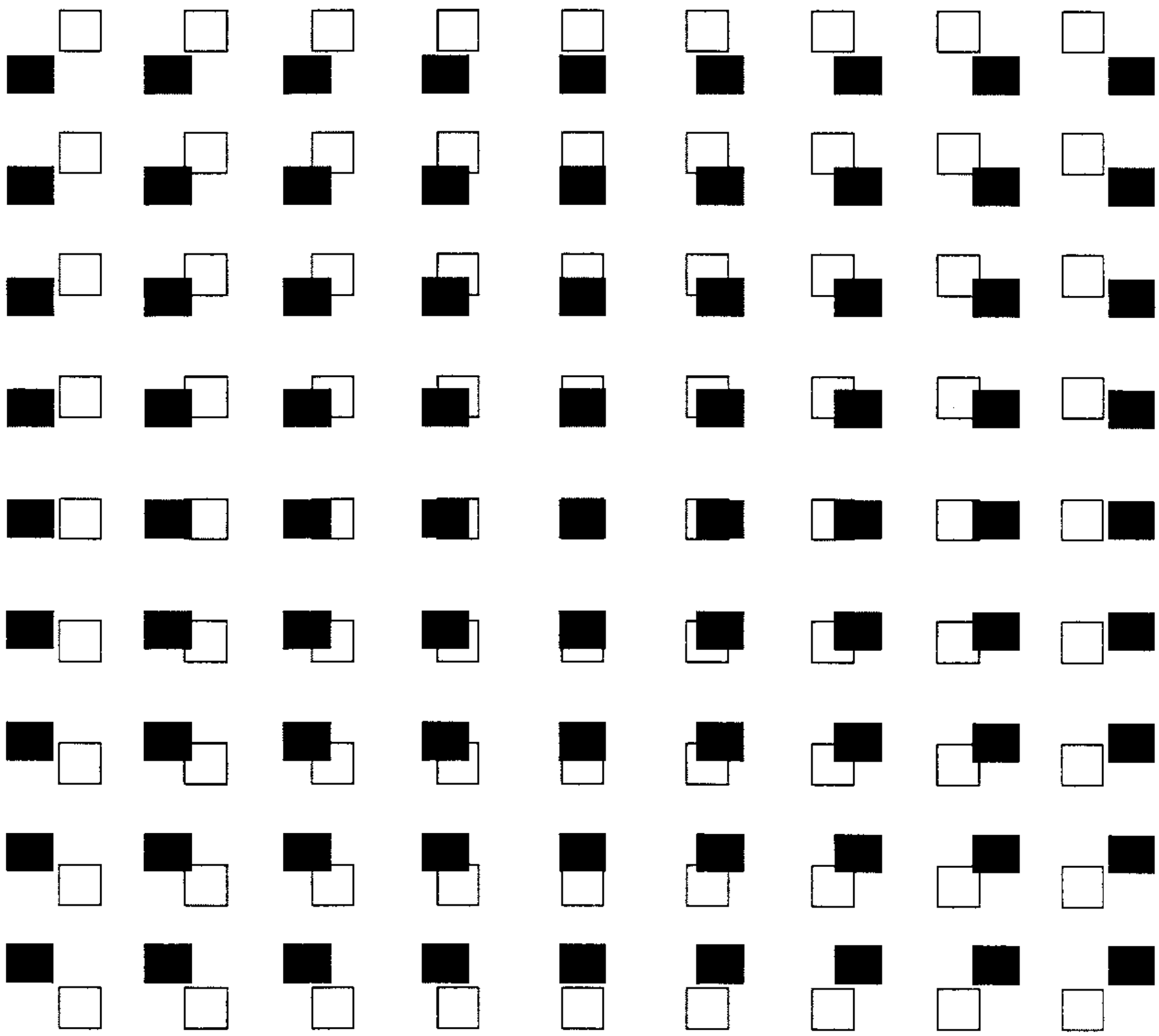
III. Practice (Tues 11 Aug)
- real instruments
- What does the future hold?

Outline of Lecture II

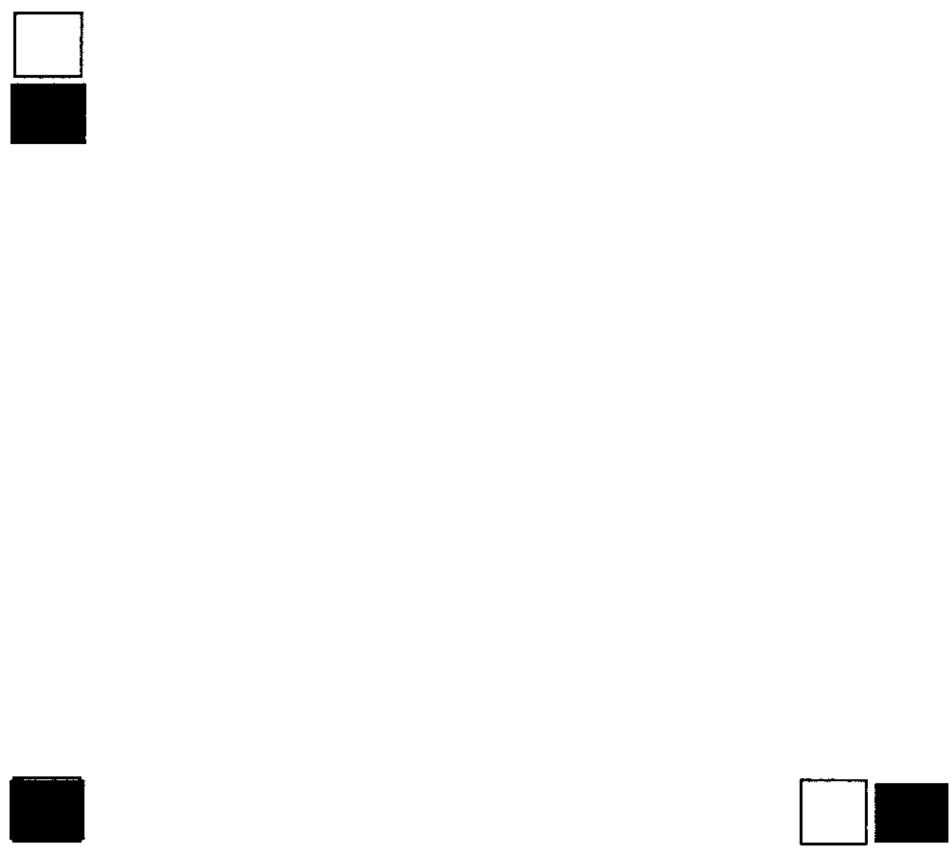
A. Interferometric detectors
- transduction mechanism
- frequency response
- noise reduction, signal extraction

