PRECISION ELECTROWEAK MEASUREMENTS AND "NEW PHYSICS"

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ABSTRACT

The status of several precisely measured electroweak parameters is reviewed. Natural relations among them are shown to constrain the Higgs mass, m_H, as well as various "New Physics" effects. Indications of an anomalous $Zb\bar{b}$ coupling are discussed. Constraints on excited W^* bosons are given.

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1 Fundamental Parameters and Natural Relations

The $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ electroweak sector of the standard model contains 17 or more fundamental parameters. They include gauge and Higgs field couplings as well as fermion masses and mixing angles. In terms of those parameters, predictions can be made with high accuracy for essentially any electroweak observable. Very precise measurements of those quantities can then be used to test the standard model, even at the quantum loop level, or search for small deviations from expectations which would indicate "New Physics".

Some fundamental electroweak parameters have been determined with extraordinary precision. Foremost in that category is the fine structure constant α. It can best be obtained by comparing the measured[1] anomalous magnetic moment of the electron, $a_e \equiv (g_e - 2)/2$

$$a_e^{\mathrm{exp}} = 1159652188(3) \times 10^{-12} \tag{1}$$

with the calculated 4 loop QED prediction[2]

$$\begin{aligned} a_e^{\mathrm{th}} &= \frac{\alpha}{2\pi} - 0.328478444 \left(\frac{\alpha}{\pi}\right)^2 + 1.181234 \left(\frac{\alpha}{\pi}\right)^3 - 1.5098 \left(\frac{\alpha}{\pi}\right)^4 \\ &\quad + 1.66 \times 10^{-12} \end{aligned} \tag{2}$$

where the 1.66×10^{-12} comes from small hadronic and weak loop effects. Assuming no significant "new physics" contributions to a_e^{th}, it can be equated with (1) to give

$$\alpha^{-1} = 137.03599959(40) \tag{3}$$

That precision is already quite extraordinary. Further improvement by a factor of 10 appears to be technically feasible and should certainly be undertaken. However, at this time such improvement would not further our ability to test QED. QED tests require comparable measurements of α in other processes. Agreement between two distinct α determinations tests QED and probes for "new physics" effects. After a_e, the next best (direct) measurement of α comes from the quantum Hall effect

$$\alpha^{-1}(qH) = 137.03600370(270) \tag{4}$$

which is not nearly as precise. Nevertheless, the agreement of (3) and (4) (at the 1.50 level) is a major triumph for QED up to the 4 loop quantum level.

In terms of probing "new physics", one can search for a shift in a_e by m_e^2/Λ_e^2 where Λ_e is the approximate scale of some generic new short-distance effect. Current comparison of $a_e \to \alpha$ and $\alpha(qH)$ explores $\Lambda_e \lesssim 100$ GeV. To probe the much more interesting $\Lambda_e \sim \mathcal{O}$ (TeV) region would require an order of magnitude improvement in a_e and about two orders of magnitude error reduction in some direct precision determination of α such as the quantum Hall effect. Perhaps the most likely possibility is to use the already very precisely measured Rydberg constant in conjunction with a much improved m_e determination to obtain an independent α.

The usual fine structure constant, α, is defined at zero momentum transfer as is appropriate for low energy atomic physics phenomena. However, that definition is not well suited for short-distance electroweak effects. Vacuum polarization loops screen charges such that the effective (running) electric charge increases at short-distances. One can incorporate those quantum loop contributions into a short-distance[4] $\alpha(m_Z)$ defined at $q^2 = m_Z^2$. The main effect comes from lepton loops, which can be very precisely calculated, and somewhat smaller hadronic loops. The latter are not as theoretically clean and must be obtained by combining perturbative calculations with results of a dispersion relation which employs $\mathcal{O}(e^+e^- \to \text{hadrons})$ data. A recent study by Davier and Höcker finds[5]

$$\alpha^{-1}(m_Z) = 128.933(21) \tag{5}$$

where the uncertainty stems from low energy hadronic loops. Although not nearly as precise as α^{-1}, the uncertainty quoted in (5) is impressively small and a tribute to the effort that has gone into reducing it. (When I first studied this issue in 1979, I estimated[4] $\alpha^{-1}(m_Z) \simeq 128.5 \pm 1.0$.) However, the error in (5) is still somewhat controversial, primarily because of its reliance on perturbative QCD down to very low energies. For comparison, an earlier study by Eidelman and Jegerlehner,[6] which relied less on perturbative QCD and more on e^+e^- data found

$$\alpha^{-1}(m_Z) = 128.896(90) \qquad (\text{E \& J } 1995) \tag{6}$$

That estimated uncertainty is often cited as more conservative and therefore employed in m_H and "new physics" constraints. As we shall see, the smaller uncertainty in (5) has very important consequences for predicting the Higgs mass. I note that a more recent study[7] by Eidelman and Jegerlehner finds

$$\alpha^{-1}(m_Z) = 128.913(35) \qquad (\text{E \& J } 1998) \tag{7}$$

which is in good accord with (5) and also exhibits relatively small uncertainty. In my subsequent discussion, I employ the result in (5), but caution the reader that a more conservative approach would expand the uncertainty, perhaps even by as much as a factor of 4 or 5.

