Unification and Gravity

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Particle physics has traditionally concerned itself with the interactions visible in nuclear processes

- strong, weak, electromagnetic interactions
- interactions responsible for quark + lepton masses

Why should gravity become involved ?

The Standard Model gives an excellent description of strong, weak, & electromagnetic interactions in terms of a local quantum field theory

It's time to move on to the next set of questions.

Here we find circumstantial evidence for the role of gravity.

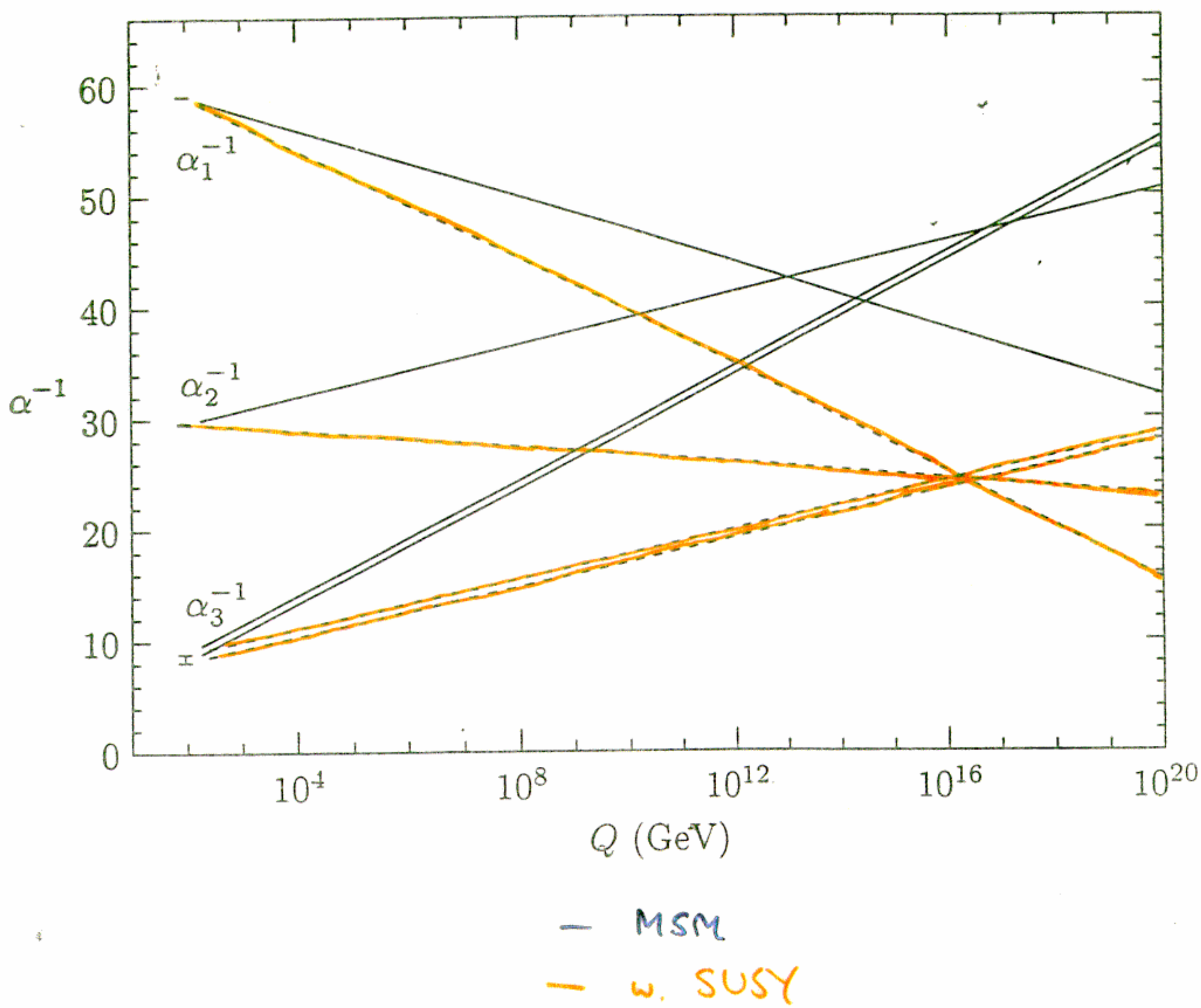
1. the relative strength of the gauge interactions is explained by grand unification and the running of couplings

$$\alpha_i^{-1}(m_Z) = \alpha_U^{-1} - \frac{b_i}{2\pi}\log\left(\frac{m_U}{m_Z}\right) + \cdots$$

but

this requires $\log\left(\frac{m_U}{m_Z}\right) \sim 20$

i.e. m_U near the Planck scale



60
50
40
30
20
10
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
10^4
10^8
10^{12}
10^{16}
10^{20}
Q (GeV)
— MSM
— w. SUSY

