
Gravitational Waves: Sources & Signatures

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Review

- ◆ Characterizing Radiation
 - Waveform
 - Total energy radiated
 - Energy spectrum
- ◆ Characterizing Detectors
 - Antenna pattern
 - Noise statistics
 - Noise power spectrum
- ◆ Characterizing detection
 - Bayesian & Frequentist statistics
 - Signal-to-noise ratio

Today's Outline

- ◆ Periodic sources
- ◆ Stochastic sources

Periodic Sources

- ◆ Tapping the pulsar flywheel

Non-axisymmetric rotator

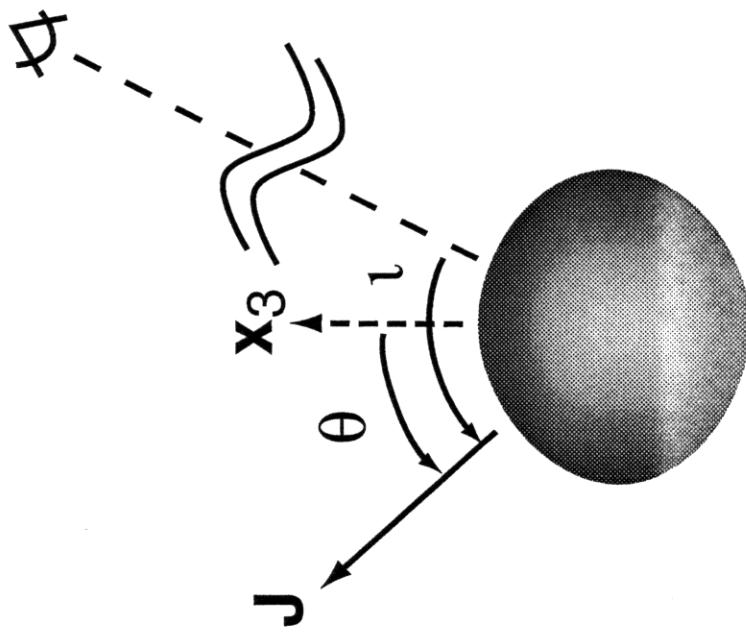
- ◆ Lumpy NS rotating about principal axis
 - $\varepsilon = \Delta I/I$, $I = \text{mom.inertia}$
 - » limited by crust strength
- ◆ Problem:
 - what generates asymmetry?

$$h(t) \approx h_0 \cos 4\pi f t$$

$$\overline{\rho^2} = 62 \left(\frac{h_0}{5 \times 10^{-26}} \right)^2 \frac{10^{-46} \text{ Hz}^{-1}}{S(2f)} \frac{T_{\text{obs}}}{\text{month}}$$
$$h_0 = 4.8 \times 10^{-26} \frac{I}{10^{45} \text{ g cm}^2} \frac{\varepsilon}{10^{-6}} \frac{10 \text{ Kpc}}{r} \left(\frac{f}{300 \text{ Hz}} \right)^2$$

Free Precession

- ◆ Non-axisymmetric NS
- ◆ Special case: axisym. precession
 - Geometry is observable
 - » $\cos(\mathbf{J}, \mathbf{x}_3)$
 - » $\cos(\mathbf{J}, \mathbf{n})$



$$\begin{aligned} h_+ \\ h_x \end{aligned} = \frac{h_0}{4} \begin{cases} \sin 2\varphi (-4 \sin^2 \theta \cos 4\pi ft + \sin 2\theta \sin 2\pi ft) \\ 4 \cos \varphi \sin^2 \theta \cos 4\pi ft - \sin \varphi \sin 2\theta \sin 2\pi ft \end{cases}$$

Quasi-periodic oscillators: QPOS

- ◆ Pulsed, modulated X-ray emission
 - ~ 1 KHz pulses, up to ~ 700 Hz modulation
 - X-rays from accretion
 - » $\dot{M} \geq 10^{-11} M_{\odot} \text{yr}^{-1}$
 - Modulation is beating
 - » NS spin vs. accreting matter orbital period
 - $V_B \sim 260 - 589$ Hz for fast QPOS
- ◆ Bildsten 1998
 - non-axis. accretion → differential heating
 - differential heating → diff. e-capture rate
 - diff. e-capture → non-axis. mass distribution
 - » but mass redistributes...
 - non-axis. mass distr. → GW emission
 - GW emission limits V_B
 - » $h_c \leq 10^{-26}$ for obs.QPOS

Neutron star R-Modes

- ◆ R-modes

- oscillating currents in

- rotating stars

$$\vec{\delta V} \approx aR\Omega \left(\frac{r}{R}\right)^\ell \vec{Y}_{\ell\ell}^B e^{i\omega t}$$

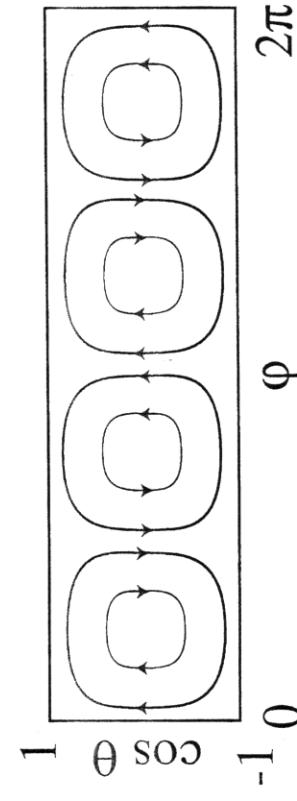
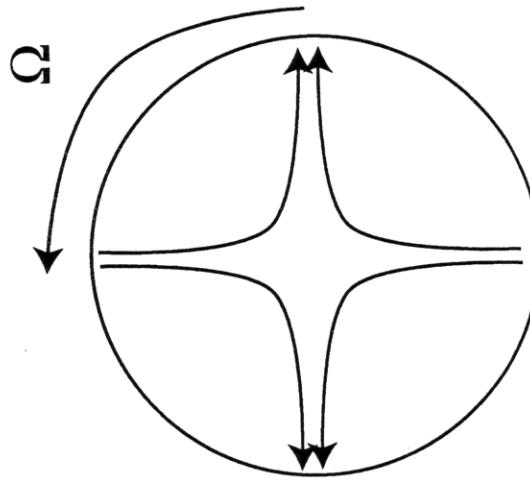
$$\omega \approx \frac{(\ell-1)(\ell+2)}{\ell+1} \Omega$$

