Gravity at the Planck Length

- To understand the weak, strong, and electromagnetic interactions, it is probably necessary to include gravity as well.

Why?

- Supersymmetry
- Unification of couplings

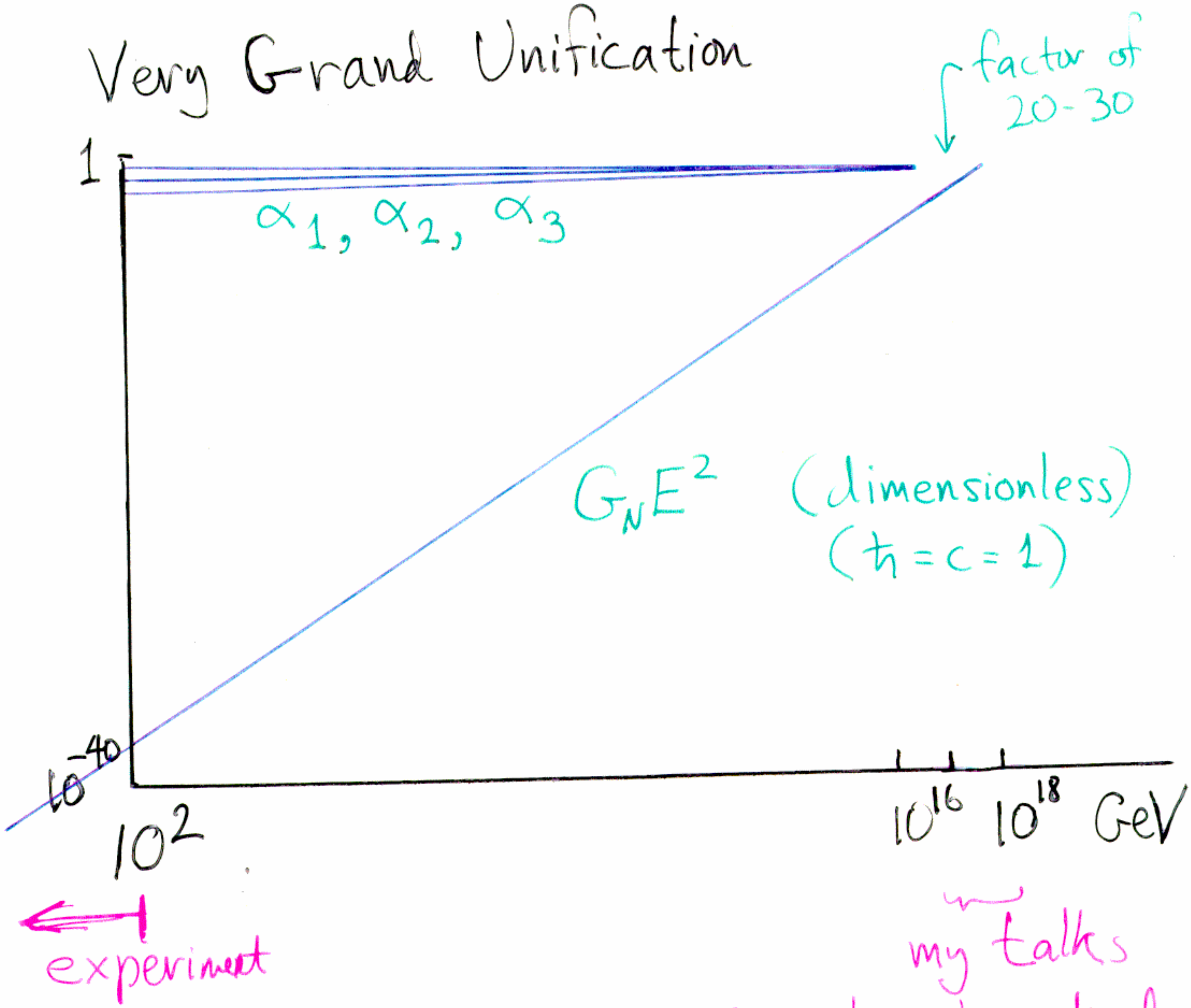
. . .

What is supersymmetry?