2

the idea of spontaneous symmetry breaking poses serious problems of fine-tuning:

- precision electroweak ↓

Higgs is probably an elementary scalar

Can we calculate its potential?

Why is $|\mu|^2 \sim 10^{-28}\, m_U^2$?

the best solutions invoke supersymmetry, a modified space-time structure.

- vacuum energy shifts should contribute to the cosmological constant:

Veltman

	ΔV	R_0
electroweak	$(100\,GeV)^4$	10 cm
strong interaction chiral symmetry breaking	$(100\,MeV)^4$	10^7 cm
λ	$(10^{-3}\,eV)^4$	10^{29} cm

3 the remaining mysterious parameters of the Standard Model cannot be determined within quantum field theory

- gauge theories are consistent, even nonperturbatively for a range of choices of n_g

- Higgs Yukawa couplings are "renormalizable" i.e. adjustible input parameters

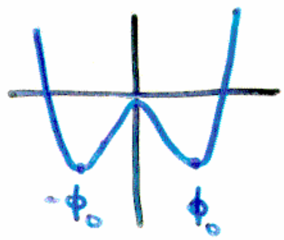
- the Standard Model is chiral ; why ?

[nonperturbative gravity gets stuck here ...]

The solution of this last set of problems seems to require a mechanical model that produces massless fermions in 4-d.

Here is a simple one ...

Consider the Dirac eq. in 5-dimensions, coupled to a scalar field ϕ with a double-well potential



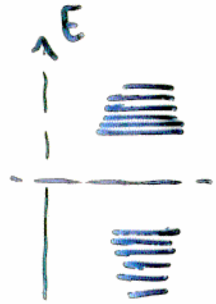
$$\left[i\gamma^M \partial_M - \lambda \phi(x) \right] \Psi = 0$$

$$\gamma^M = \left\{ \underset{\gamma^0}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}, \underset{\gamma^i}{\begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}}, \underset{\gamma^4}{\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}} \right\}$$

for $\phi(x) = \phi_0$

we have a 5-d fermion w. mass

$$m = \lambda \phi_0$$



look for a solution which obeys the equation for a massless spinor in 4-d.

4-d L-fermion:

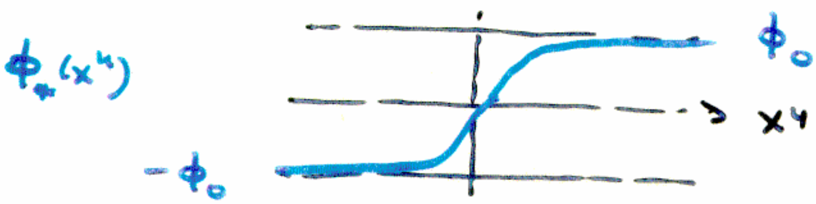
$$[\, i\partial_0 - i\vec{\sigma}\cdot\vec{\partial}\,]\,\psi_L = 0$$

$$\psi_L = e^{-ik\cdot x}\, u_k \qquad (k^0 + \vec{k}\cdot\vec{\sigma})\, u_k = 0$$

for the 5-d problem, look for a solution

$$\Psi = e^{-ik\cdot x} \begin{pmatrix} u_k \\ 0 \end{pmatrix} \cdot w(x^4)$$

We can find a solution of this kind in a scalar field configuration



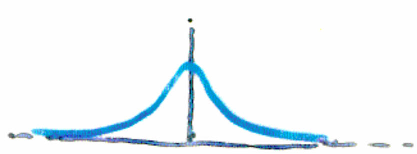
then $\quad [i\gamma^M \partial_M - \lambda\phi]\,\Psi$

becomes: $\quad \begin{pmatrix} -\partial_4 - \lambda\phi_*(x^4) \\ i\partial_0 - i\vec{\sigma}\cdot\vec{\partial} \end{pmatrix} \cdot e^{-ik\cdot x}\begin{pmatrix} u_L \\ 0 \end{pmatrix} \omega(x^4)$