B. Resonant-mass detectors
- frequency response
- transduction mechanism
- noise reduction, signal extraction

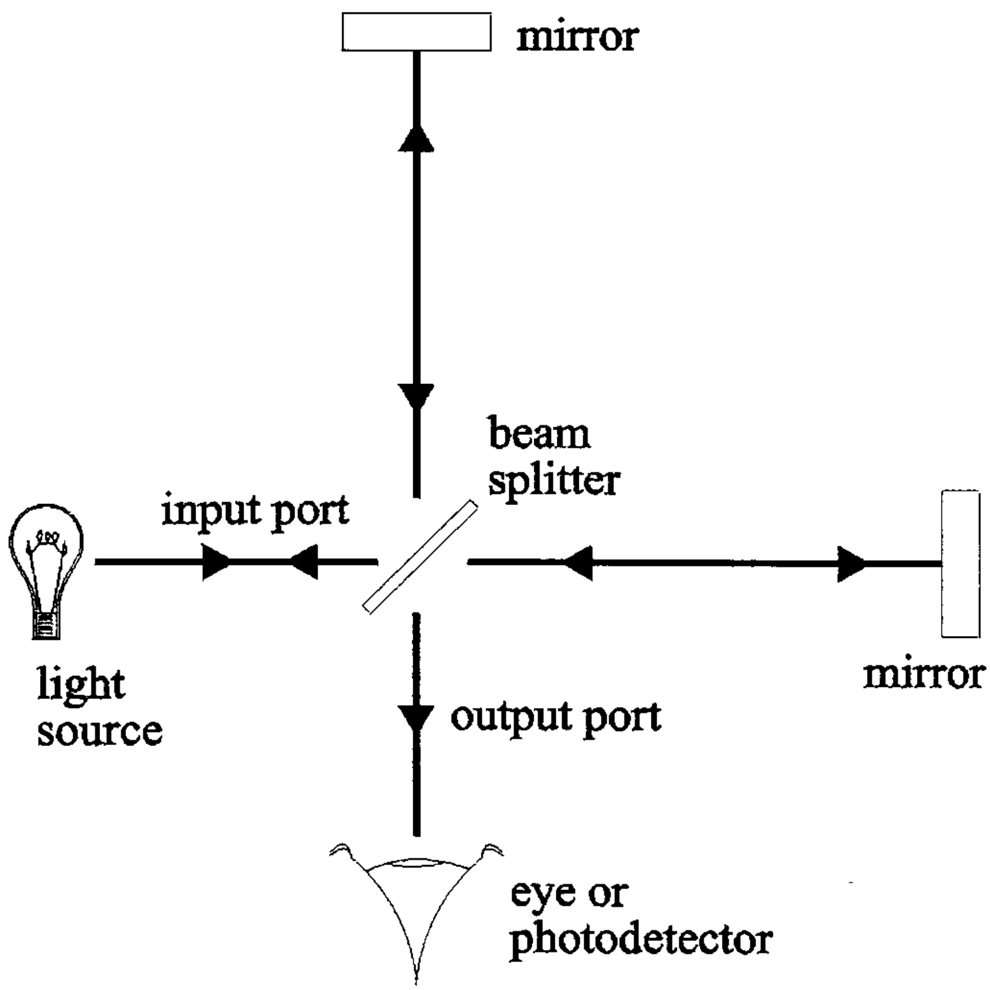
Gravitational wave creates shear strain



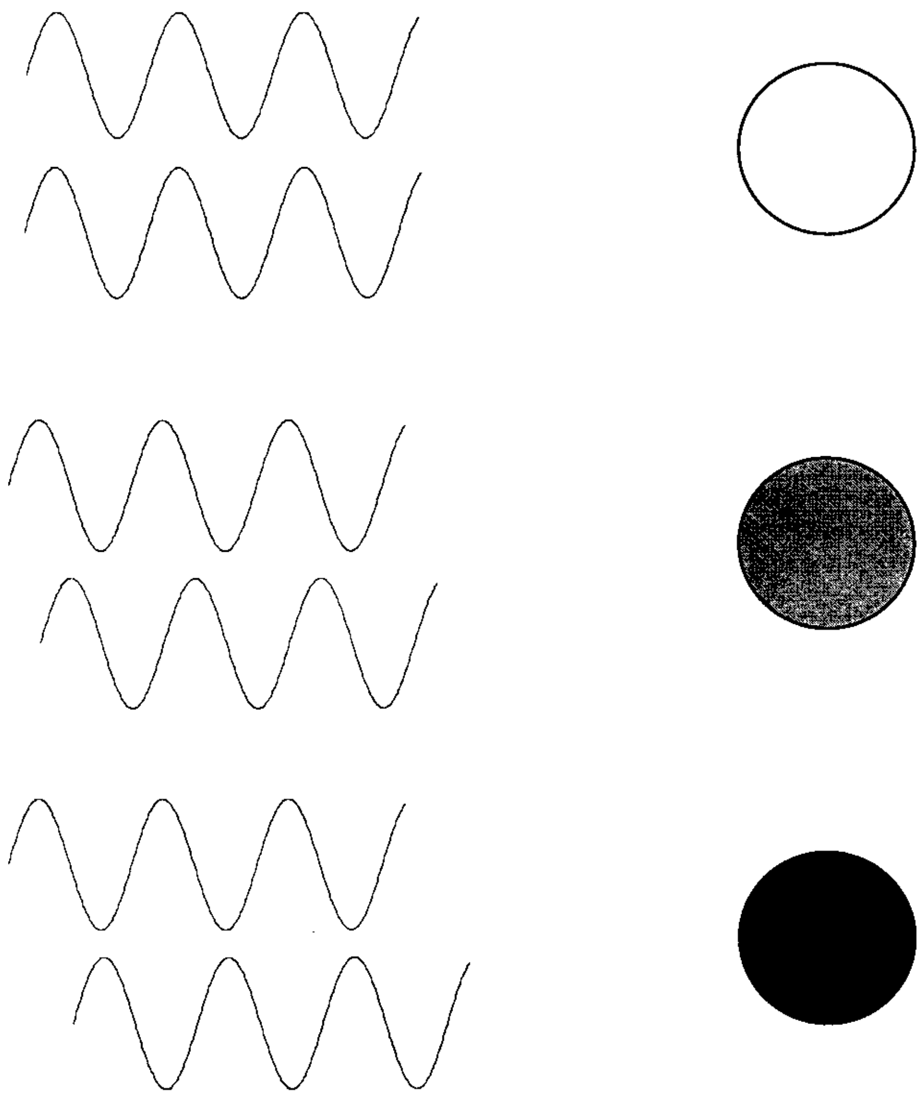
3 test masses, with gravitational wave



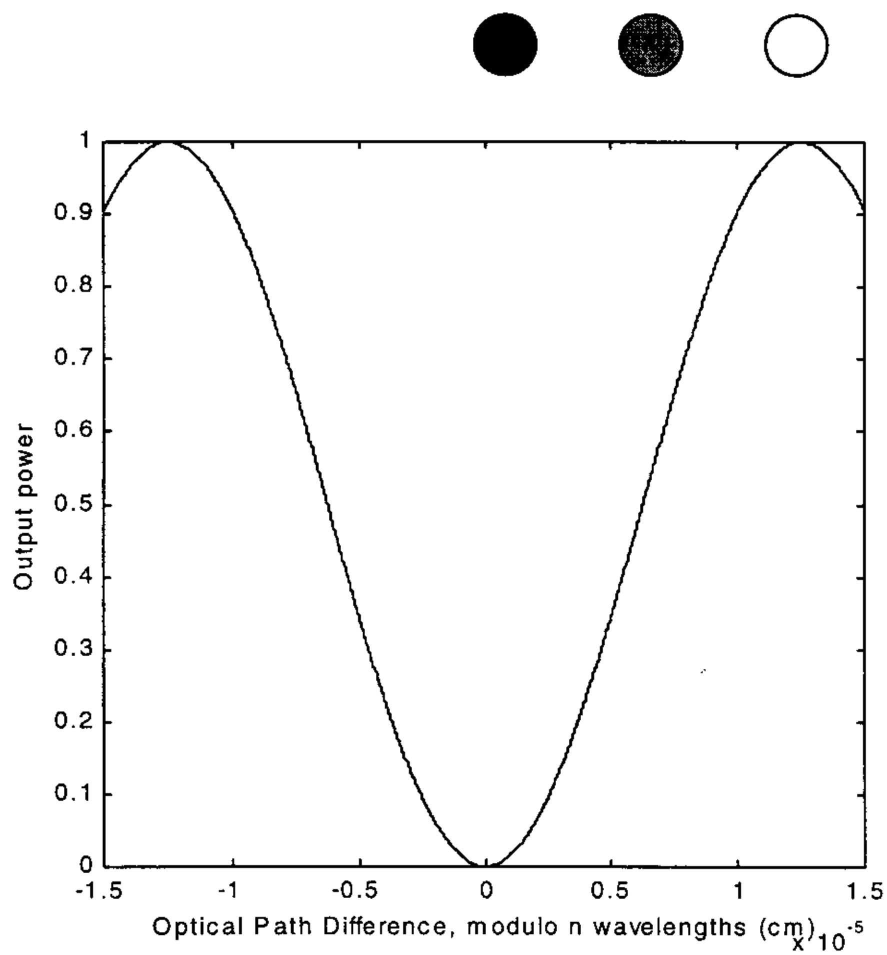
Michelson interferometer



Interference vs. path length difference



Interferometer: transducer from length difference to power



Mechanical frequency response

For free masses

$$x(t) = \frac{1}{2} h(t) \times L.$$

Far above pendulum resonance, the masses are effectively free, so the response is flat.

Optical frequency response

Optical response $d\phi/dh$ is indep. of f for $f << c/2NL \equiv \tau^{-1}$, grows with τ until $\tau \sim (2\pi f)^{-1}$.

Arms usually use Fabry-Perot cavities to increase storage time, and hence response.

For long storage time, response has single pole roll-off, "automatically optimized" vs. f.

An interferometer is intrinsically broad-band

If noise allows, could directly read $h(t)$ from output, at high bandwidth.

Q: What is noise spectrum?

Strong low-frequency noise will limit useful bandpass.