A related short-distance coupling, $\alpha(m_Z)_{\overline{MS}}$, can be defined by modified minimal subtraction at scale $\mu = m_Z$. It is particularly useful for studies of coupling unification in grand unified theories (GUTS) where a uniform comparitive definition ($\overline{MS}$) of all couplings is called for.[8] The quantities $\alpha(m_Z)$ and $\alpha(m_Z)_{\overline{MS}}$ differ by a constant, such that[9]

$$\alpha^{-1}(m_Z)_{\overline{MS}} = \alpha^{-1}(m_Z) - 0.982 = 127.951(21) \tag{8}$$

In weak interaction physics, the most precisely determined parameter is the Fermi constant, G_μ, as obtained from the muon lifetime. One extracts that quantity by comparing the experimental value

$$\tau_\mu = 2.197035(40) \times 10^{-6} s \tag{9}$$

with the theoretical prediction

$$\begin{aligned} \tau_\mu^{-1} = \Gamma(\mu \to \text{all}) &= \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \text{R.C.})\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2}\right) \\ f(x) &= 1 - 8x + 8x^3 - x^4 - 12x^2 \ell n x \end{aligned} \tag{10}$$

In that expression R.C. stands for Radiative Corrections. Those terms are somewhat arbitrary in the standard model. The point being that G_μ is a renormalized parameter which is used to absorb most loop corrections to muon decay. Those corrections not absorbed into G_μ are explicitly factored out in R.C. For historical reasons and in the spirit of effective field theory approaches, R.C. has been chosen to be QED corrections to the old V-A four fermion description of muon decay.[10] That definition is practical, since the QED corrections to muon decay in the old V-A theory are finite to all orders in perturbation theory. In that way, one finds

$$\text{R.C.} = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{\alpha}{\pi}\left(\frac{2}{3}\ell n\frac{m_\mu}{m_e} - 3.7\right) + \left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{9}\ell n^2\frac{m_\mu}{m_e} - 2.0\ell n\frac{m_\mu}{m_e} + C\right)\cdots\right) \tag{11}$$

The leading $\mathcal{O}(\alpha)$ terms in that expression have been known for a long time from the pioneering work of Kinoshita and Sirlin[11] and Berman.[12] Coefficients of the higher order logs can be obtained from the renormalization group constraint[13]

$$\left(m_e \frac{\partial}{\partial m_e} + \beta(\alpha)\frac{\partial}{\partial \alpha}\right) \text{R.C.} = 0$$
$$\beta(\alpha) = \frac{2}{3}\frac{\alpha^2}{\pi} + \frac{1}{2}\frac{\alpha^3}{\pi^2}\cdots \tag{12}$$

The -3.7 two loop constant in parenthesis was very recently computed by van Ritbergen and Stuart.[14] It almost exactly cancels the leading log two loop correction obtained from the renormalization group approach (or mass singularities argument) of Roos and Sirlin.[13] Hence, the original $\mathcal{O}(\alpha)$ correction in (9) is a much better approximation than one might have guessed. Comparing (9) and (10), one finds

$$G_\mu = 1.16637(1) \times 10^{-5}\ \text{GeV}^{-2} \tag{13}$$

There have been several experimental proposals to reduce the uncertainty in τ_μ and G_μ by a factor of 10. Such improvement appears technically feasible and, given the fundamental nature of G_μ, should certainly be undertaken. However, from the point of view of testing the standard model, the situation is similar to α. G_μ is already much better known than the other parameters it can be compared with; so, significant improvement must be made in other quantities before a more precise G_μ is required. This point should become clearer subsequently when I describe other indirect Fermi constant determinations and their uncertainty (about 100 times worse than (13)).

Let me emphasize the fact that lots of interesting loop effects have been absorbed into the renormalization of $g_{2_0}^2/4\sqrt{2}m_W^{0^2}$ which we call G_μ. Included are top quark[15] and Higgs loop corrections[16] to the W boson propagator as well as potential "new physics" from SUSY loops, Technicolor etc. Even tree level effects of possible more massive gauge bosons such as $W^{*^\pm}$ bosons are effectively incorporated into G_μ. To uncover those contributions requires comparison of G_μ with other precisely measured electroweak parameters which have different quantum loop (or tree level) dependences. Of course, those quantities must be related to G_μ in such a way that short-distance divergences cancel in the comparison.