- ◆ GW emission

- destabilizes r-modes

- ◆ Viscosity

- stabilizes r-modes



R-modes

- ◆ Young, hot NS . . .
 - born rotating rapidly (period ~ 1 ms)
 - bulk viscosity ($\sim T^6$) stabilizes mode
- ◆ cools . . .
 - becomes unstable to GW emission
 - » mode grows until GW amplification = viscous loss
 - ◆ and cools more
 - shear viscosity ($\sim T^2$) stabilizes mode

Angular velocity vs. temperature

- ◆ Mode unstable for ~ 1 yr after birth

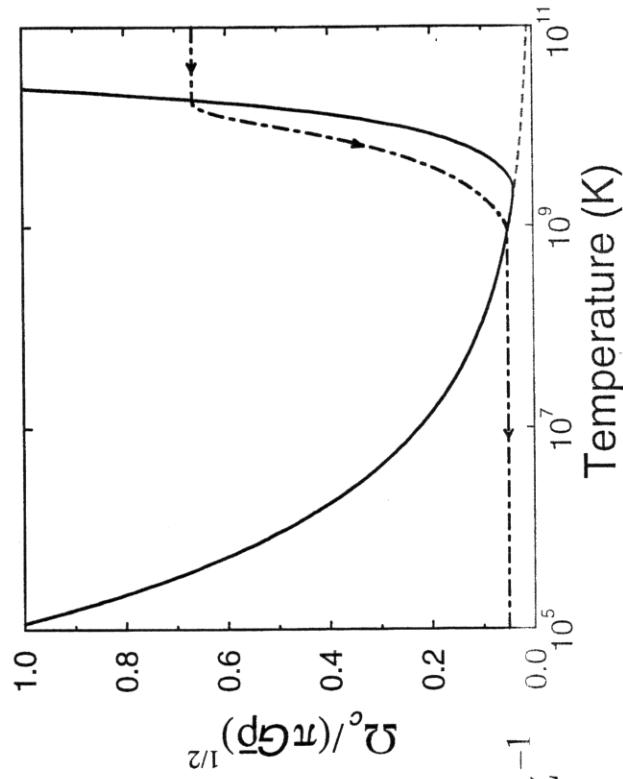
– Period changes from
 ~ 1 ms (birth) to ~ 13 ms (one yr)

» Lindblom et al. PRL **80**,
 4843-4846

$$\tilde{h}(f) = 5.7 \times 10^{-25} \sqrt{\frac{1 \text{ KHz}}{f \text{ Hz}}}^{1/2}$$

$$\rho \approx 8.8 \left(\frac{I}{10^{45} \text{ g cm}^2} \right)^{1/2} \frac{20 \text{ Mpc}}{r} \frac{120 \text{ Hz}}{f_{\min}}$$

» Owen et al. gr-qc/
 9804044



Detection challenges

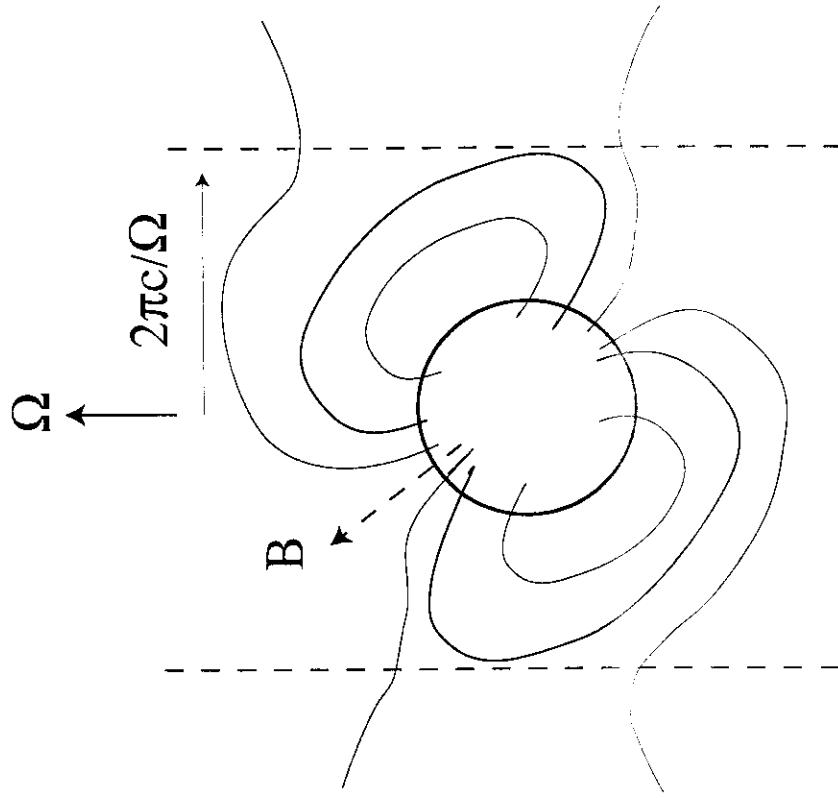
- ◆ Earth rotates, orbits
 - redshift induced frequency modulation
- ◆ Three cases
 - f, θ, φ known: tractable
 - targeted (*e.g.*, galactic center): tractable, but intense
 - blind search: intractable