Interferometer readout noise: photon shot noise

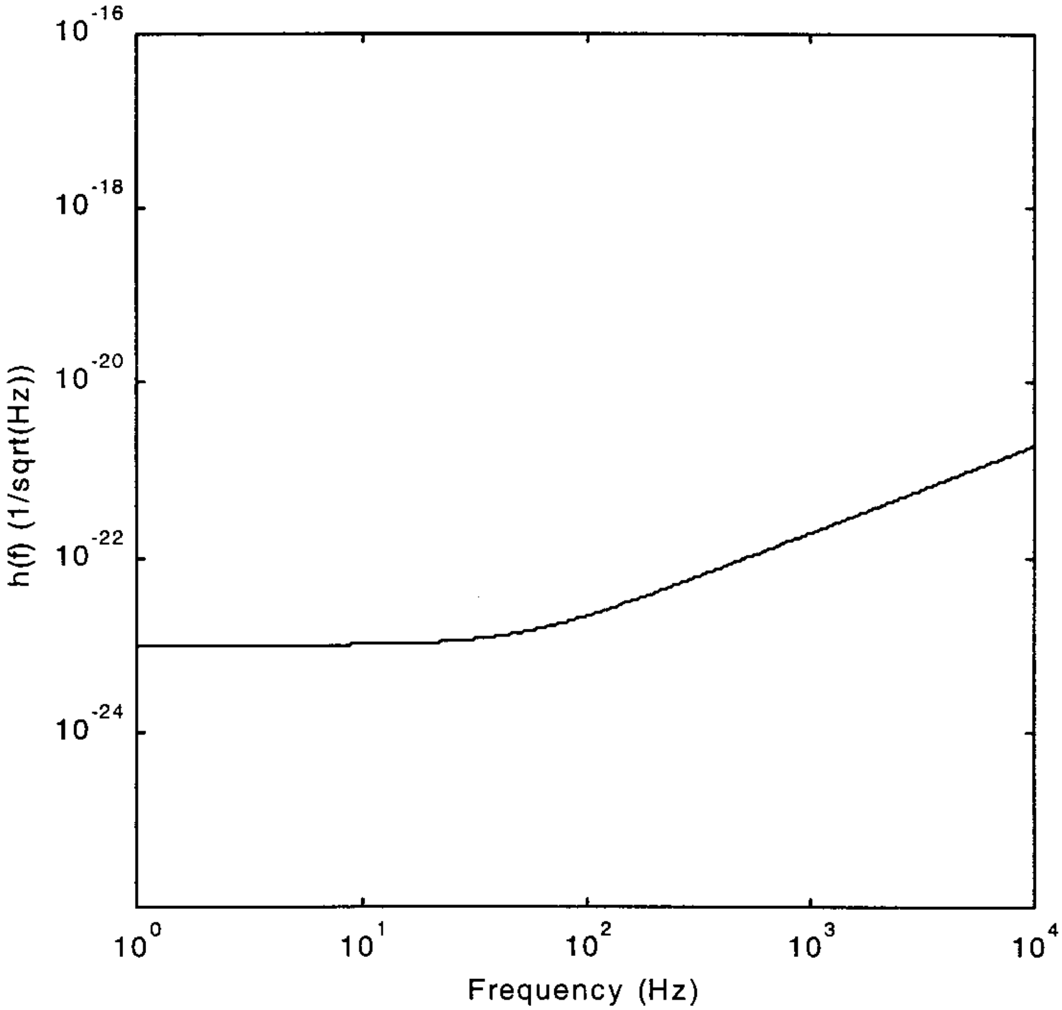
Shot noise: Poisson statistics in photon arrivals, calibrated wrt *dP/dh*.

$$h_{shot}(f) = \frac{1}{NL}\sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

If P = 100 W, L = 4 km, N = 180, then $h_{shot}(f) = 1 \times 10^{-23}/\mathrm{Hz}^{1/2}$.

(This is ~LIGO I level, and can be made substantially lower.)

Shot noise



(includes Fabry-Perot response)

Interferometer thermal noise: pendulum mode and internal modes

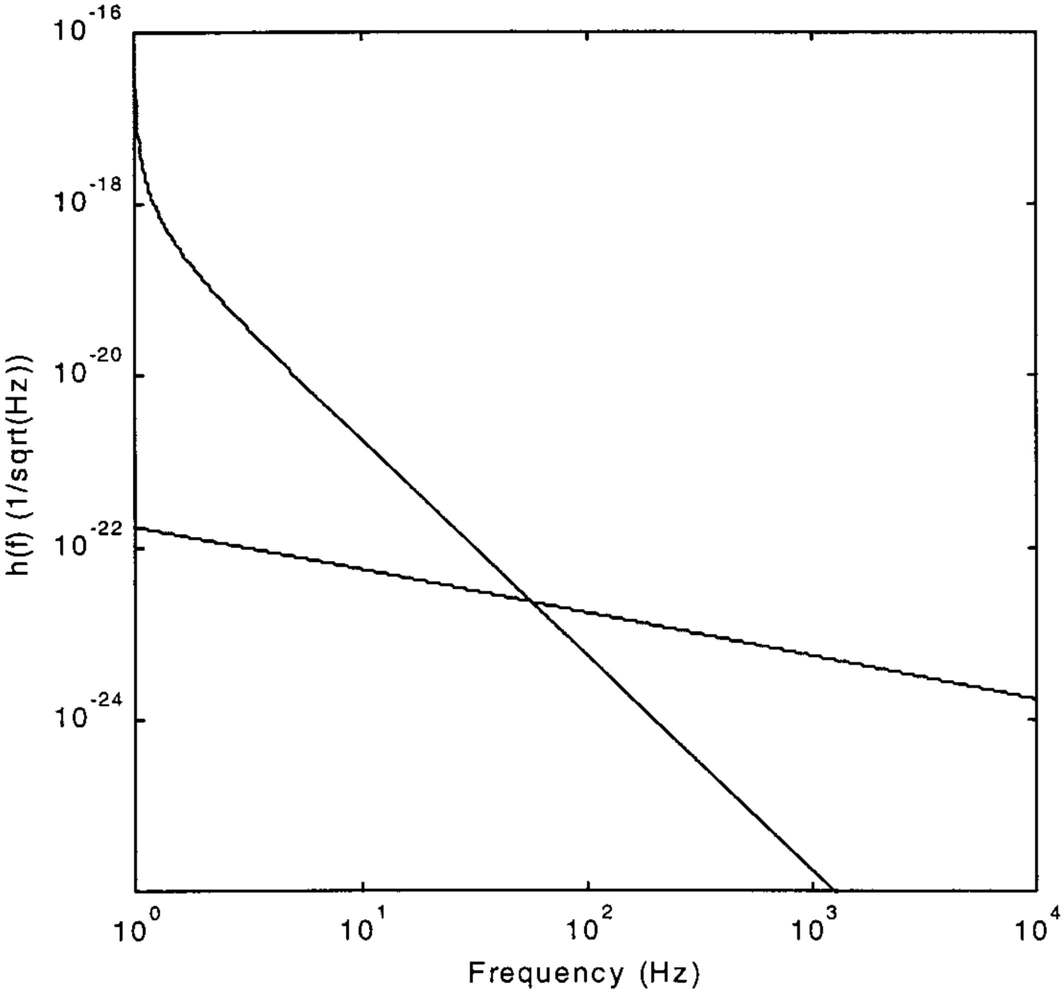
Between pendulum mode ω_{pend} at 1 Hz and internal modes ω_i starting at 10 kHz, thermal noise is given by