Fortunately, due to an underlying global SU(2)$_V$ symmetry in the standard model, there exist natural relations among various bare parameters[17]

$$\sin^2\theta_W^0 = \frac{e_0^2}{g_{2_0}^2} = 1 - (m_W^0/m_Z^0)^2 \tag{14}$$

Each of those bare unrenormalized expressions contains short-distance infinities, but the divergences are the same. Therefore, those relations continue to hold for renormalized quantities, up to finite, calculable radiative corrections.[17] The residual radiative corrections contain very interesting effects such as m_t and m_H dependence as well as possible "new physics". So, for example, one can relate

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}m_W^2(1 - m_W^2/m_Z^2)}(1 + rad.\, corr.) \tag{15}$$

and test the predicted radiative corrections, if m_Z and m_W are also precisely known.

Gauge boson masses are not as well determined as G_μ, but they have reached high levels of precision. In particular, the Z mass has been measured with high statistics Breit-Wigner fits to the Z resonance at LEP with the result

$$m_Z = 91.1867(21)\ \mathrm{GeV} \tag{16}$$

That determination is so good that one must be very precise regarding the definition of m_Z. (Remember the Z has a relatively large width ~ 2.5 GeV.) The quantity in (16) is related to the real part of the Z propagator pole, m_Z (pole), and full width, Γ_Z, by[18]

$$m_Z^2 = m_Z^2(\mathrm{pole}) + \Gamma_Z^2 \tag{17}$$

The two mass definitions m_Z and m_Z (pole) differ by about 34 MeV, which is much larger than the uncertainty in (16). Hence, one must specify which definition is being employed in precision studies. I note, that the m_Z in (16) is also more appropriate for use in low energy neutral current amplitudes.

In the case of the $W^\pm$ bosons, the renormalized mass, m_W, is similarly defined by

$$m_W^2 = m_W^2(\mathrm{pole}) + \Gamma_W^2 \tag{18}$$

That quantity is obtained from studies at $p\bar{p}$ colliders, $m_W = 80.41(9)$ GeV, as well as $e^+e^- \to W^+W^-$ at LEPII, $m_W = 80.37(9)$ GeV. Together they give

$$m_W = 80.39(6)\ \mathrm{GeV}\ . \tag{19}$$

The current level of uncertainty, ± 60 MeV, is large compared to Δm_Z. It is expected that continuing efforts at LEPII and Run II at Fermilab's Tevatron should reduce that

error to about ± 30 MeV. A challenging but worthwhile goal for future high energy facilities would be to push Δm_W to ± 10 MeV or better. At that level, all sorts of interesting "new physics" effects are probed (as I later illustrate). I note that the m_W defined in (19) is also the appropriate quantity for low energy amplitudes such as muon decay.

Another important quantity for precision standard model tests is m_t, the top quark mass. Measurements from CDF and D$\emptyset$ at Fermilab give

$$m_t(\text{pole}) = 174.3 \pm 5.1 \text{ GeV} \tag{20}$$

Reducing that uncertainty further is important as we shall subsequently see. Future Tevatron efforts are expected to reduce the uncertainty in m_t to about ± 2 GeV. LHC and NLC studies should bring it well below ± 1 GeV.

In addition to masses, the renormalized weak mixing angle plays a central role in tests of the standard model. That parameter can be defined in a variety of ways, each of which has its own advocates. I list three popular examples

$$\begin{aligned} &\sin^2\theta_W(m_Z)_{\overline{MS}} && (\overline{MS} \text{ definition at } \mu = m_Z) && (a) \\ &\sin^2\theta_W^{\text{eff}} && (Z\mu\bar{\mu} \text{ vertex}) && (b) \\ &\sin^2\theta_W \equiv && 1 - m_W^2/m_Z^2 && (c) \end{aligned} \tag{21}$$