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Science pay-off

- ◆ Precession:
 - test pulsar emission mechanism models
- ◆ Bildsten Stars:
 - test QPO beat-frequency model
- ◆ R-modes:
 - Observe early remnant
 - shear, bulk viscosity, remnant cooling



Summary

- ◆ Periodic Sources
- rapidly rotating NSs
 - » axisymmetry & precession
 - » non-axisymmetric
 - unknowns
 - » non-axisymmetry origin?
 - » QPOs promising; R-modes less so
 - detection challenges
 - » long integration times;
 - » long integration times; frequency modulation
- ◆ Stochastic signals
 - exotic sources
 - » primordial, strings, inflation, *etc.*
 - non-exotic sources
 - » “embarrassment of riches”
 - » spectrum observation is census taking
 - LISA, galactic binaries & binary stellar evolution

Stochastic Signals

- ◆ Exotic and mundane

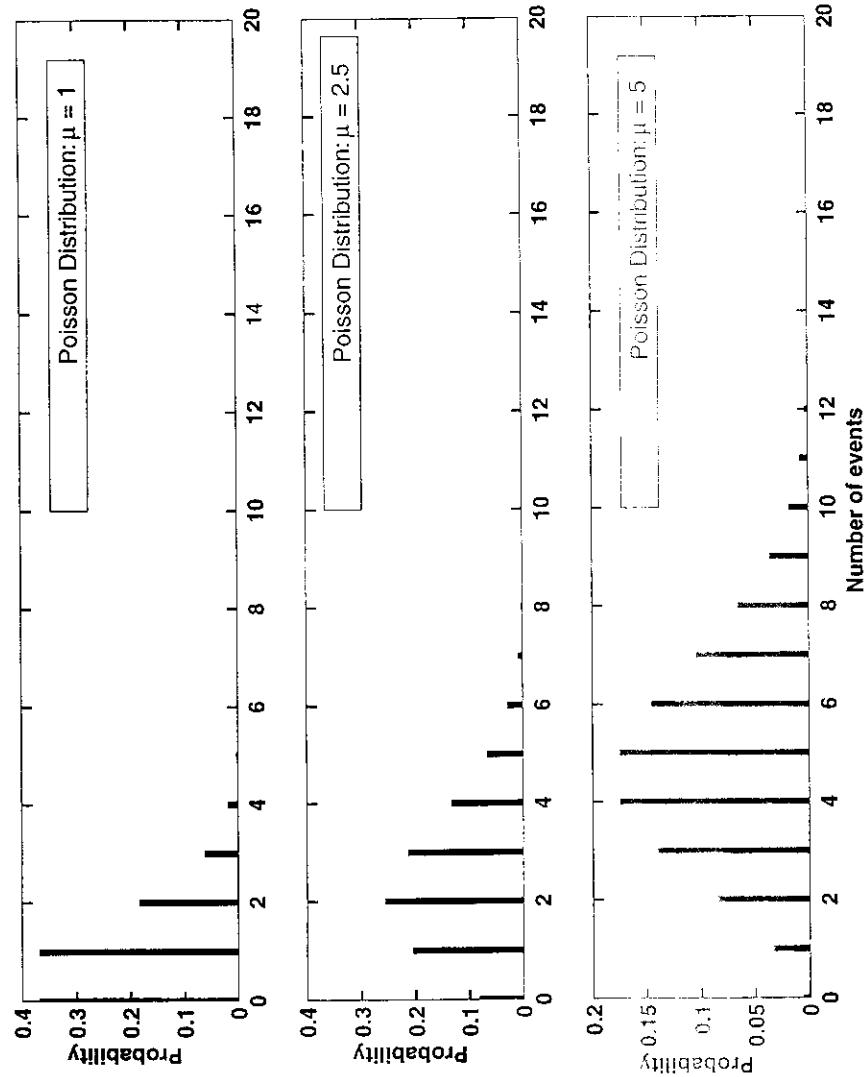
Stochastic signals?