$$x(f) = \sqrt{\frac{4k_B T \omega_{pend}^2}{mQ\omega^5}},$$

combined with

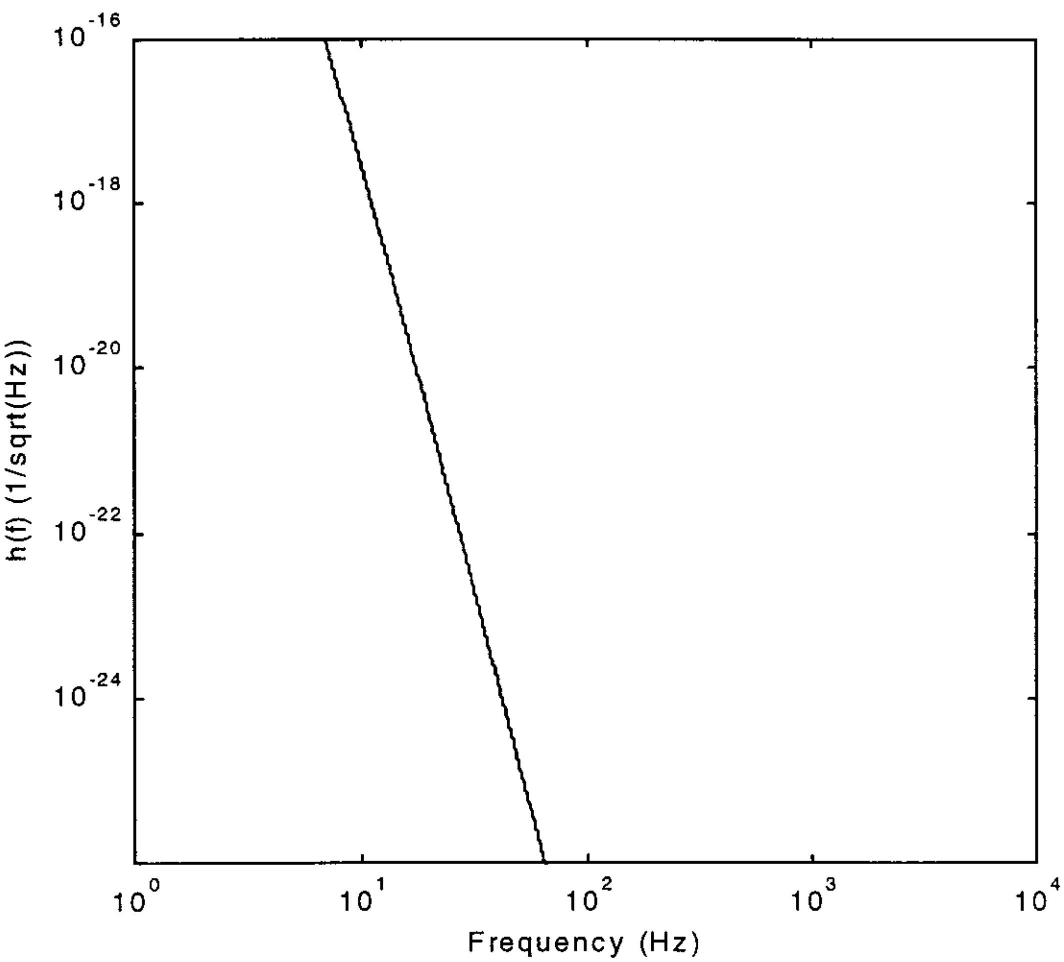
$$x(f) = \sum_i \sqrt{\frac{4k_B T}{mQ\omega_i^2 \omega}}.$$

Thermal noise

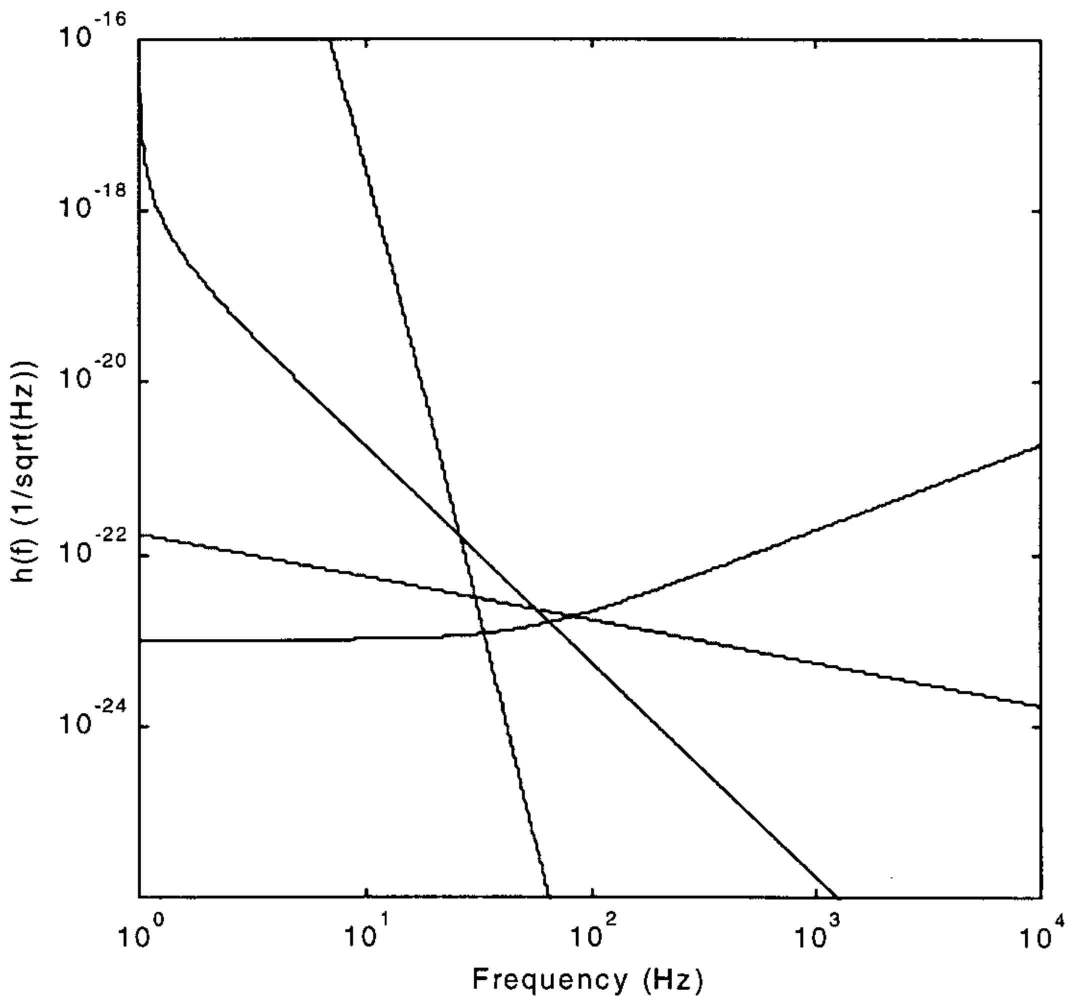


These are ~LIGO I levels, and can be substantially improved.