A. A lot of new particles.

B. A spacetime symmetry (like general coordinate invariance), the first new one since General Relativity $\Rightarrow$ particle interactions closer to gravity.

Very Grand Unification



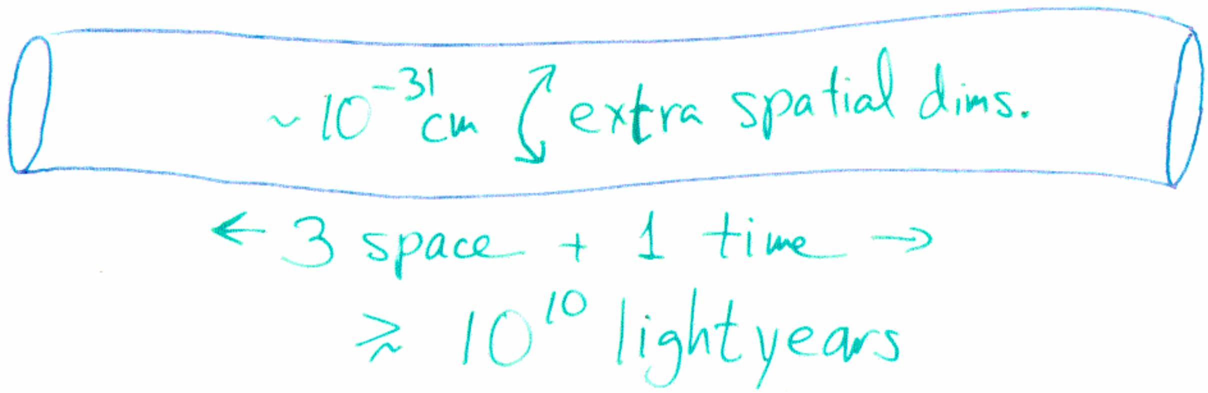
my talks (mostly theoretical, see also talks by Peskin, Arkani-Hamed)

Main topics:

- Beyond four dimensions
- String theory
- Duality in field and string theory (and D-branes, M-theory, ...)
- An alternative to string theory?
- Black hole quantum mechanics
- Large-N_c gauge theory?!
- Outlook

Higher Dimensions

Possible geometry for spacetime:



Wavelengths > size of small dimensions see only large dimensions, so low energy physicist sees:

- Why this is a natural idea
- Why it is a good idea

Why extra dimensions are natural:

- Cosmological argument – the four dimensions we see were once smaller and highly curved. Perhaps there are others that <u>remain</u> small and highly curved.

- Symmetry breaking – most symmetry in nature is spontaneously broken, or otherwise hidden; e.g.

$$SU(3) \times SU(2) \times U(1) \Rightarrow U(1),$$

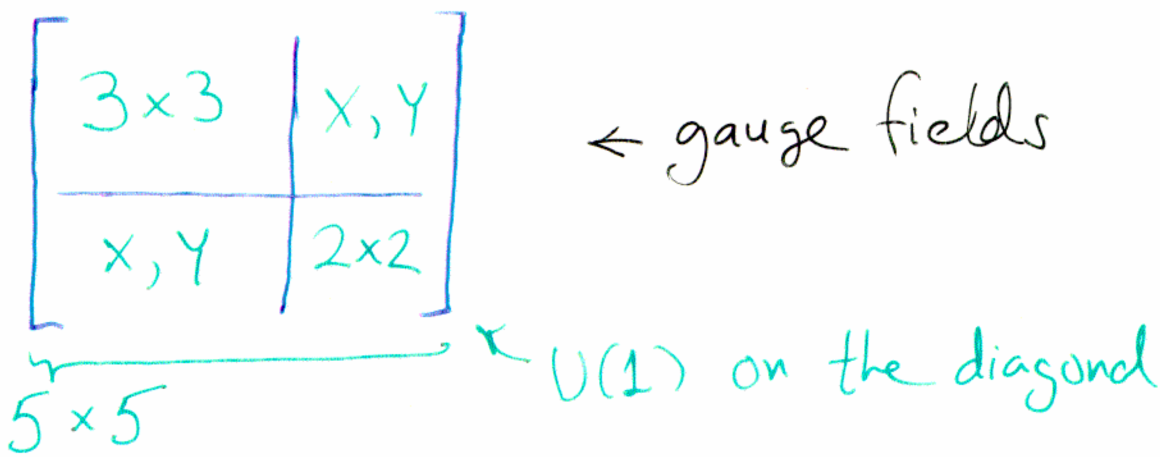
etc., etc. Perhaps the same is true of spacetime symmetries,

$$SO(3,1) \quad \text{(Lorentz inv., rotations + boosts)}$$
$$\subset SO(3+n,1),$$

