Detection of Gravitational Waves I. Principles

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Overview of 3 Lectures

I. Principles (Today, Fri 7 Aug)

- **nature of gravitational waves**
- **experimental principles**

II. Strategies (Mon 10 Aug)

- resonant-mass detector systems
- interferometer systems

III. Practice (Tues 11 Aug)

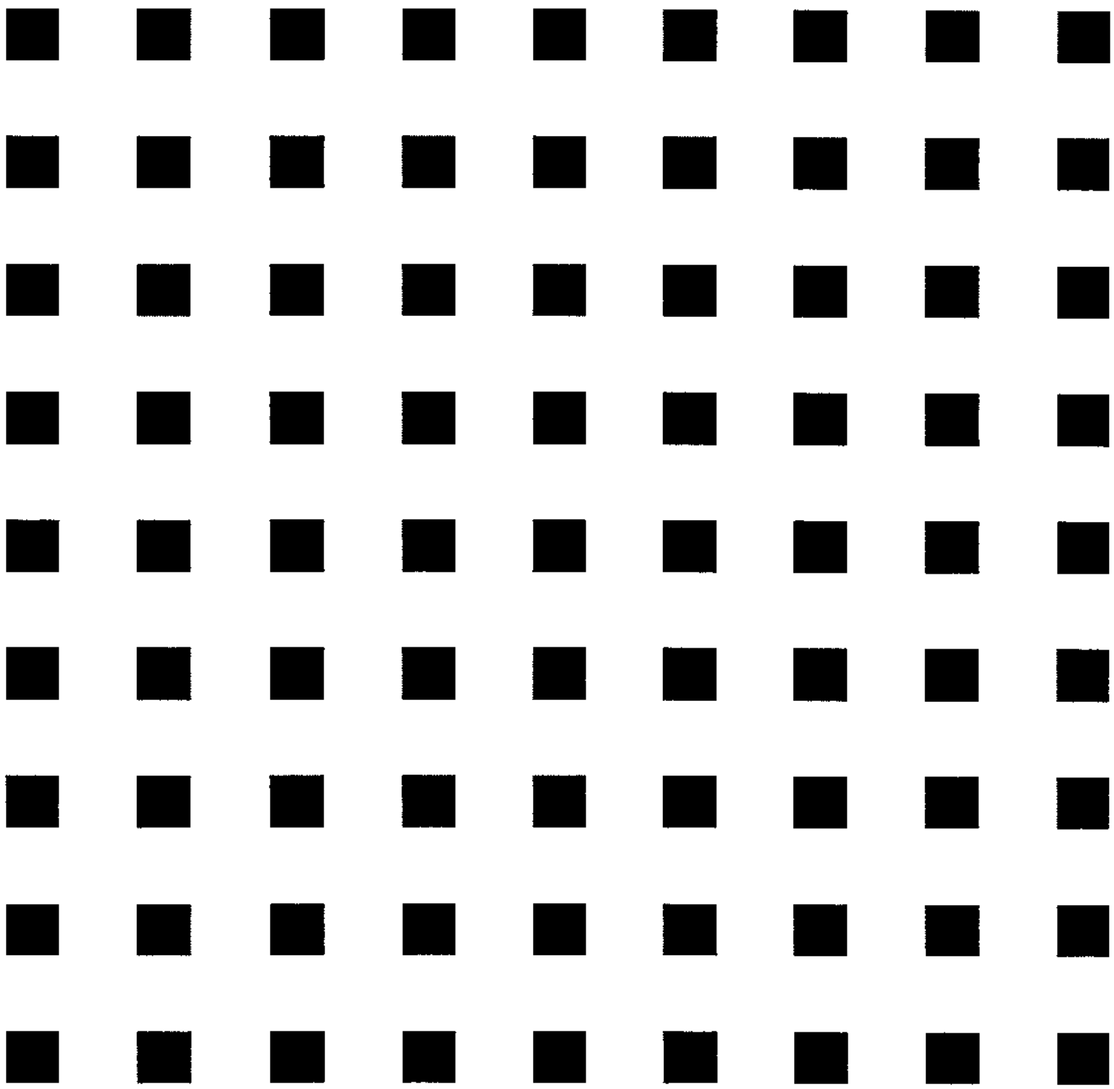
- real instruments
- What does the future hold?