They differ by finite $\mathcal{O}(\alpha)$ loop corrections. The $\overline{MS}$ definition is particularly simple, being defined as the ratio of two $\overline{MS}$ couplings $\sin^2\theta_W(m_Z)_{\overline{MS}} \equiv e^2(m_Z)_{\overline{MS}}/g_2^2(m_Z)_{\overline{MS}}$. It was introduced for GUT studies,[8] but is useful for most electroweak analyses. The effective, $\sin^2\theta_W^{\text{eff}}$, weak angle was invented for Z pole analyses. Roughly speaking, it is defined by the ratio of vector and axial-vector components (including loops) for the on-mass-shell $Z\mu\bar{\mu}$ vertex $\rightarrow 1\text{–}4\sin^2\theta_W^{\text{eff}}$. Although conceptually rather simple, analytic electroweak radiative corrections expressed in terms of $\sin^2\theta_W^{\text{eff}}$ are complicated and ugly. Numerically, it is close to the $\overline{MS}$ definition[19]

$$\sin^2\theta_W^{\text{eff}} = \sin^2\theta_W(m_Z)_{\overline{MS}} + 0.00028 \tag{22}$$

but the analytic structure of the difference is quite complicated. For those intent on employing $\sin^2\theta_W^{\text{eff}}$, a strategy might be to calculate radiative corrections in terms of $\sin^2\theta_W\ (m_Z)_{\overline{MS}}$ and then translate to $\sin^2\theta_W^{\text{eff}}$ via (22). But why not simply use $\sin^2\theta_W\ (m_Z)_{\overline{MS}}$?

Currently, Z pole studies at LEP and SLAC give

$$\begin{aligned} \sin^2\theta_W(m_Z)_{\overline{MS}} &= 0.23100 \pm 0.00022 \\ \sin^2\theta_W^{\rm eff} &= 0.23128 \pm 0.00022 \end{aligned} \tag{23}$$

That result includes measurements of the left-right asymmetry, A_{LR}, at SLAC as well as the various lepton asymmetries at LEP and SLAC. The A_{LR} contribution had for some time given a relatively low value for the weak mixing angle, but as statistics have increased it has moved pretty much in line with (23). Currently, the $Z \to b\bar{b}$ forward-backward asymmetries at LEP give a higher $\sin^2\theta_W^{\rm eff}$ and would bring up the average, if included. However, the $Zb\bar{b}$ coupling appears to be somewhat anomalous; so, one should be cautious when including such results. I return to this problem later.

Future higher statistics running at SLAC could reduce the uncertainty in $\sin^2\theta_W^{\rm eff}$ below ± 0.0002, mainly from improvements in A_{LR}. There are very good reasons to do even better. One could imagine redoing A_{LR} at a future polarized lepton-lepton (e^+e^- or $\mu^+\mu^-$) collider, but with very high statistics. In principle, one might reduce the uncertainty in $\sin^2\theta_W^{\rm eff}$ to ± 0.00004 or lower, an incredible achievement if possible.

The so-called on-shell or mass definition[20] in (21c) also has its advocates. It can be directly obtained from m_W and m_Z determinations. Indeed, at hadron colliders, the ratio m_W/m_Z can have reduced systematic uncertainties. One could imagine that the current uncertainty in

$$\sin^2\theta_W = 1 - m_W^2/m_Z^2 = 0.2228 \pm 0.0012 \tag{24}$$

might be reduced by a factor of about 4 at the LHC. Such a reduction is extremely interesting since the comparison of $\sin^2\theta_W$ and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ provides a clean probe of "new physics". It is also possible (because of a subtle cancellation of certain loop effects[21]) to measure $\sin^2\theta_W$ in deep-inelastic $\nu_\mu N$ scattering. Indeed, a recent Fermilab experiment found[22]

$$\sin^2\theta_W = 0.2253 \pm 0.0019 \pm 0.0010 \tag{25}$$

where the first error is statistical and the second systematic. That single measurement is quite competitive with (24) and complements it nicely. One might imagine a future high statistics effort significantly reducing the error in (25), but that would require a new high energy neutrino beam.

All of the above precision measurements can be collectively used to test the standard model, predict the Higgs mass, and search for "new physics" effects. That ability stems from the natural relations in (14) and calculations[20,23] of the radiative corrections to them. Parametrizing those radiative corrections by Δr, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta\hat{r}$, one finds[24]

$$\begin{aligned}
\frac{\pi\alpha}{\sqrt{2}G_\mu m_W^2} &= \left(1-\frac{m_W^2}{m_Z^2}\right)(1-\Delta r) && (a)\\
\frac{\pi\alpha}{\sqrt{2}G_\mu m_W^2} &= \sin^2\theta_W(m_Z)_{\overline{MS}}(1-\Delta r(m_Z)_{\overline{MS}}) && (b)\\
\frac{4\pi\alpha}{\sqrt{2}G_\mu m_Z^2} &= \sin^2 2\theta_W(m_Z)_{\overline{MS}}(1-\Delta\hat{r}) && (c)
\end{aligned} \tag{26}$$

Those expressions contain all one loop corrections to α, muon decay, m_W, m_Z and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ and incorporate some leading two loop contributions. The quantities Δr and $\Delta\hat{r}$ are particularly interesting because of their dependence on m_t and m_H. In addition, all three quantities provide probes of "new physics".