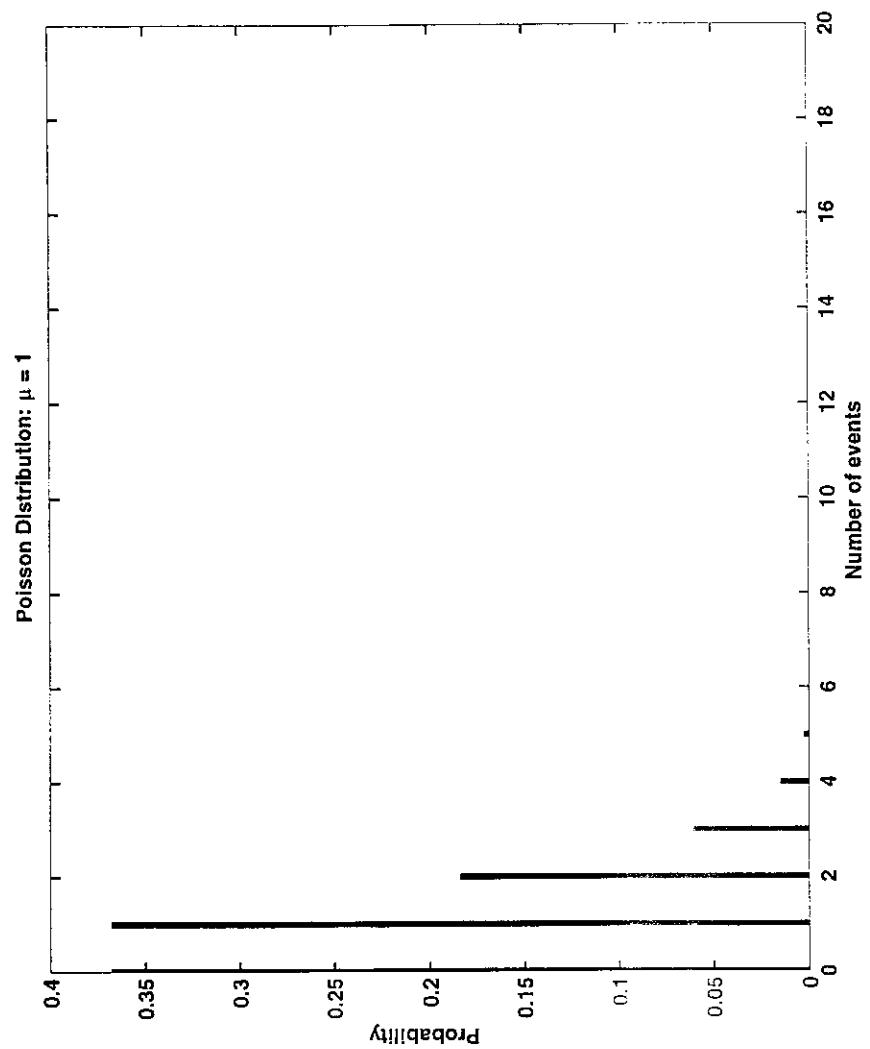
- ◆ Predictable waveform sources
 - Binary inspiral chirp, black hole ringdown, rapidly rotating neutron star, etc.
- ◆ Unpredictable waveform sources
 - EM. examples: Johnson noise, radio hiss, CMBR
 - GW examples: primordial background, cosmic strings, inflation