Outline of Lecture I

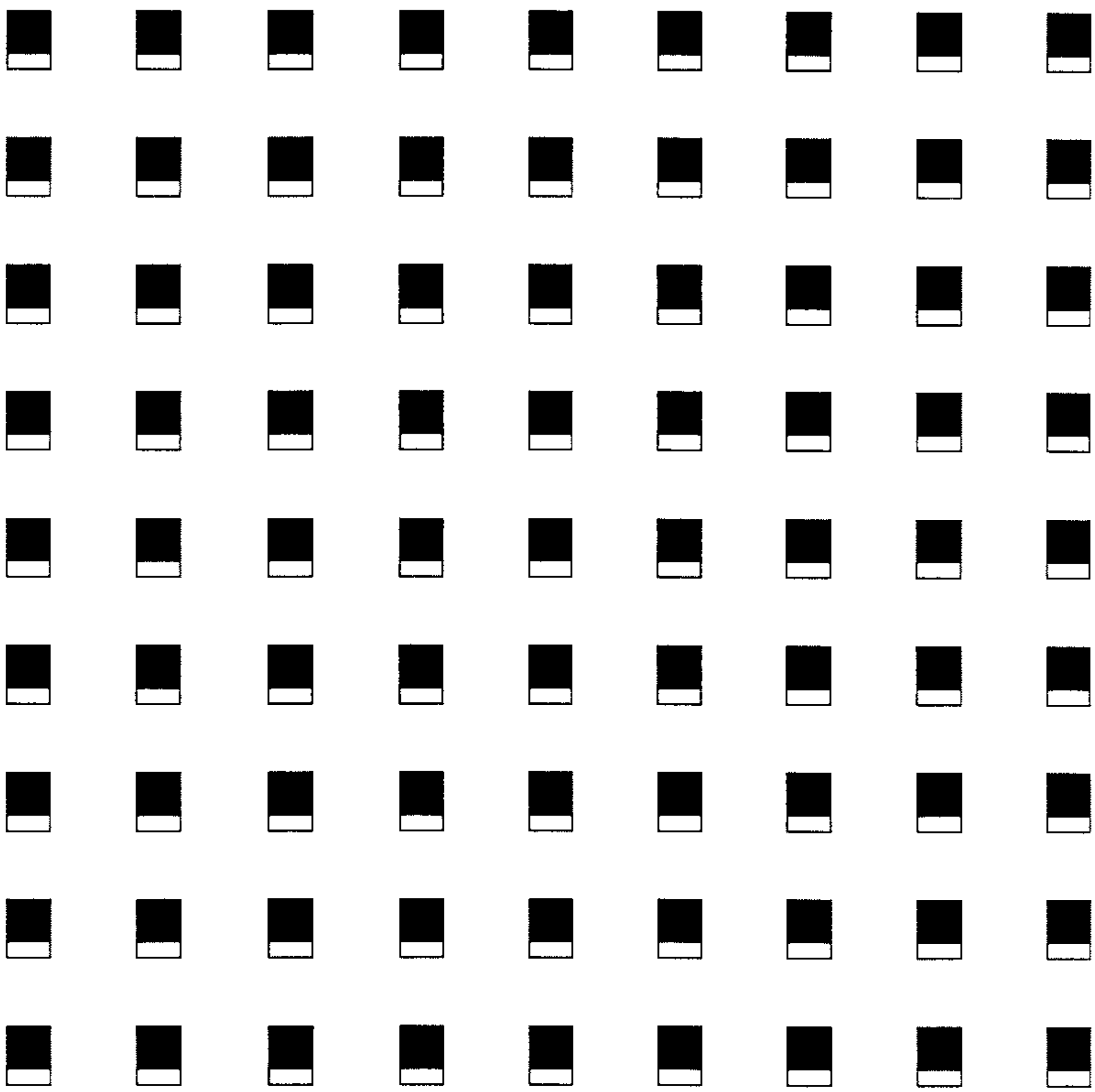
A. Nature of gravitational waves
- character of phenomenon
- expected strength
- sources