Numerically, all three radiative corrections in (26) contain a significant contribution from vacuum polarization effects[4] in α, about $+7\%$. They are basically the same as the corrections that enter into the evolution of α to $\alpha(m_Z)$. Leptonic loops contribute a significant part of that effect and can be very accurately computed. Hadronic loops are less clean theoretically and lead to a common uncertainty in Δr, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta\hat{r}$ of

$$-\alpha\Delta\alpha^{-1}(m_Z) \tag{27}$$

For $\Delta\alpha^{-1}(m_Z) = 0.021$ as in (5), that amounts to a rather negligible ± 0.00015 error. However, for $\Delta\alpha^{-1}(m_Z) = \pm 0.090$ as in (6), it increases to ± 0.00066. That large an uncertainty would impact precision tests. If one wishes to avoid that low energy hadronic loop uncertainty, dependence on α can be circumvented by considering

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = \left(1-\frac{m_W^2}{m_Z^2}\right)(1-\Delta r+\Delta r(m_Z)_{\overline{MS}}) \tag{28}$$

Currently, that comparison is not competitive in constraining m_H. However, future significant improvements in m_W could make it very interesting.

Using $m_t = 174.3 \pm 5.1$ GeV as input, one can compute the radiative corrections in (26) as functions of m_H. Those results are illustrated in table 1. Note that Δr is

most sensitive to changes in m_H but also carries the largest uncertainty from $\Delta m_t = \pm 5.1$ GeV (± 0.0020). Hence, efforts to determine m_H from m_W will require a better measurement of m_t. On the other hand, determining m_H from $\sin^2\theta_W(m_Z)_{\overline{MS}}$ via $\Delta\hat{r}$ is less sensitive to Δm_t but more sensitive to $\Delta\alpha^{-1}(m_Z)$. Those dependences are illustrated by the following approximate relations[25] obtained from (26a) and (26c)

$$m_W = (80.385 \pm 0.032 \pm 0.003 \text{ GeV}) \left(1 - 0.00072\ell n\left(\frac{m_H}{100 \text{ GeV}}\right) - 1\times 10^{-4}\ell n^2\left(\frac{m_H}{100 \text{ GeV}}\right)\right) \quad (29)$$

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = (0.23112 \pm 0.00016 \pm 0.00006)\left(1 + 0.00226\ell n\left(\frac{m_H}{100 \text{ GeV}}\right)\right) \quad (30)$$

where the errors correspond to $\Delta m_t = \pm 5.1$ GeV and $\Delta\alpha^{-1}(m_Z) = \pm 0.021$ respectively. Note that increasing $\Delta\alpha^{-1}(m_Z)$ to ± 0.090 would significantly compromise the utility of $\sin^2\theta_W(m_Z)_{\overline{MS}}$ for determining m_H but have less of an impact on m_W. Predictions for m_W and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ are illustrated in table 2 for various m_H values.[26]

Table 1: Values of Δr, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta\hat{r}$ for various m_H. A top quark mass of 174.3 ± 5.1 GeV and $\alpha^{-1}(m_Z) = 128.933(21)$ are assumed.

m_H (GeV)	Δr	$\Delta r(m_Z)_{\overline{MS}}$	$\Delta\hat{r}$
	$\pm 0.0020 \pm 0.0002$	$\pm 0.0001 \pm 0.0002$	$\pm 0.0005 \pm 0.0002$
75	0.03402	0.06914	0.05897
100	0.03497	0.06937	0.05940
125	0.03575	0.06955	0.05974
150	0.03646	0.06964	0.06000
200	0.03759	0.06980	0.06042
400	0.04065	0.07005	0.06144

Employing $m_W = 80.39 \pm 0.06$ GeV and $\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.23100 \pm 0.00022$, one finds

$$m_H = 92^{+141+75+5}_{-67-35-5} \text{ GeV} \qquad (\text{from } m_W) \quad (31)$$

$$m_H = 79^{+41+28+9}_{-27-21-8} \text{ GeV} \qquad (\text{from } \sin^2\theta_W(m_Z)_{\overline{MS}}) \quad (32)$$

Table 2: Predictions for m_W and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ for various m_H values.

m_H (GeV)	m_W (GeV)	$\sin^2\theta_W(m_Z)_{\overline{MS}}$
75	80.401	0.23097
100	80.385	0.23112
125	80.372	0.23124
150	80.360	0.23133
200	80.341	0.23148
400	80.289	0.23184

where the second and third errors correspond to $\Delta m_t = \pm 5.1$ GeV and $\Delta\alpha^{-1}(m_Z) = \pm 0.021$. Several features of those predictions are revealing. The first is that $\sin^2\theta_W(m_Z)_{\overline{MS}}$ currently gives a very good (best) determination of m_H. Note, however, the uncertainties scale as the central value; so, the relatively small value, 79 GeV, helps reduce the uncertainties. Also, a larger $\Delta\alpha^{-1}(m_Z) = \pm 0.090$ would significantly increase the overall uncertainty.[27] In the case of m_W, one needs a better measurement of that parameter along with improvement in m_t, if it is to pinpoint m_H.