Seismic noise: a low frequency "wall"



Total interferometer noise spectrum



How to deal with strong low-frequency noise?

Strong noise at low frequencies threates to move operating point through many fringes.

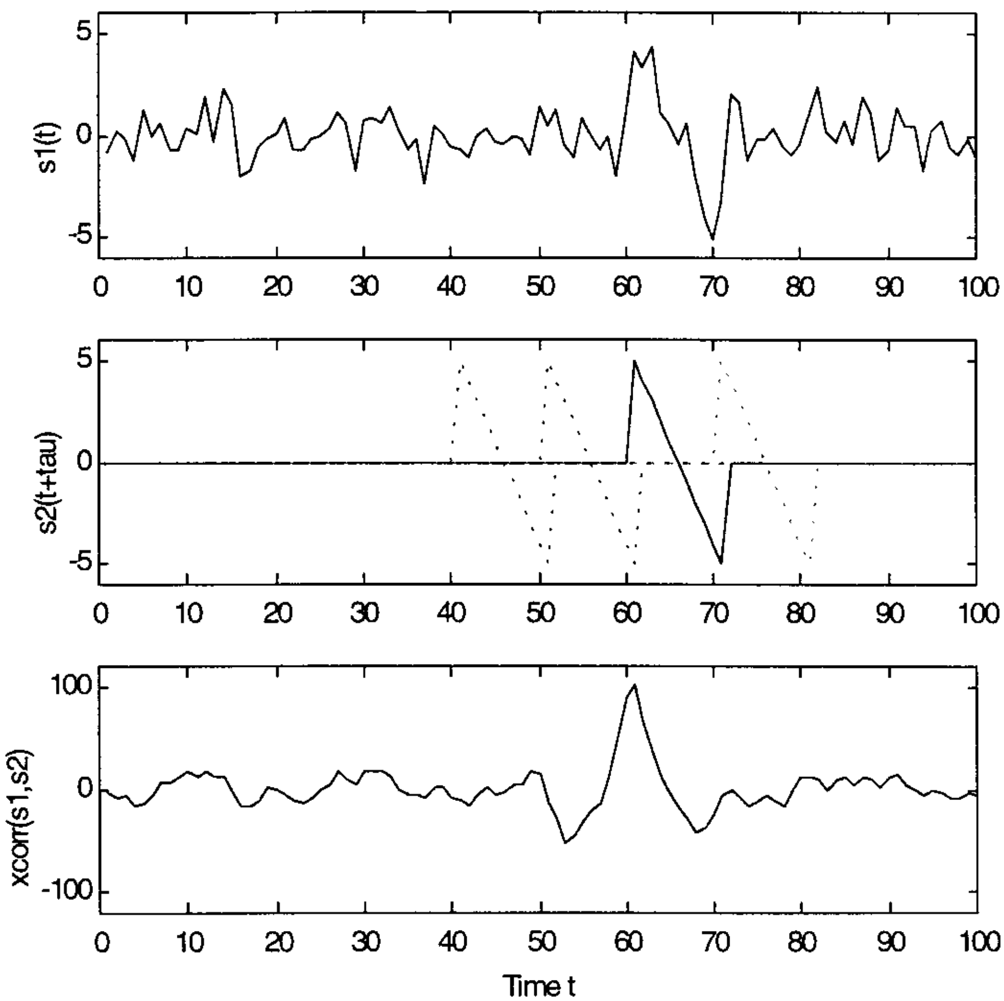
Solve by locking interferometer to chosen operating point, using feedback.

This null servo guarantees linear response.

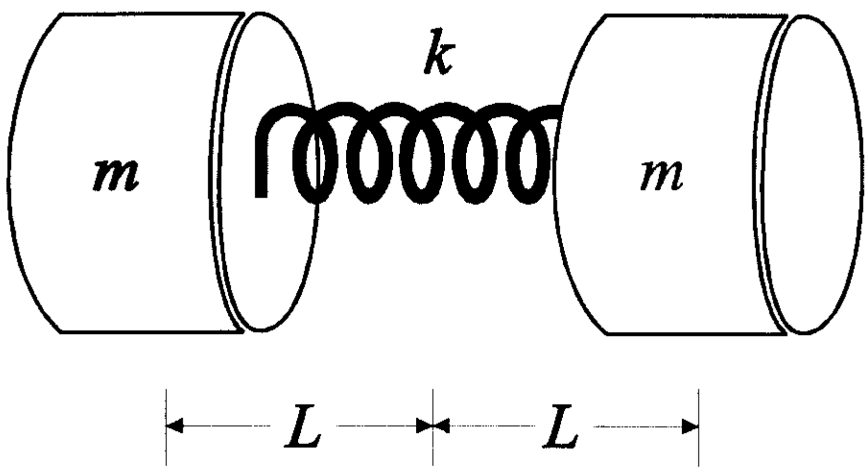
Then superposition allows us to look for small high-f signals.