B. Experimental principles
- how to set up test masses
- how to make the signal large
- how to make the noise small

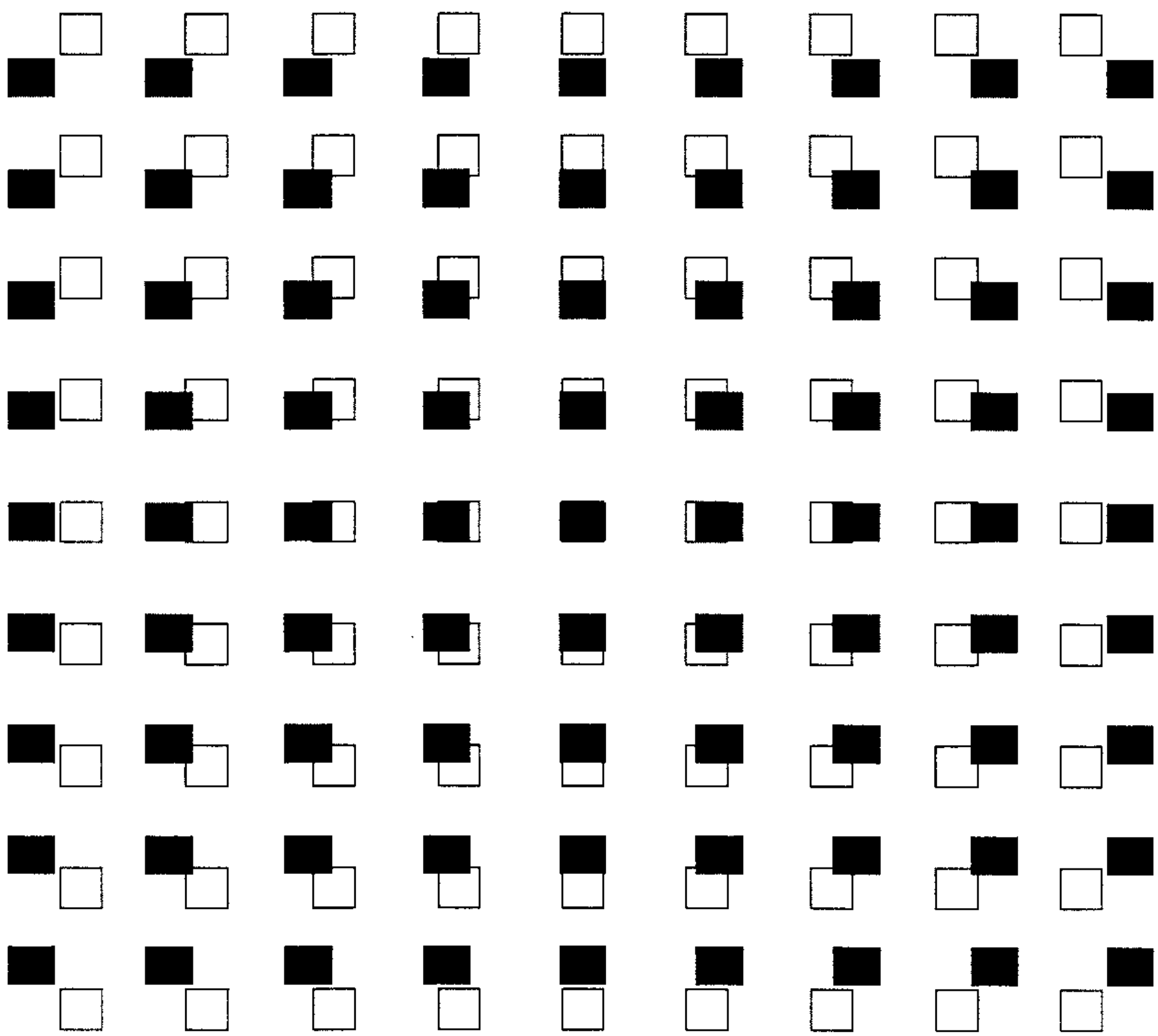
An array of test bodies



E/M wave moves charged bodies



Gravitational wave creates shear strain



Observable effect

Free masses move with wave,

$$\Delta x(t) = \frac{1}{2} h(t) \times L$$

and *h(t)* is proportional to $\ddot{I}$.