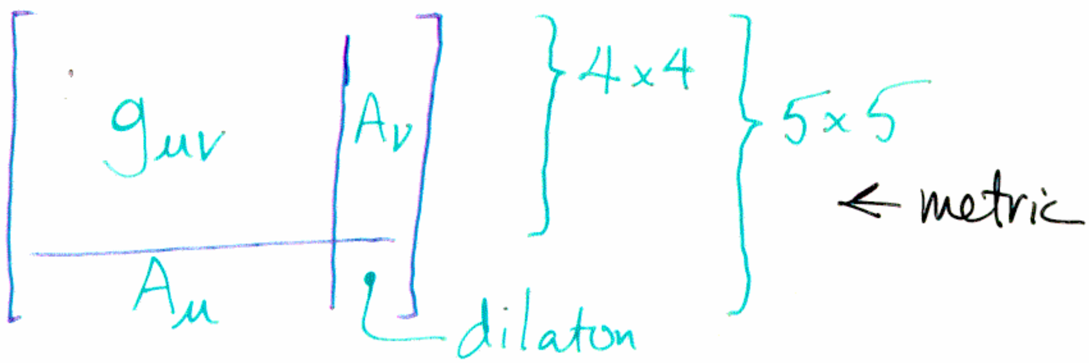
$\Rightarrow$ n extra spatial dimensions

Why extra dimensions are likely —

- Cartoon version of Grand Unification:



- Cartoon version of Kaluza-Klein theory:



Einstein's equation in 5 dimensions →

Einstein + Maxwell in 4.

• Higher-dimensional spinor fields → generations, repeated copies of the same gauge quantum numbers.

• Spacetime symmetry of string theory is $SO(9,1)$. (Λ_{CW}: $SO(10,1)$)

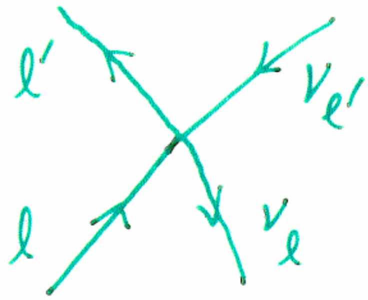
Main signature of higher-dimensional threshold:

At $E < \frac{1}{R}$ ← size of small dimension, one excites only states with wavefunctions 'independent' of small dimensions. At $E \gtrsim \frac{1}{R}$, many new particles

- String Theory

UV Problem of Quantum Gravity —

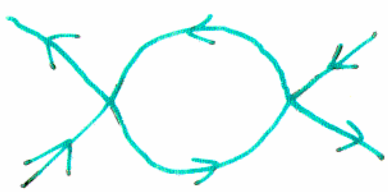
recall UV problem of 4-fermi weak interaction —



coupling = G_F, units $(\text{energy})^{-2}$ ($\hbar = c = 1$).

Dimensionless coupling = $G_F E^2$ $\Rightarrow$

Perturbation theory breaks down at high energy; nonrenormalizable ∞'s

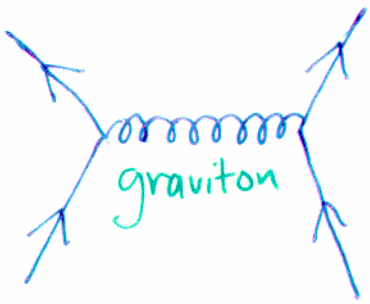


← diverges at short distance

Indication that theory is breaking down, new physics smears out interaction —

<u>spontaneously broken Yang-Mills</u>
(Weinberg-Salam)