Taken together, (31) and (32) are very suggestive of a relatively light Higgs scalar not far from the current LEPII bound from non observation of $e^+e^- \to ZH$

$$m_H > 89.8 \text{ GeV} \qquad (\sqrt{s} = 183 \text{ GeV data}) \tag{33}$$

Preliminary studies of $\sqrt{s} = 189$ GeV e^+e^- data indicate that bound will soon rise to ~ 95 GeV. Future upgrades to $\sqrt{s} \simeq 200$ GeV will push the Higgs discovery potential to ~ 105 GeV. In addition, searching for the Higgs via associated $W^\pm H$ and ZH at the Fermilab $p\bar{p}$ collider during Run II promises discovery up to $m_H \sim 115$ GeV, perhaps even higher. Higgs discovery may soon be at hand.

2 The $Zb\bar{b}$ Problem

Currently, the only real anomaly in Z pole measurements seems to involve $b\bar{b}$ final states. The LEP $b\bar{b}$ forward-backward asymmetry and SLAC $b\bar{b}$ left-right forward-backward asymmetry are consistent with a 3σ deviation for A_b from the standard model

expectation

$$\begin{aligned} A_b^{\rm exp}/A_b^{\rm theory} &= 0.96 \pm 0.02 \\ A_b &= \frac{g_L^2(b) - g_R^2(b)}{g_L^2(b) + g_R^2(b)} \end{aligned} \tag{34}$$

where $g_L(b)$ and $g_R(b)$ are the Z couplings to left and right-handed b quarks, normalized to have standard model values (at tree level)

$$\begin{aligned} g_L(b) &= -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \simeq -0.423 \\ g_R(b) &= \frac{1}{3}\sin^2\theta_W \simeq +0.077 \end{aligned} \tag{35}$$

At the same time, the quantity $R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ exhibits very good accord with standard model expectations

$$R_b^{\rm exp}/R_b^{\rm theory} = 1.004 \pm 0.004 \tag{36}$$

and thereby leads to the constraint

$$g_L^2(b) + g_R^2(b) = 0.1858 \pm 0.0010 \tag{37}$$

Solving (34) and (37) together gives

$$\begin{aligned} g_L^2(b) &= 0.1746 \pm 0.0020 \\ g_R^2(b) &= 0.0112 \pm 0.0018 \end{aligned} \tag{38}$$

In terms of deviations from the standard model, $\delta g_L(b)$ and $\delta g_R(b)$, one finds (ignoring the negative $g_R(b)$ and positive $g_L(b)$ solutions)

$$\begin{aligned} \delta g_L(b) &= 0.005 \pm 0.002 \\ \delta g_R(b) &= 0.0287 \pm 0.0088 \end{aligned} \tag{39}$$

The $\delta g_L(b)$ deviation on its own amounts to only a - 1% shift and could probably be interpreted as a "new physics" quantum loop correction; however, such a large $\delta g_R(b)$ shift of 40% is very difficult to explain. For that reason, most theorists have dismissed

the above 3σ deviation as experimental in origin, i.e. stemming from a statistical or systematic effect, rather than an indication of "new physics". Nevertheless, it is amusing to contemplate other potential consequences of non-zero $\delta g_L(b)$ and $\delta g_R(b)$ of the magnitude in (39). First, I note that deviations of similar magnitude cannot occur in $\delta g_L(d)$ and $\delta g_R(d)$; otherwise they would have been observed in atomic parity violation and $\nu_\mu N$ experiments. Furthermore, it is unlikely that they are present in $\delta g_R(s)$ and $\delta g_L(s)$. If that were the case, one would expect (but could avoid) induced $s \to d$ flavor-changing weak neutral currents which could significantly enhance $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\bar{\nu}$ etc; and that seems not to be the case. If one concludes that the anomaly occurs only in $Zb\bar{b}$, it is still likely that related new flavor changing $b \to s$, $b \to d$, and $s \to d$ weak neutral currents would occur. The predicted magnitude of those effects depends on the degree and nature of quark mixing; however, generically interesting observable consequences almost certainly result. It will be interesting to see if anomalies in $Z \to b\bar{b}$ asymmetries persist as the data is further scrutinized and whether FCNC b (and K) decays will be in accord with Standard Model expectations or also exhibit anomalies.