We see history of source's shape.

Can also think in terms of a force

$$F_x(t) = \frac{1}{2} m L \ddot{h}_{xx}(t).$$

(Useful when test masses feel non-gravitational forces too.)

E/M vs. Grav waves

charge	mass
E and ***B*** fields	h = shear strain
c	c
Maxwell 1867	Einstein 1916
Hertz 1886-91	?
pretty strong	very weak

The quadrupole formula

Leading order radiating multipole is quadrupole term. (Dipoles cancel by conservation of ***p***, ***L***.)

Radiated field is given by

$$h_{\mu\nu} = \left(\frac{2G}{c^4 R}\right)\ddot{I}_{\mu\nu}$$

where $I_{\mu\nu}$ is

$$I_{\mu\nu} \equiv \int dV\left(x_\mu x_\nu - \delta_{\mu\nu} r^2 / 3\right)\rho(r).$$

A gravitational Hertz experiment is hard to do

Hertz set up transmitter, receiver on opposite sides of room.

Two 1-ton masses, separated by 2 meters, spun at 1 kHz, has

$$\ddot{I} = 1.6 \times 10^{11} \text{ kg m}^2\text{s}^{-2}.$$

At distance of 1 $\lambda = 300$ km,

$$h = 9 \times 10^{-39}.$$

Not very strong.

Waves from naturally occurring sources

For a binary star,

$$h = r/R,$$

where

R = distance to source, and

$r = r_{S1} r_{S2}/a$, with

$r_{S1,2}$ Schwarzschild radii of the stars,

and a is their separation.

Often, $r \sim 1$ km (neutron star binary).

At Virgo Cluster, $R \sim 10^{21}$ km.

Hence, expect $h \sim 10^{-21}$

(on msec time scales!)

Required sensitivity: is it possible?

$h \sim 10^{-21}$ is $x \sim 5 \times 10^{-22}$ m in 1 m, or $x \sim 2 \times 10^{-18}$ m in 4 km.

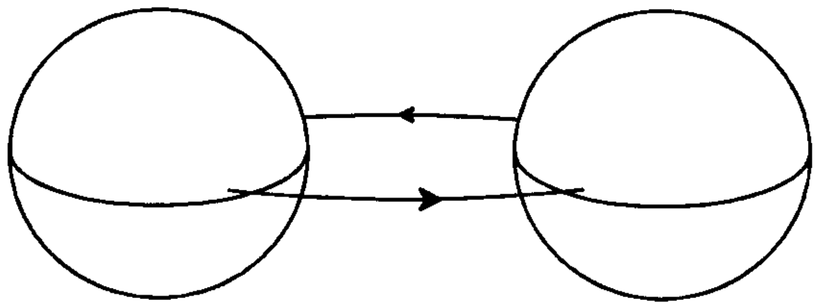
Compare to atom, 10^{-10} m.

But we sense collective state of many kg worth of atoms.

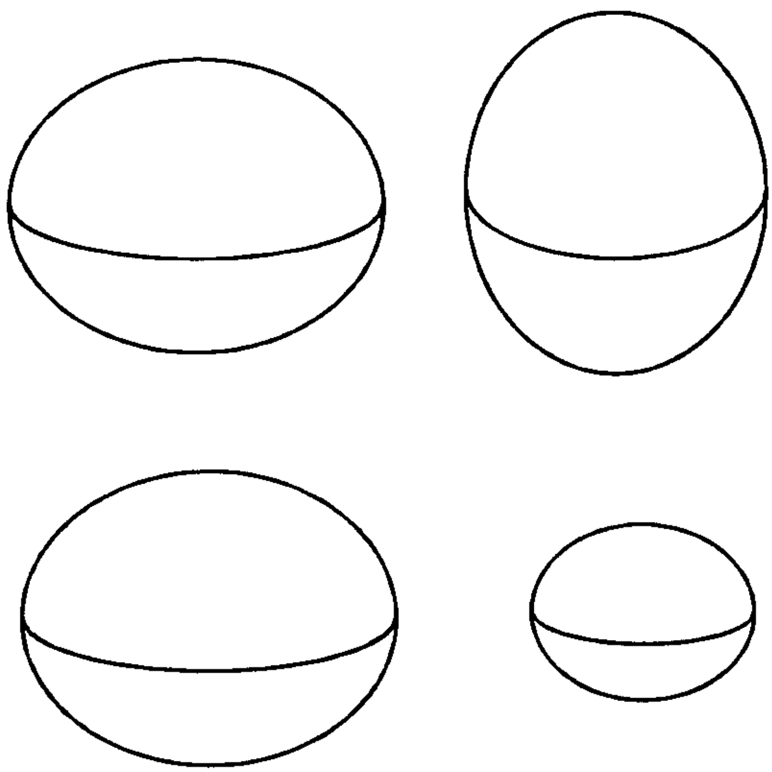
(Relevant numbers:

- 1 ton of aluminum has 2×10^{28} atoms.
- 10 cm^2 of light beam on mirror surface averages over 10^{17} atoms.)

Time-varying quadrupoles



(e.g. Hulse-Taylor Binary Pulsar)



Astronomical sources

Supernovae

– formation of **neutron star** or **black hole**

Orbital motion

– **neutron star** or **black hole** binaries

Collisions

– **neutron stars** or **black holes**

Early Universe

Coalescing binaries

Neutron star binaries are common.

Black hole or mixed binaries may be, too.

Orbital motion is calculable, with great precision. (Good for matched filtering.)

Interesting physics in final stages (neutron matter EOS, γ bursts?)

An ideal target to search for.

Black hole modes

Black holes consist of strong pure gravity.

The "hydrogen atom" of General Relativity.

A disturbed black hole vibrates at characteristic frequencies, its "quasi-normal modes". ("Quasi" because strongly damped by gravitational rad'n.)

$$f_0 \approx 0.7 \frac{c}{2GM}.$$

Sensitivity benchmarks

- "Efficient" supernovae
  - $h \approx 10^{-18}$ in Milky Way (1/30 yr^{-1})
  - $h \approx 10^{-21}$ in Virgo Cluster (few per year), ~ LIGO I sensitivity
  - but how efficient are they?
- Neutron star binary coalescences at several hundred Mpc (~1 yr^{-1})
  - guaranteed
  - a challenging goal for sensitivity, $h_{eff} \leq 10^{-22}$

Possible strong sources of gravitational waves

- Don’t be blind to surprises!
- Gamma ray bursts may be neutron star binaries, or perhaps black hole binaries.
- X-ray sources like Sco X-1 might be “lopsided” neutron stars.

How to think about the gravitational wave business

First task is detection.

Then study signals to learn about sources.

Maximizing the signal to noise ratio (SNR) is the name of the game.

Three ways to maximize SNR:

- make signal large
- make noise small
- be smart about extracting signals from noise

What does gravitational wave detection entail?

Construct a set of test masses.

Isolate them from disturbances.

Instrument them so that very small motions are discernible.

Watch the (scalar) time series corresponding to test mass differential motion.

Wait for a large enough gravitational wave to arrive.

Check other detectors for coincidence.