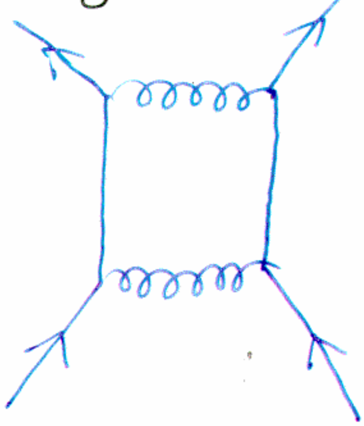
Gravity:



Coupling $G_N \sim (\text{energy})^{-2}$

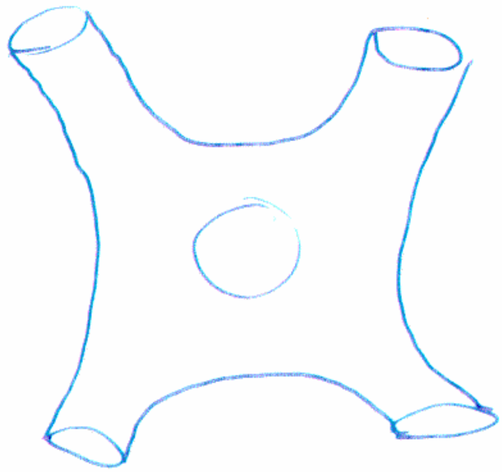
Dimensionless: $G_N E^2$

⇒ breakdown of theory, nonrenormalizable divergences at high energy:



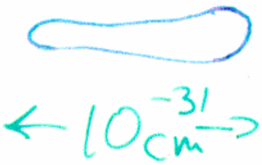
diverges when interactions become coincident

New physics needed to smear out interaction; the only known way is:



String theory:

...



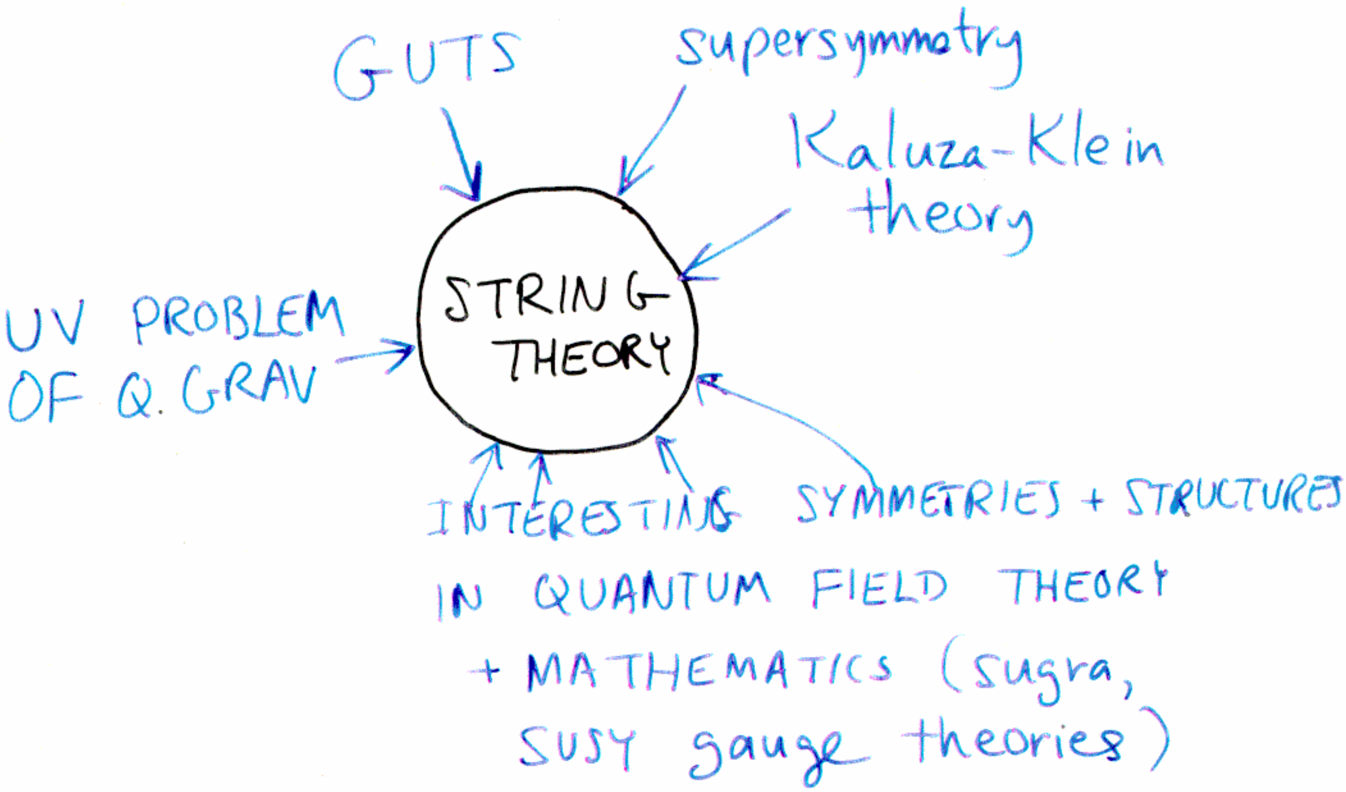
Different internal states $\Rightarrow$ graviton, gauge bosons, spin-$\frac{1}{2}$ and spin-0 $\Longrightarrow$ all of Standard Model from one building block. One interaction:

different components $\rightarrow$ gauge, gravitational, Yukawa interactions.

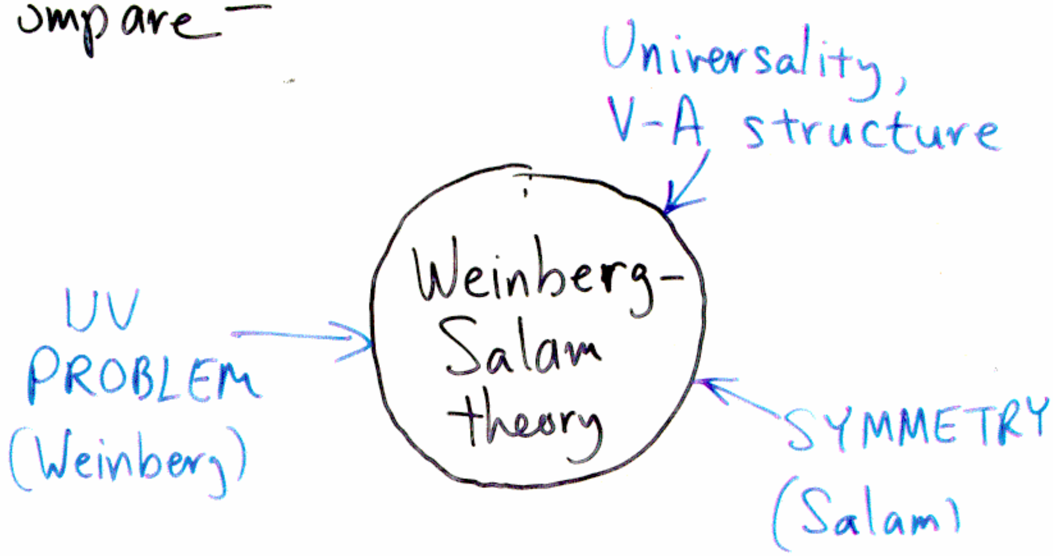
Very restrictive: string must have massless spin-2 state, whose long-distance interactions are governed by General Relativity.

String must move in "superspace".

All roads lead to string theory —



Compare —



Main problem: four dimensional physics depends on shape of compact dimensions, and there are many classical solutions. All but a few are destabilized by quantum effects, but these are not fully understood.

Deeper problem: we understand small numbers of strings interacting weakly (perturbation theory), but we need to understand many degrees of freedom w/ strong interactions. In quantum field theory these give:

spontaneous symmetry breaking
dynamical symmetry breaking
quark confinement
dimensional transmutation

These dynamical effects play a key role in the Standard Model (falsifiability; Pauli).

We expect these + many more in string theory.