How to find weak signals in strong noise

Matched filtering

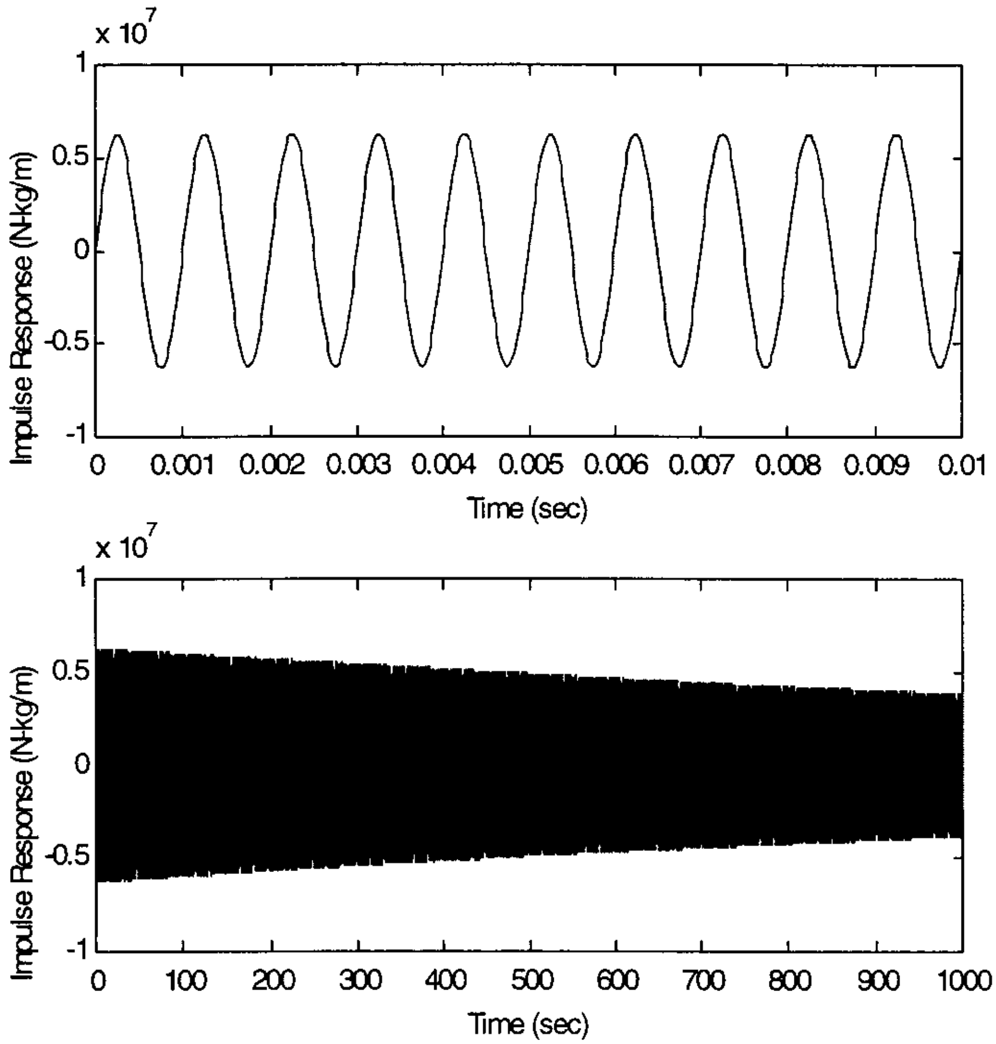


Two test masses and a spring

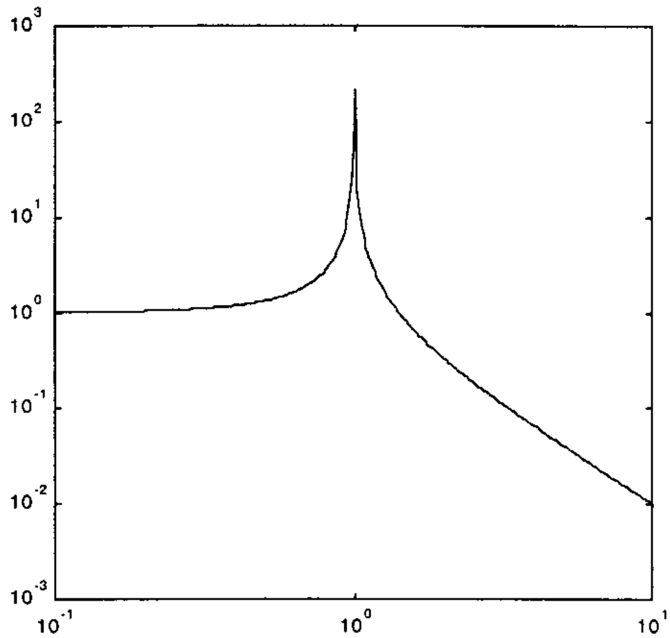


Resonant-mass detector response

Impulse excites high-Q resonance



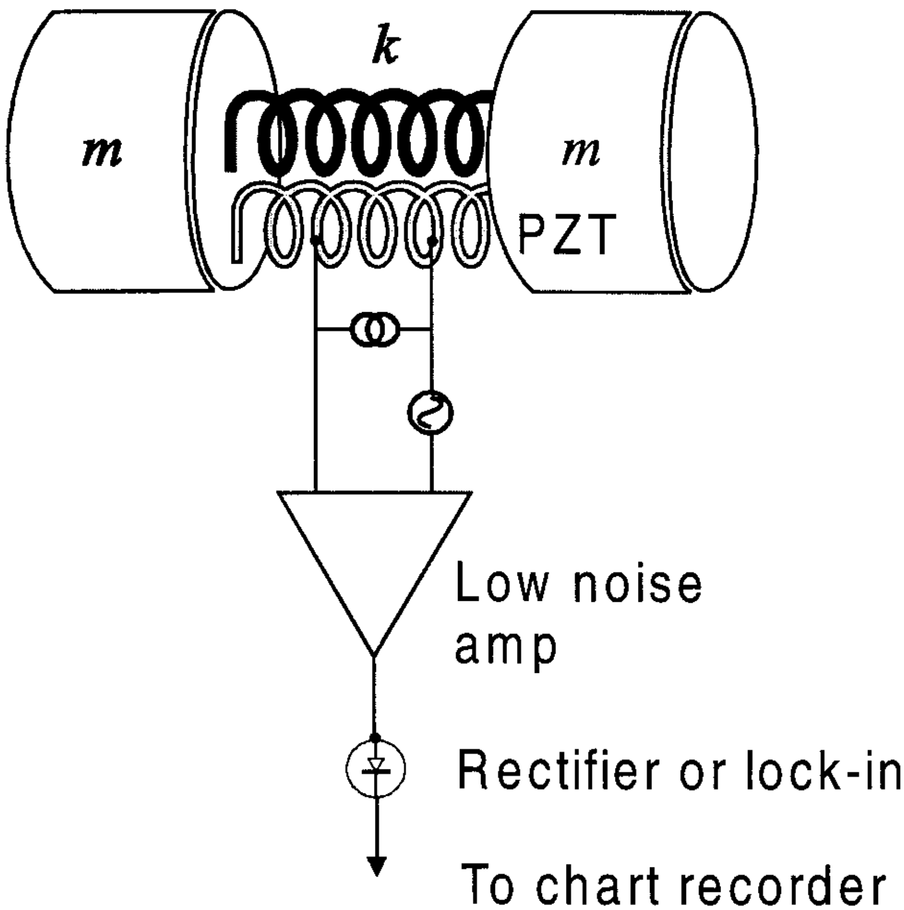
Resonant-mass frequency response



WHY?