the Dirac eq. is satisfied for

$$\partial_4\,\omega = -\lambda\,\phi_*(x^4)\cdot\omega$$

$$\omega = \exp\left[-\lambda\int_0^{x^4} dz\;\phi_*(z)\right]$$



if we seek a solution for a right-handed 4-d fermion, we find

$$\Psi = e^{-ik\cdot x} \begin{pmatrix} 0 \\ u_R \end{pmatrix} \bar{\omega}(x^4)$$

where $\quad (+\,\partial_4 - \lambda\,\phi_*(x^4))\,\bar{\omega} = 0$

this gives $$\bar{\omega} = \exp\left[+ \int_0^{x^4} dz\, \phi_*(z) \right]$$

which $\to \infty$ as $x^4 \to \pm\infty$;

this is not physically sensible

thus,

the domain wall $\phi_*(x)$ leads to a

left-handed chiral fermion in 4-d !

generalize ! consider

$$i\gamma^M D_M \Psi = 0$$

coupling to gauge + gravitational fields in extra dimensions

$$\gamma^M = \{ \gamma^\mu \otimes 1 , \quad \gamma^5 \otimes \gamma^\eta \}$$

$\mu = 0123 \qquad \eta : 1 - d$

solution :

$$\Psi = e^{-ikx} u_L \cdot \omega(y)$$

is a massless L-fermion in 4-d

if ω is a chiral fermion in d-d

satisfying

$$i\gamma^\eta D_\eta \omega = 0$$

Now we can invoke the

Atiyah - Singer Index Theorem

in a general gauge and gravitational background field configuration

$$i\gamma\cdot D\ \omega = 0$$

has n_+ chiral and n_- antichiral solutions with

$$\Delta n = n_+ - n_- = \text{a topological number of the background field conf.}$$

2-D flux tube

$$\Delta n = \frac{\Phi}{\Phi_0}$$

4-D instanton

$$\Delta n = 2\cdot\frac{\int F\tilde{F}}{32\pi^2}$$

6-D manifold w. $R_{\mu\nu} = 0$

$$\Delta n = \frac{1}{2}\chi_E$$

In this way, the result

$$n_g = 3$$

counts the topological number

of a field config. in extra dimensions

Higgs Yukawa couplings

are overlaps of the wavefunctions $\omega(y)$

$$\sim \int d^d y \; \omega_Q(y)\, \omega_\phi(y)\, \omega_{d_R}(y)$$

It seems that we have reduced the problem
of generations to that of finding
solutions of classical field eqs. in extra dimensions

but,
it is not necessarily so simple . . .

We must use a consistent theory of
quantum gravity

There is only one (so far) :
String theory!

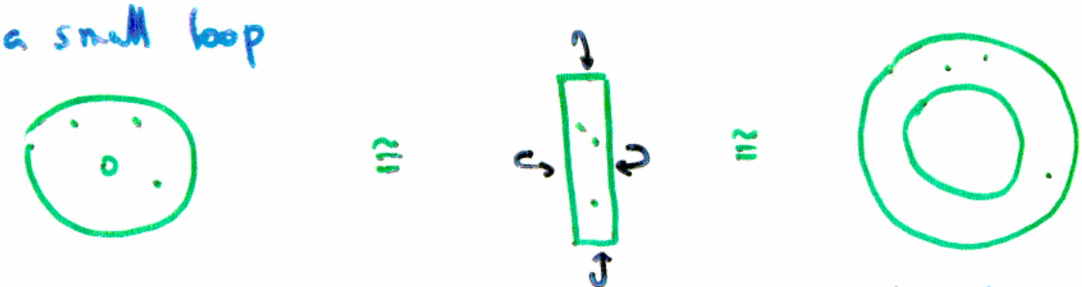
String theory is intrinsically a quantum theory
which arose originally from particle physics ;
its use in gravity leads to profound changes
in our picture of space-time .