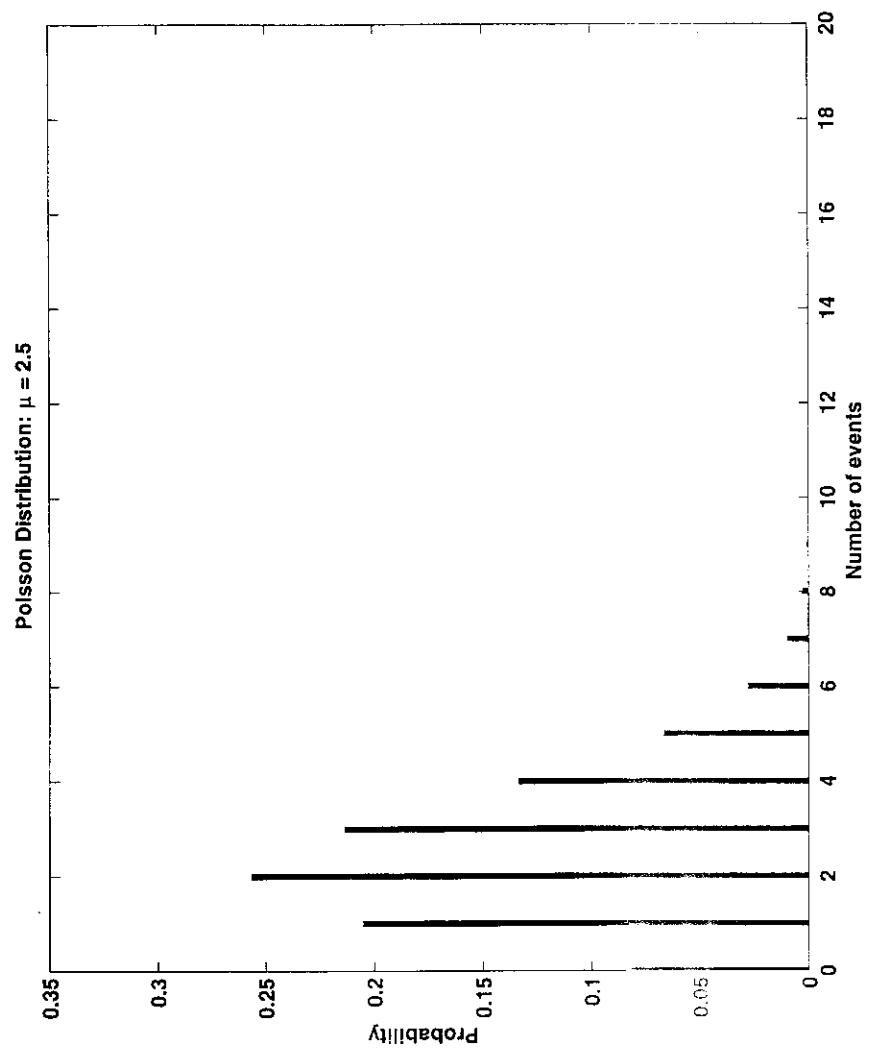
Rates and arrival times

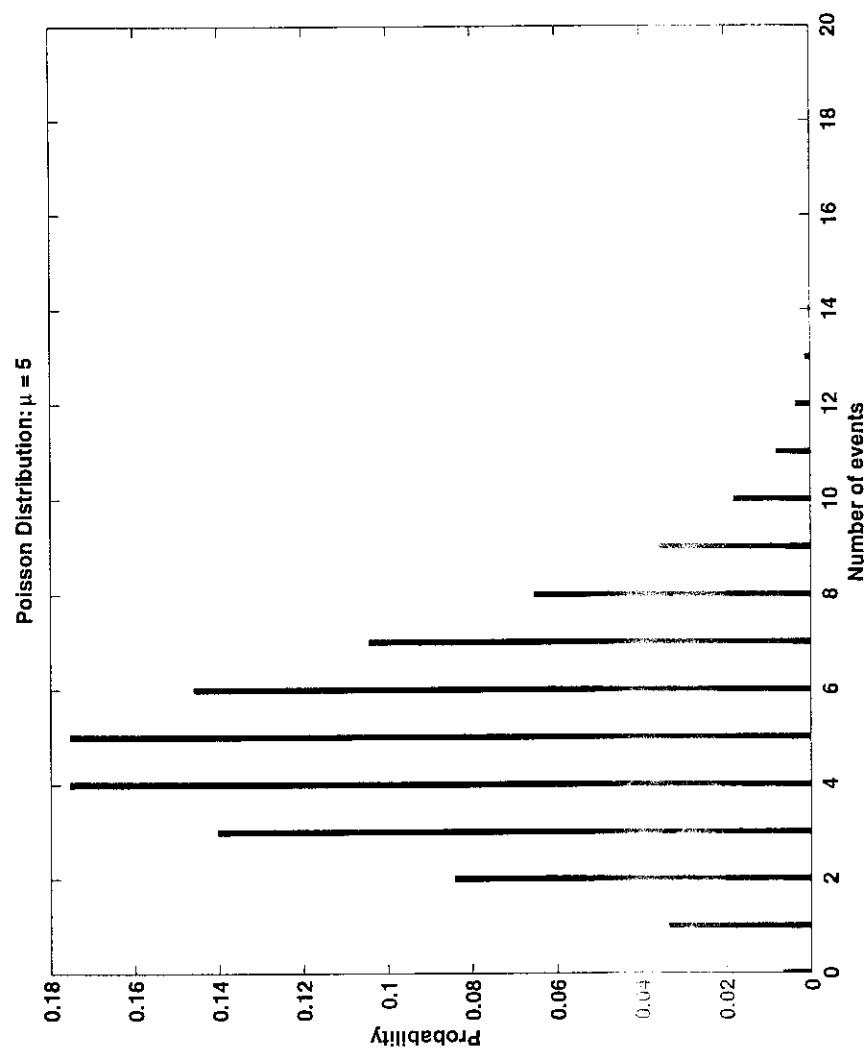
- ◆ Rate
 - Average # per unit time
 - ◆ Rate n
 - Poisson distribution:
probability of N events
in interval τ :
- $$P(N \mid n\tau) = \frac{(n\tau)^N}{N!} e^{-n\tau}$$
- | interval time | # signal starts | # signal "rate" |
|---------------|-----------------|-----------------|
| 0-5 s | 3 | 0.6/s |
| 5-10 s | 4 | 0.8/s |
| 10-15 s | 11 | 2.2/s |
| 15-20 s | 2 | 0.4/s |
| 20-25 s | 5 | 1.0/s |
| 0-25 s | 25 | 1.0/s |
- long-run rate
- » Only defined for non-negative integer N

Poisson probability distribution

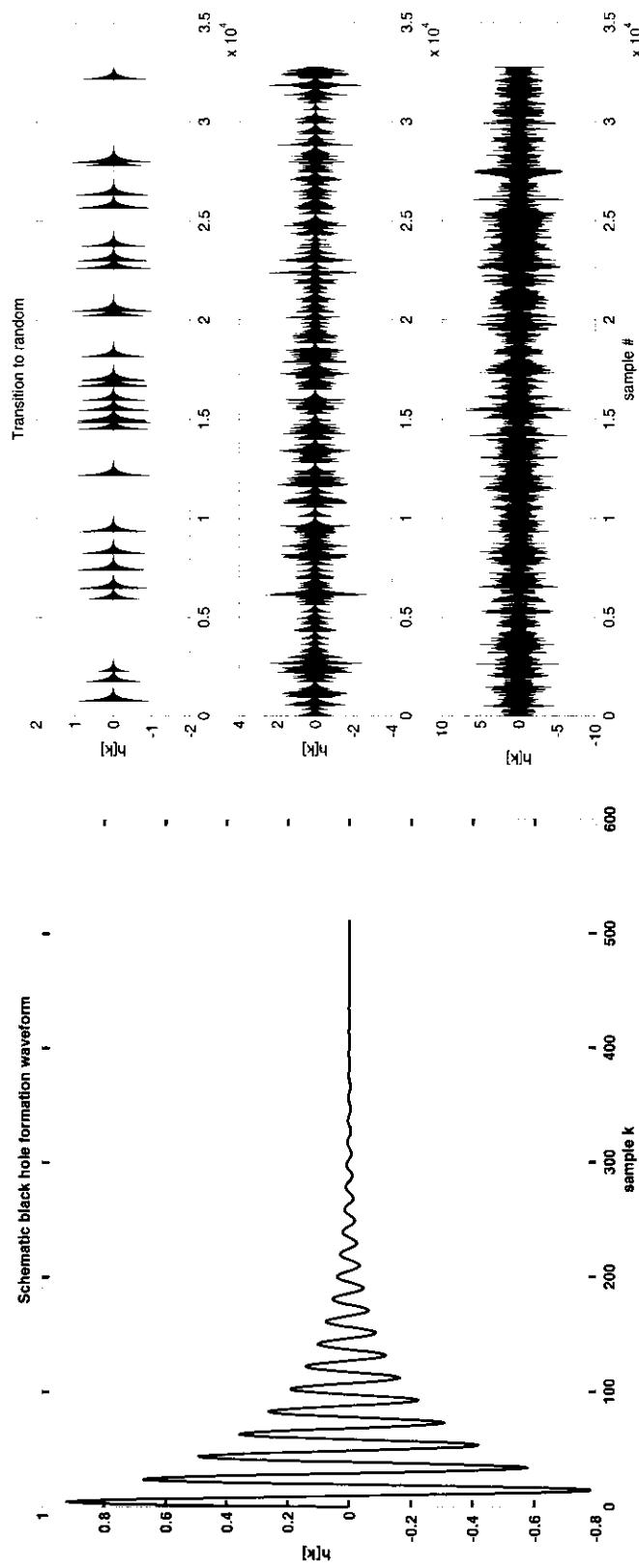








From “signal” to “noise”