- Match to signal frequency (possible in special case)
- **A strategy against readout noise.**

First generation resonant-mass system



Room temperature, $h_{rms} \sim 10^{-16}$

Resonant-mass detector viewed as a kind of matched filtering

A brief impulse sets the detector ringing for a long time.

Thus it is converted from $\delta(t-t_0)$ to a nearly sinusoidal function, from broad-band to narrow-band form.

Amplifier noise is ~ white. Applying a narrow-band filter at resonant frequency rejects most amplifier noise, admits signal.

Clever.

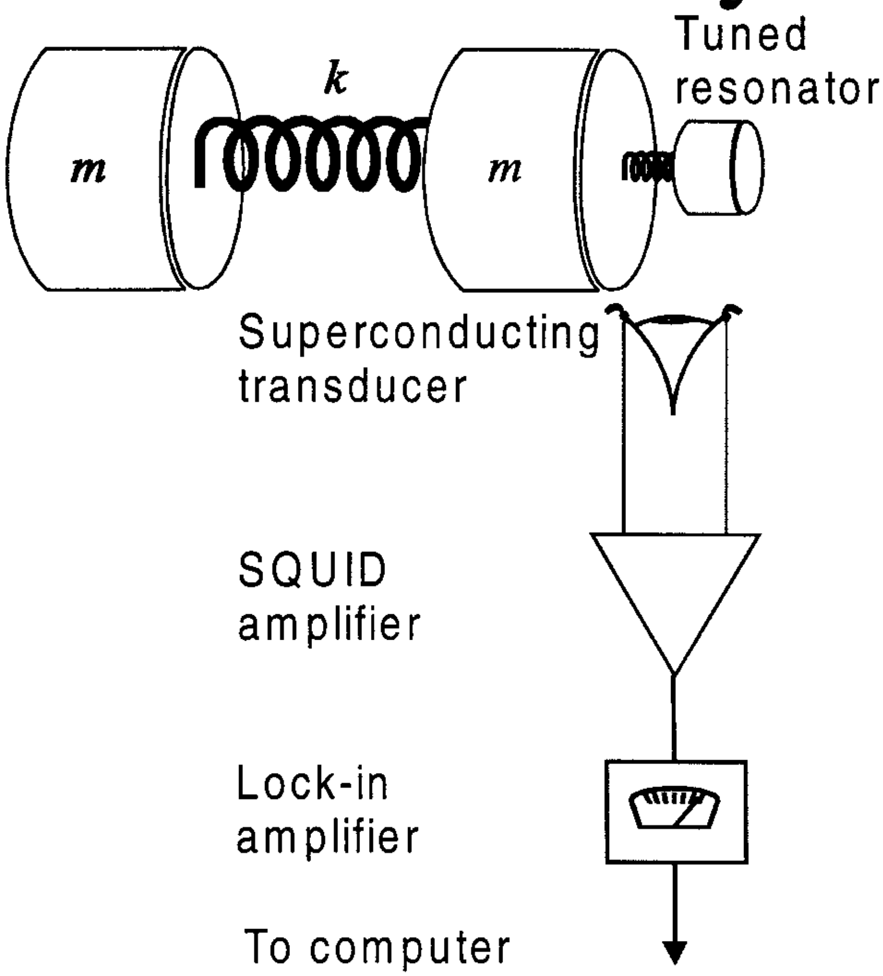
Burst energy as natural measurable

When optimal filter averages over resonant response for a long time, all that one learns is that the resonator has had its excitation changed.

What one measures is the energy *E* that the wave imparted, which depends only on the wave's Fourier transform at the resonance frequency,

$$E = \frac{M v_s^4}{L^2} \left| \tilde{h}(f_0) \right|^2$$

Second generation resonant-mass system



$$T = 4\ \mathrm{K},\ h_{\mathrm{rms}} \sim 10^{-18}$$

Resonant transducer (Paik '76)

It is hard to make a transducer strong enough to efficiently transform resonator's energy to electrical form.

The job is easier if a mechanical impedance transformation is made, by sensing motion of much smaller tuned resonator, which after the beat period moves by an amount larger by $2\sqrt{M/m}$.

Thermal noise in resonant-mass detectors

High Q resonance also reduces thermal noise.

No benefit if you try to integrate for whole damping time of resonance, since then successive measurements scatter by $kT/2$.

But if integration is shorter than damping time, only fraction of thermal noise contributes.

Best integration time is a compromise.

Optimum with thermal and readout noise

Minimum detectable energy (1 σ):

$$\Delta E_{\min} \approx 2k_B T \frac{\tau}{\tau_d} + 4k_B T_{amp} \frac{1}{\beta\tau\omega_0}$$

where T is temperature of bar,

T_{amp} is noise temp of amplifier,

τ is sampling time,

τ_d is the damping time, and

β is the fraction of bar energy at the amplifier terminals.

Amplifier back action, via reciprocal transducer

Transducer is reciprocal, so electrical noise from amplifier "acts back" on bar, making it noisier.

(If amplifier were at quantum limit, this is what would enforce the UncertaintyPrinciple.)

Thus,

$$\Delta E_{\min} \approx 2k_B T \frac{\tau}{\tau_d} + k_B T_{amp}\left(\frac{4}{\beta\tau\omega_0} + \frac{\beta\tau\omega_0}{2}\right).$$

Is a resonant-mass detector intrinsically narrow-band?

No, not if thermal noise were the main noise source. Then, you could sample quickly, and correct for resonant response.

But if resonance is being used to help minimize readout noise, then the whole point is to integrate for a while.

Still, bandwidth could, ideally, be a substantial fraction of ω_0.

What is the best way to compare bars and ifos?

- sensitivity
- bandwidth
- complexity
- maturity
- improvability

Preview of final lecture

LSU's ALLEGRO

Prototype interferometers

- MPQ's 30-m ifo
- LIGO's 40-m ifo
- LIGO's Phase Noise Ifo

LIGO I

The future: large interferometer development, spheres, interferometers in space.