3 Muon Decay and the S, T, U Parameters

As previously discussed, muon decay provides a very precise determination of $G_\mu = 1.16637(1) \times 10^{-5}$ GeV^{-2} which contains within it potential "new physics" effects. For example, heavy chiral fermions present in 4th generation models or technicolor theories would contribute to gauge boson self energies. Those loop effects would show up in the Δr, $\Delta r(m_Z)_{\overline{MS}}$ and $\Delta\hat{r}$ of (26) as additional contributions. One way to unveil or constrain such effects is to define Fermi constants in terms of α, $\sin^2\theta_W(m_Z)_{\overline{MS}}$, m_W, and m_Z via (26)

$$
\begin{aligned}
G_F^{(1)} &= \frac{\pi\alpha}{\sqrt{2}m_W^2(1 - m_W^2/m_Z^2)(1 - \Delta r)} && (a) \\
G_F^{(2)} &= \frac{\pi\alpha}{\sqrt{2}m_W^2 \sin^2\theta_W(m_Z)_{\overline{MS}}(1 - \Delta r(m_Z)_{\overline{MS}})} && (b) \quad (40) \\
G_F^{(3)} &= \frac{4\pi\alpha}{\sqrt{2}m_Z^2 \sin^2 2\theta_W(m_Z)_{\overline{MS}}(1 - \Delta\hat{r})} && (c)
\end{aligned}
$$

Comparison of those quantities with G_μ tests the standard model and probes for possible "new physics" in the Δr, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta\hat{r}$. If there is no "new physics" one

should find $G_\mu = G_F^{(1)} = G_F^{(2)} = G_F^{(3)}$.

To examine the situation, we take $m_H = 125$ GeV as our central value and allow for the range $75 < m_H < 200$ GeV. Then from the values in table 1 and the measured m_W, m_Z, $\sin^2\theta_W(m_Z)_{\overline{MS}}$ and α, one finds

$$\begin{aligned} G_F^{(1)} &= 1.1676(55) \times 10^{-5}\ \mathrm{GeV}^{-2} && (a) \\ G_F^{(2)} &= 1.1671(21) \times 10^{-5}\ \mathrm{GeV}^{-2} && (b) \\ G_F^{(3)} &= 1.1672(14) \times 10^{-5}\ \mathrm{GeV}^{-2} && (c) \end{aligned} \qquad (41)$$

where the uncertainties reflect errors in m_t, m_H, $\Delta^{-1}\alpha(m_Z)$, m_W, and $\sin^2\theta_W(m_Z)_{\overline{MS}}$. The excellent agreement between those quantities and $G_\mu = 1.16637(1) \times 10^{-5}$ GeV^{-2} obtained from muon decay is quite remarkable. It shows no indication of "new physics". Note also that the uncertainty in even the most precise $G_F^{(3)}$ is more than 100 times the current error in G_μ. Hence, improving G_μ further would not sharpen such tests, improving $\sin^2\theta_W(m_Z)$, m_W, m_t and measuring m_H would.

As an example of the utility of (41), consider the deviations expected from heavy chiral fermion doublets. The appendage of such particles to the standard model modifies gauge boson self-energies. Those effects shift the radiative corrections in (26). Such shifts are conveniently parametrized by the S, T, and U parameters of Peskin and Takeuchi[28,29]

$$\begin{aligned} \delta\Delta r &= 0.0166S - 0.0258T - 0.0195U && (a) \\ \delta\Delta r(m_Z)_{\overline{MS}} &= 0.0084(S+U) && (b) \\ \delta\Delta\hat{r} &= 0.011S - 0.00782T && (c) \end{aligned} \qquad (42)$$

Hence, if S, T and $U \neq 0$, one expects the relationships

$$\begin{aligned} G_\mu &= G_F^{(1)}(1 + 0.0166S - 0.0258T - 0.0195U) && (a) \\ G_\mu &= G_F^{(2)}(1 + 0.0084(S+U)) && (b) \\ G_\mu &= G_F^{(3)}(1 + 0.011S - 0.00782T) && (c) \end{aligned} \qquad (43)$$

In technicolor models, one has the generic prediction[28] $S \sim \mathcal{O}(+1)$ which would lead to about a 1% difference between the G_μ and $G_F^{(i)}$. However, (41) exhibits no such effect. In fact it constrains that quantity at $\mathcal{O}(0.1\%)$. A global fit to all electroweak data (for $m_H \simeq 100$ GeV) gives[30]

$$
\begin{aligned}
S &= -0.17^{+0.17}_{-0.12} && (a) \\
T &= -0.16^{+0.15}_{-0.18} && (b) \qquad (44) \\
U &= 0.19 \pm 0.21 && (c)
\end{aligned}
$$

Those constraints are consistent with (43) and (41). If one assumes $m_H \sim \mathcal{O}(1\text{ TeV})$ as would be more appropriate for technicolor, one finds $S = -0.29 \pm 0.14$ which is even more incompatible with $S \sim \mathcal{O}(+1)$. Therefore, if dynamical electroweak symmetry breaking is to be consistent with precision measurements, the dynamics must be very novel to render $S \sim 0$.