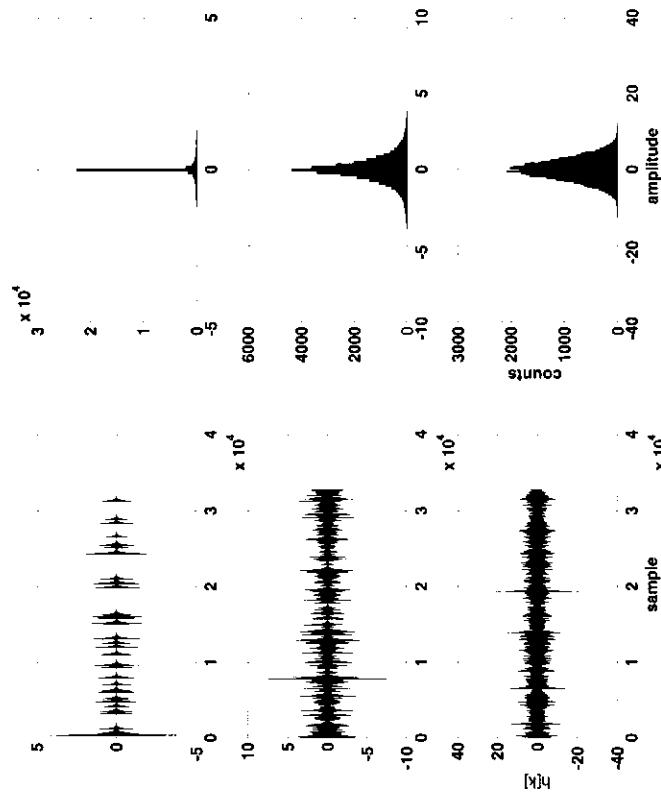
Transition to Normal distribution

- ◆ The Central Limits

Theorem in action

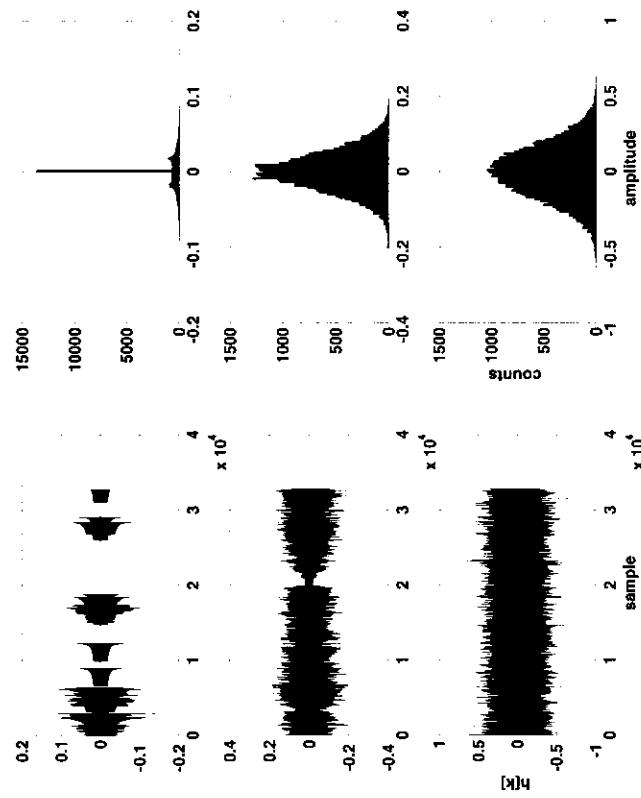
$$N(x | \sigma) = 2\pi\sigma \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right]$$