Test masses

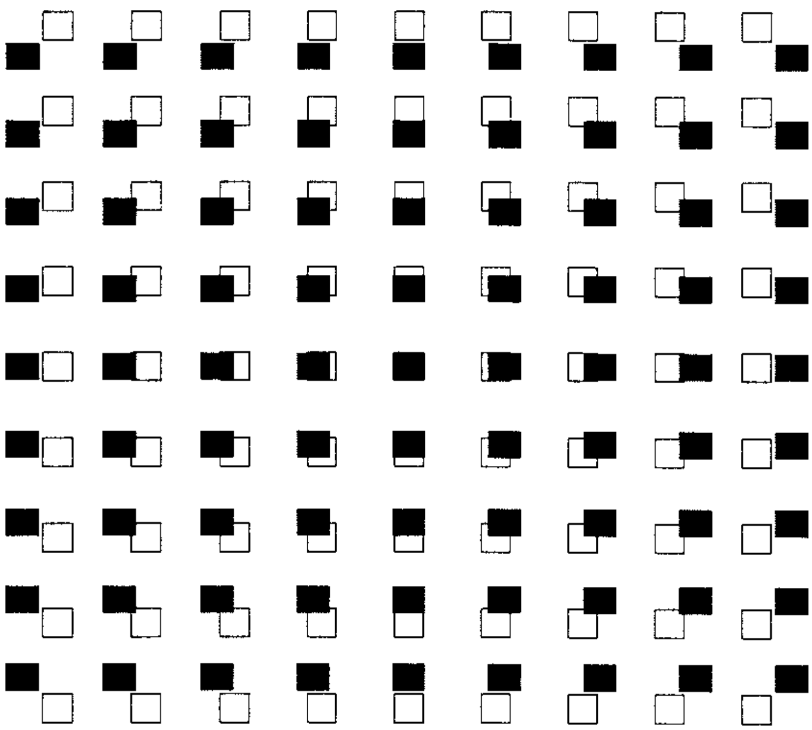
1. Nearly free
   - Cylinder (~10 kg) of fused silica.
   - Suspended as 1 Hz pendulum.
   - Essentially free, for $f \gg 1$ Hz.
   - Three (or more), in large `L'.
2. Resonant
   - Single large (several ton) resonator (of Al, Nb, …)
   - Gravest resonance ~1 kHz.
   - Acts like pair of test masses, separated by size of resonator.

Making the signal large

Only one trick here:

Use widely separated test masses.

Interferometers use this trick.



Making the noise small

Many noises, many tricks.

Three kinds of noise dominate:

- thermal noise (Brownian motion)
- readout noise
  - including back action and the "quantum limit"
- seismic noise

Describing noise by its spectral density

Random time series described by **power spectrum** $S(f)$.

It is the mean square fluctuation in a 1 Hz bandpass filter, as a function of frequency.

Has units of V^2/Hz, m^2/Hz, etc.

Integrate it for noise in wider band.

(Sometimes use its square root, the **amplitude spectral density**, with units $m/\sqrt{Hz}$.)

Thermal noise

A fundamental non-grav-wave reason for test masses to move.

A broadband force noise spectrum applied to test masses,

$$S_F(f) = 4kTR_{\mathrm{mech}}.$$

Tricks: reduce T, reduce R_{mech}.

Thermal noise rms

Rms Brownian motion is large.

Equipartition Theorem:

$$kx^2/2 = k_B T/2$$

- Interferometer:

  10 kg, 1 Hz, $T = 300$ K

$$x_{\mathrm{rms}} = 3 \times 10^{-12}\ \mathrm{m}$$

- 3rd gen. resonant-mass detector:

  1 ton, 1 kHz, $T = 50$ mK:

$$x_{\mathrm{rms}} = 4 \times 10^{-18}\ \mathrm{m}$$

Effective thermal noise is smaller than rms

In resonant mass detectors, look only for excitations on times short compared to damping time. Only small fraction of $k_B T$ contributes.

In interferometers, measurements made at frequencies far from mechanical resonances. Only small thermal noise power in "wings" of resonance.

Readout noise

Measurement precision is always limited.

Ex: interferometer with “1 fringe” precision

$\lambda / L \sim 4 \times 10^{-11}$

“Splitting a fringe” is required!

Fringe splitting is possible.

Limit is not wavelength, but shot noise in photon arrivals at photodetector.

Readout noise II

- Details always instrument-specific:
  - interferometer: photon shot noise
  - electrical amplifiers: voltage and current noise
- Importance in measurement depends on transducer gain
  - ($h = 10^{-21}$ corresponds to how many picoamps?)