String theory cures the divergences of quantum gravity. How?

in field theory:  diverges as the loop becomes small

in string theory: 

a small loop

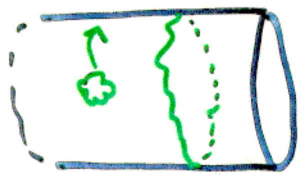


is equivalent to a long loop

and is not counted separately

similarly, very small spatial loops are omitted in string theory

large loop



$$E \sim \left\{ \left|\frac{2\pi n}{R}\right|, \ |mRT| \right\}$$

small loop

$$E \sim \left\{ \left|\frac{2\pi n}{R'}\right|, \ |mR'T| \right\}$$

$R' \ll \frac{1}{\sqrt{T}}$ is equivalent to

$$R = \frac{2\pi}{TR'} \gg \frac{1}{\sqrt{T}}$$

At strong coupling, string theory contains
even more counterintuitive space-time rearrangements

Witten:

10-D closed string theory contains localized solutions
w. charge n and mass

$$m = \frac{|n|}{g} \qquad \text{(exact result!)}$$

Let $g \to \infty$; reinterpret this as
the spectrum of momenta about a compact dimension

then we find

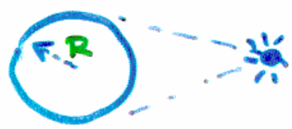
$$R_{11} \sim g$$

opening up as $g \to \infty$

Strominger

the compact space dimensions may become singular

as



S^2

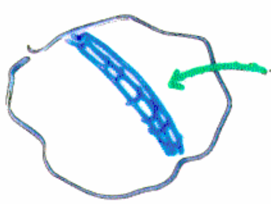
but, string theory contains 2-d surfaces that can wrap around this sphere

$R \to 0$ extra SU(2) gauge symmetry

$R < 0$ spontaneously broken SU(2)

$\approx$ topology change

Kachru - Silverstein



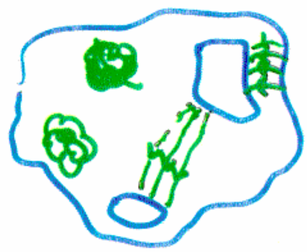
4-d instanton of size ρ containing a quark/lepton generation

as $\rho \to 0$ we come to a singular configuration

continue to $\rho < 0$,

the singularity lifts out into the 11th dimension

Thus, we find a picture of the higher dimensions as highly curved & filled with structure:



When supersymmetry is unbroken, many such distinct configurations are degenerate

The final favored configuration is selected by spontaneous breaking of supersymmetry

The parameters of these configurations become effective scalar fields; the potential may give these only small masses

Dimopoulos + Giudice

$$\mathcal{L} = \int d^4x \; V_d \left\{ \frac{1}{2} m_{str.}^2 (\partial_\mu \sigma)^2 - \mathcal{E}(\sigma) \right\}$$

σ = dim-less

$$= \int d^4x \left\{ \frac{1}{2}(\partial_\mu \Sigma)^2 - \frac{1}{2} m^2 \Sigma^2 \right\}$$

with

$$m \sim \left[\frac{\mathcal{E}}{m_{str}^2} \right]^{\frac{1}{2}} \sim \frac{\Lambda^2}{m_{str.}}$$

$m_{str} \sim 10^{18}$ GeV $\qquad \Lambda \sim$ energy scale of $\mathcal{E}(\sigma)$

The value of this estimate depends on the mechanism of supersymmetry breaking

gravity-mediated: $\frac{\Lambda^2}{m_{str.}} \sim 1\ \mathrm{TeV}$

gauge-mediated: $\Lambda \sim 1\ \mathrm{TeV}$

$$\downarrow$$

$$m \sim 10^{-3}\,\mathrm{eV} \sim [0.1\,\mathrm{mm}]^{-1}$$

- it is possible for m to be larger if $\mathcal{E}(\sigma)$ does not break supersymmetry
  (Krasnikov - Kaplunovsky racetrack)

- the measurement of the spectrum of superparticles will give concrete experimental information on $\mathcal{E}(\sigma)$

Note that, when we try to compute $\mathcal{E}(0)$
the cosmological constant problem
is inescapable !

Does $\mathcal{E}(0)$ relax to a minimum where $\lambda = 0$?

Is there an associated light scalar
particle ?

Is there a dual description of Nature
where $\lambda = 0$ manifestly ?

Witten : $g \to \infty$ limit of 3-d world

Kachru, Kumar, Silverstein : competing supersymmetries

Conclusions? (more questions than answers)

Our study of the traditional questions of particle physics has led us to address the deepest fundamental questions of gravitational physics

Hopefully, it has also given us new insights toward the solution of these problems ....