Transition to Normal Distribution

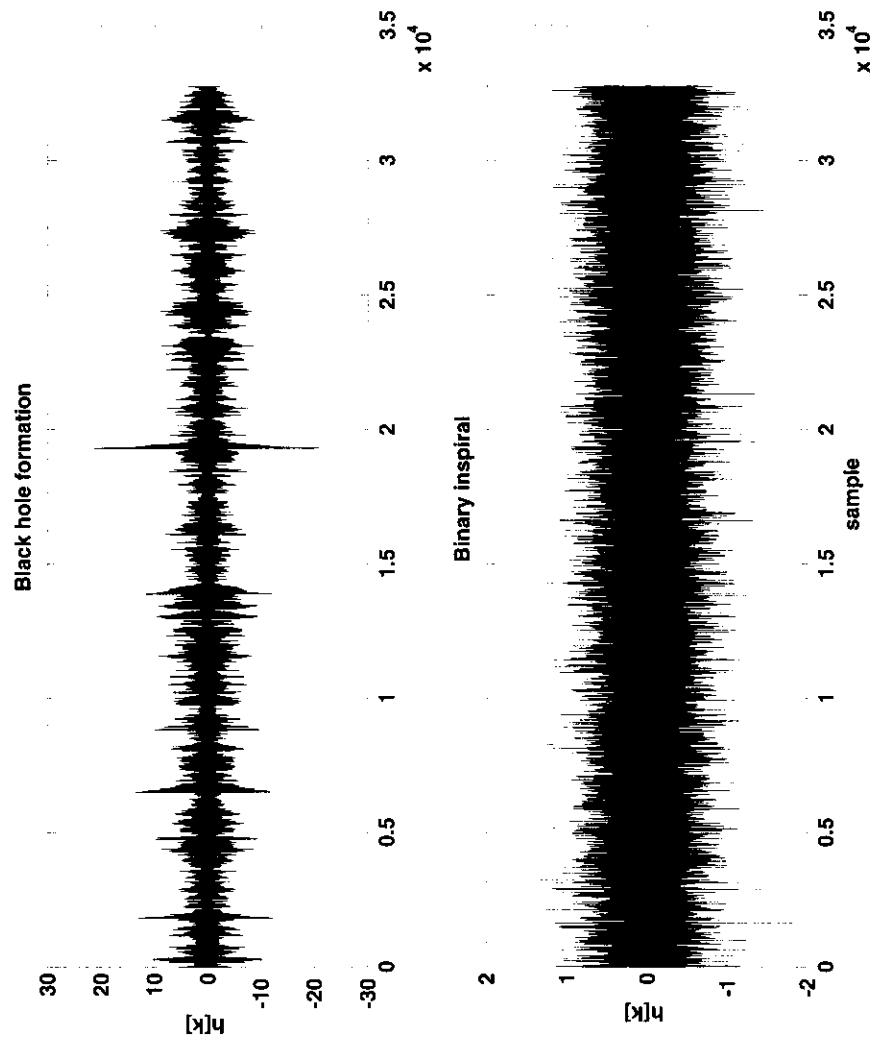


Different waveform, same result

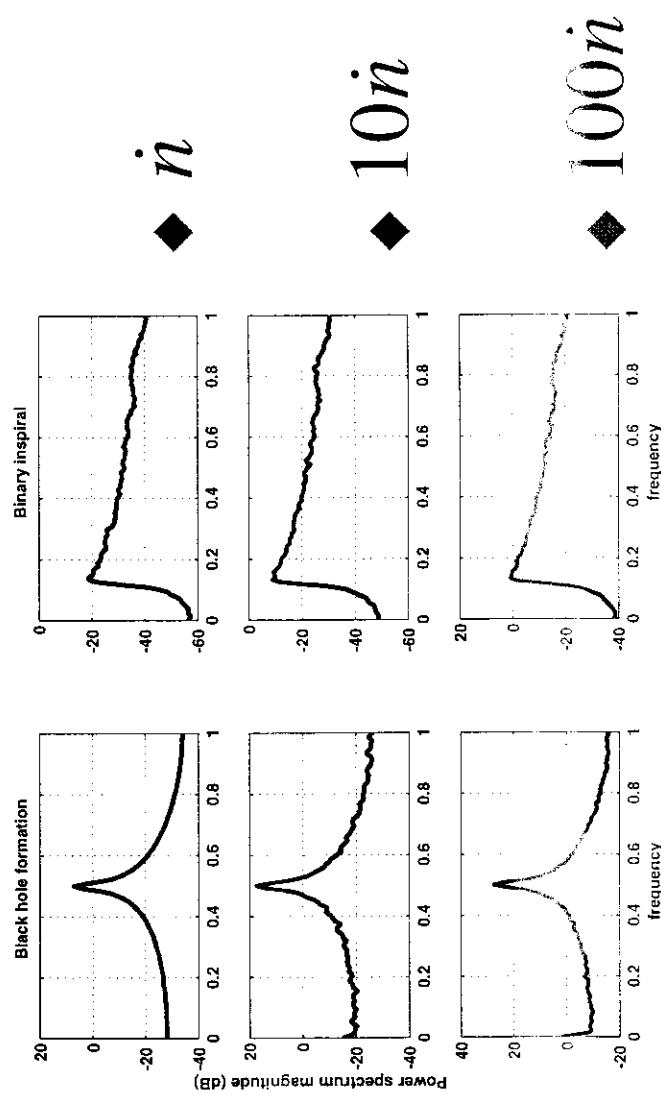
- ◆ Binary inspiral waveform



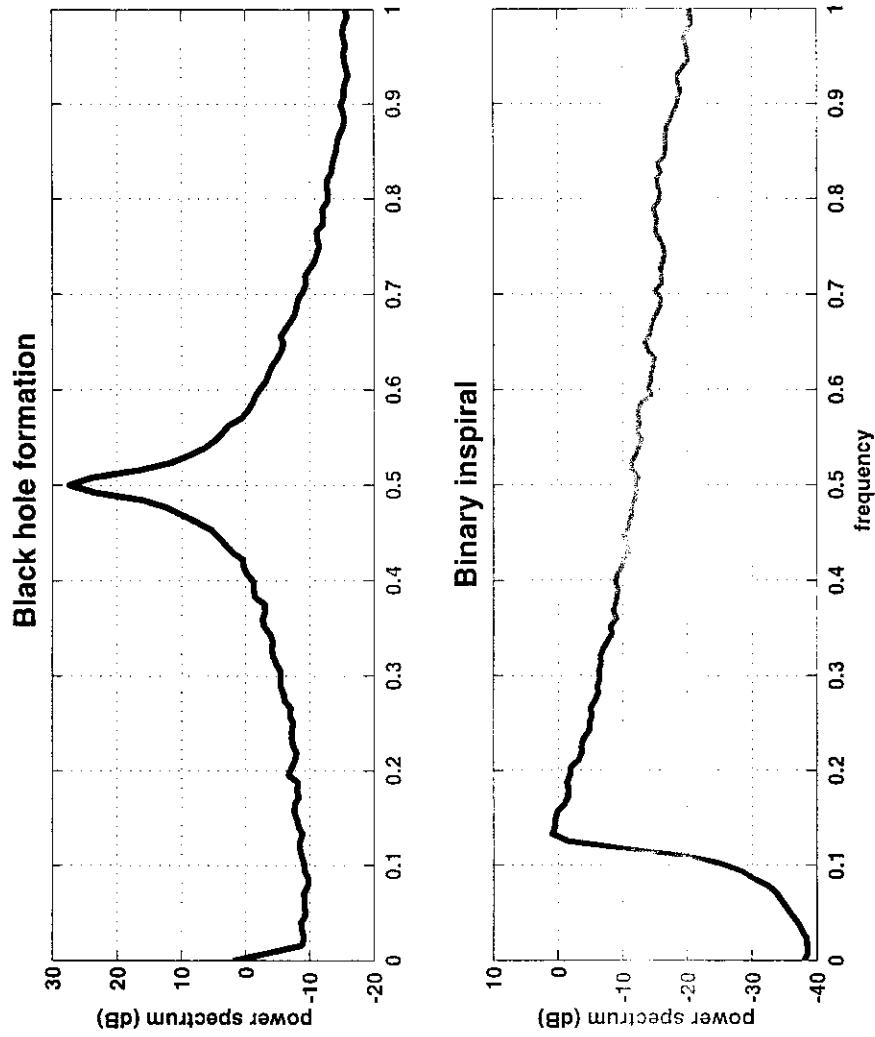
Same distribution, different signals



Power is proportional to rate



Correlations distinguishing stochastic signals

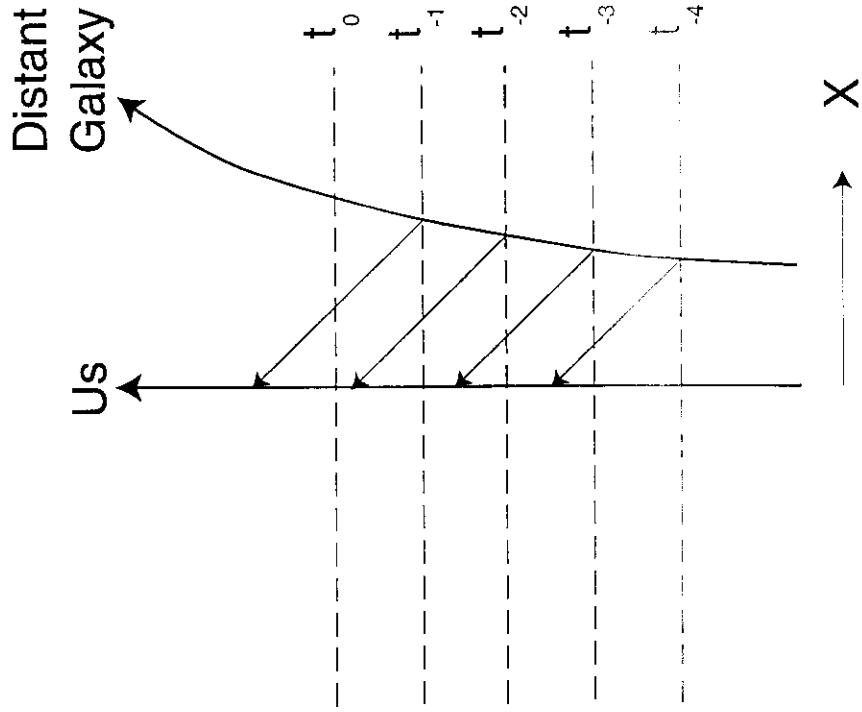


Murmuring or Deafening?

- ◆ Signals come from sources near and far
 - nearer: stronger, rarer
 - farther: weaker, more frequent
- ◆ Mean source energy
 - $1/r^2$