Quantum constraint on readout noise

Conjugate variables are always subject to Uncertainty Principle constraint on their product

$$\Delta x \Delta p \geq \hbar$$

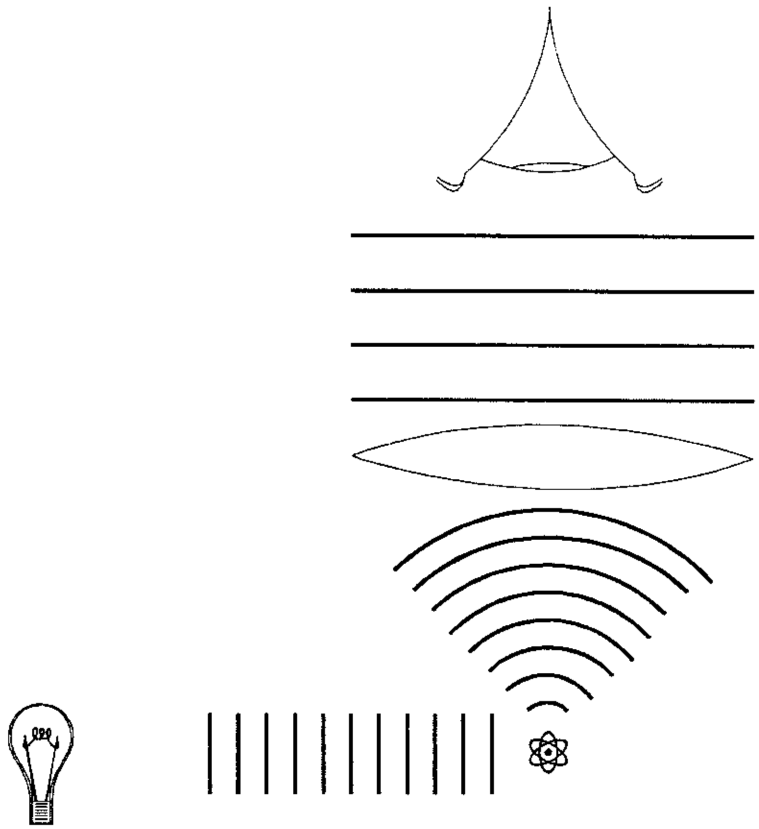
V, *I* are conjugate -- hence limit on amplifier noise temperature

$$T_{amp} = \sqrt{S_V S_I} / k_B$$

Transducer's reciprocity means noise "acts back" on test masses, enforces Uncertainty Principle.

"Heisenberg microscope" by Bohr

Photon N, ϕ are conjugate -- limit on product of position precision and recoil noise



"Heisenberg 'scope" for gravitational waves

Shot noise position precision

$$\Delta x \propto \Delta N / N = 1/\sqrt{N}$$

Shot momentum perturbation

$$\Delta p \propto \Delta F_{rad} \propto \sqrt{N}$$

Position error from Δp

$$\Delta x_{rad} = \Delta p / m\omega_0$$

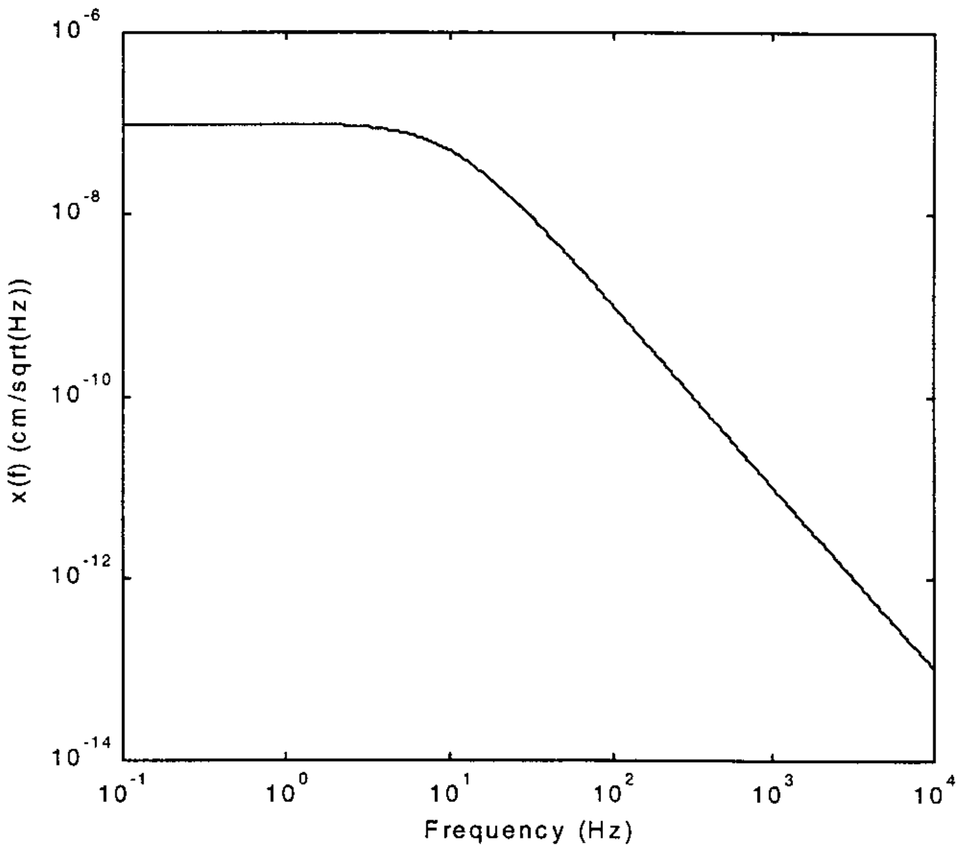
"Standard Quantum Limit"