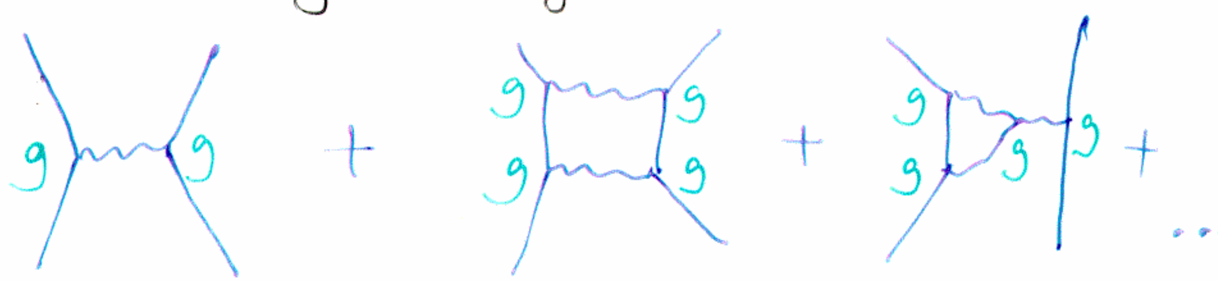
Deeper still: What is string theory? (Perturbation series doesn't converge).

- Since 1994: new methods, many new results + surprises in strongly coupled gauge + string theories.

- **Duality**: equivalence of seemingly distinct physical systems. Common in 1+1 dimensions (e.g. high temperature/low temperature duality of Ising model).

Surprise: common in 3+1 dimensions also, and in string theory.

Weak/strong duality:



Useful when g is small, useless when it is large. But in some cases there is an equivalent (dual) field theory with $g' = 1/g$.

≈ change of variables in path integral.

Electric/magnetic duality

- Maxwell's equations without source are invariant under $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$. This suggests we add magnetic source terms.

- Dirac quantization condition (1949)

$$q_{\text{Electric}}\, q_{\text{Magnetic}} = 2\pi n \hbar$$

(n = integer)

relates magnetic charge to quantization of electric charge.

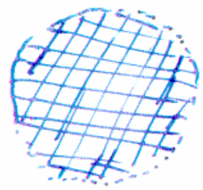
- 't Hooft, Polyakov (1974)

  unification $\Rightarrow$ magnetic monopoles.

- at weak coupling:

  - electric charges • q_E

    are small, light, weakly coupled

  - magnetic charges  q_M

    are extended (solitons), heavy, strongly coupled ($q_M = \frac{2\pi}{q_E}$, Dirac)

- conjecture (Montonen-Olive, 1977): at strong coupling everything is reversed → <u>dual</u> description in terms of magnetically coupled fields.

  Initial reaction, skepticism; since 1994, strong circumstantial evidence.

Supersymmetry: in addition to the ordinary spacetime dimensions, whose coordinates are real numbers, there are dimensions with fermionic coordinates θ_i. ($\theta_i^2 = 0$ so they have zero "size").

Relates masses and couplings of bosons, fermions.

Recall the distinction between symmetry and dynamics. Supersymmetry gives some dynamical information (exact energy eigenvalues).

Ordinary symmetry algebra:

$$[H, G] = 0$$

H = Hamiltonian, G = ordinary symmetry (electric charge, baryon#, ...)

Supersymmetry algebra:

$$[H, Q] = 0 \quad \underline{\text{and}} \quad Q^2 = H + G.$$

NOTE! all indices and coefficients are omitted. Q = supersymmetry charge.

Hamiltonian on right-hand side ↗ so symmetry constrains dynamics.

To see this we consider special states, BPS* states, which have zero supersymmetry charge,**

$$Q|\psi\rangle = 0.$$

* Bogomolnyi - Prasad - Sommerfield

** under one or more Q's, not all of them.

Then

$$0 = \langle\psi|Q^2|\psi\rangle = \langle\psi|H|\psi\rangle + \langle\psi|G|\psi\rangle$$

= energy + charge.

(LHS = 0 because $Q|\psi\rangle = 0$).

(energy) = −(charge) (coefficient omitted)

Exact calculation of energy eigenvalue, does not depend on weak coupling.

Only works for BPS states, but by using this and other information one can deduce the strongly coupled dynamics.

(Seiberg 1994 ...)

one (of many) consequences: electric/magnetic duality is true for some supersymmetric gauge theories. (though still circumstantial)