4 Extra Dimensions and $W^{*\pm}$ Bosons

Another interesting "new physics" scenario involves excited $W^{*\pm}$ bosons which may arise in theories with extra compact dimensions[31] (Kaluza-Klein excitations) or models with composite gauge bosons. Assuming fermionic couplings to $W^{*\pm}$ indentical to those of $W^{\pm}$, $g_2^* = g_2$, direct searches at the Tevatron lead to the bound[32]

$$
m_{W^*} > 720\text{ GeV} \qquad (95\%\text{ CL}) \tag{45}
$$

If such bosons exist, they would also contribute to low energy charged current processes such as muon decay and be incorporated into G_μ. They would replace g_2^2/m_W^2 in the decay amplitude by $g_2^2/\langle m_W^2 \rangle$ where

$$
\frac{1}{\langle m_W^2 \rangle} = \frac{1}{m_W^2} + \frac{(g_2^*/g_2)^2}{m_{W^*}^2} + \frac{(g_2^{**}/g_2)^2}{m_{W^{**}}^2} + \cdots \tag{46}
$$

As long as the signs are positive, the effective low energy mass $\langle m_W \rangle$ is always smaller than m_W, since the reciprocal sum acts like resistors in parallel. Therefore, if W^* bosons exist, G_μ should be larger than the $G_F^{(i)}$ in (41). However, there is no such indication. Quantitatively, one expects

$$
G_\mu = G_F^{(i)} \left(1 + C \left(\frac{g_2^*}{g_2} \right)^2 \frac{m_W^2}{m_{W^*}^2} \right) \tag{47}
$$

where

$$C = 1 + \left(\frac{g_2^{**}}{g_2^*}\right)^2 \frac{m_{W^*}^2}{m_{W^{**}}^2} + \cdots > 1 \tag{48}$$

In the simplest extra dimension theory, one might typically expect $C = \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$. More realistic scenarios can lead to even larger C. Here, I am interested only in lower bounds on m_{W^*}; so, C will not enter as long as $C \geq 1$.

Comparing (47) and (41b) gives (for $m_H \lesssim 200$ GeV)

$$m_{W^*} > 1.67 \left(\frac{g_2^*}{g_2}\right) \text{ TeV} \qquad (95\% \text{ CL}) \tag{49}$$

which is very constraining. It suggests that the radius of the extra dimensions $R \simeq 1/m_{W^*} < 1 \times 10^{-17}(g_2/g_2^*)$ cm. and that continuing searches for W^* bosons at the Tevatron are likely to yield null results. Of course, the bound can be significantly relaxed if $g_2^* << g$.

One can improve the bound in (49) by employing $G_F^{(3)}$ rather than $G_F^{(2)}$ in comparison with G_μ. One finds

$$m_{W^*} > 2.27 \left(\frac{g_2^*}{g_2}\right) \text{ TeV} \qquad (95\% \text{ CL}) \tag{50}$$

However, that bound is subject to a larger dependence on m_t, m_H, and "new physics" effects. Further improvements in $\sin^2\theta_W(m_Z)_{\overline{MS}}$ and m_W could push the m_{W^*} sensitivity to $\mathcal{O}$(5 TeV) which is competitive with LHC capabilities.

5 Conclusion

Precision electroweak measurements have tested the standard model at the $\pm 0.1\%$ level. As a byproduct, they have been used to predict the large top quark mass and now suggest a relatively light Higgs[30]

$$m_H < 255 \text{ GeV} \qquad (95\% \text{ CL}) \tag{51}$$

with values around 100 GeV favored. Discovery of the Higgs scalar may be close.

The good agreement between theory and experiment severely constrains the possible "new physics" one can append to the standard model. For example, the S parameter must be near zero. That finding leaves little room for additional chiral fermion doublets

such as a fourth generation of fermions and requires dynamical symmetry breaking scenarios to exhibit novel dynamics which respects that constraint (a difficult task). Other types of "new physics" such as relatively large extra dimensions, SUSY, Z' bosons etc. are also being constrained by such measurements. So far, there are no signs of "new physics". Nevertheless, we must continue to probe shorter distances and search for new phenomena. Surprises are certainly waiting to be unveiled.

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