$$\Delta x_{SQL} = \frac{1}{\omega_o}\sqrt{\frac{\hbar}{m}} = 5\times 10^{-23}\ \mathrm{m/Hz^{1/2}}$$

for 1 ton, 1 kHz

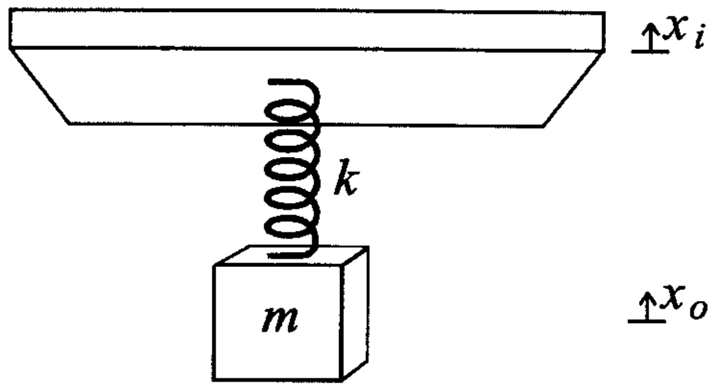
Seismic noise

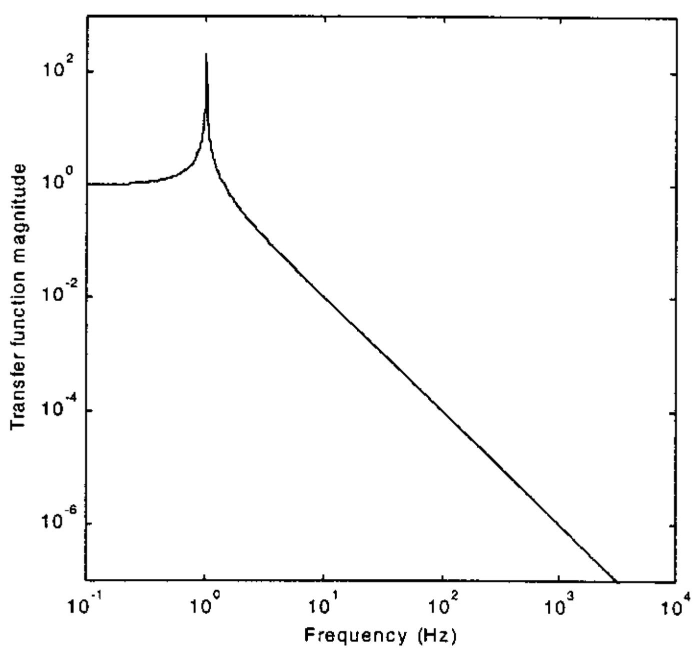
A large external reason for test masses to move



Only one trick: filter like crazy

Seismic isolation





Cascade many stages!

Preview of next lecture

- Resonant-mass and interferometric detectors
- Transduction mechanisms
- Frequency responses
- Strategies for noise reduction, extraction of signal from noise