- ◆ Power from $(r, r+dr)$
 - $\propto 4\pi r^2 dr / r^2 = 4\pi dr$
- ◆ Total power incident on detector
 - $\infty \int_0^\infty 4\pi dr = \infty ?!$
- ◆ Rate of signals arising from $(r, r+dr)$
 - $\propto 4\pi r^2 dr$

Why is the night sky dark?

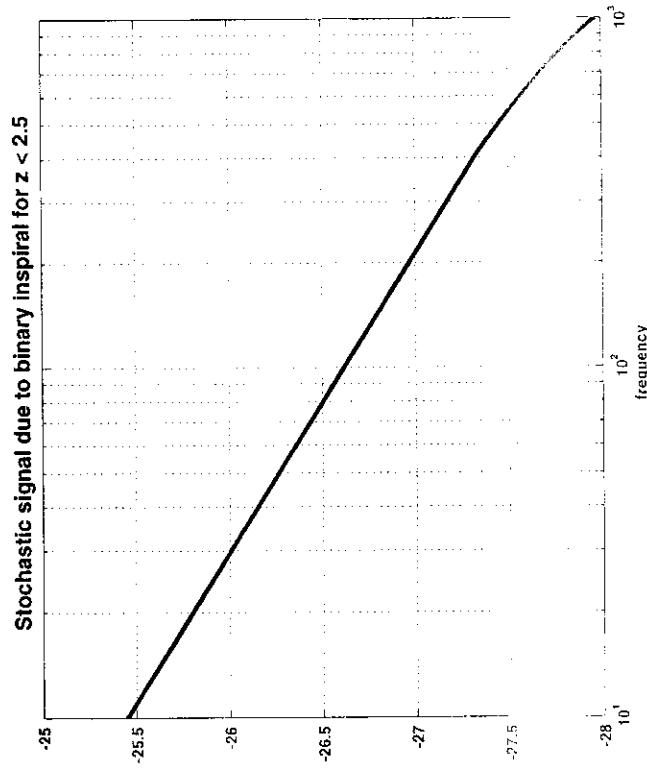
- ◆ Because the Universe is expanding:
 - expansion decreases apparent rate of distant sources
 - expansion decreases apparent energy of distant sources
 - Universe had a beginning: no sources before “date certain”



Binary inspiral stochastic signal

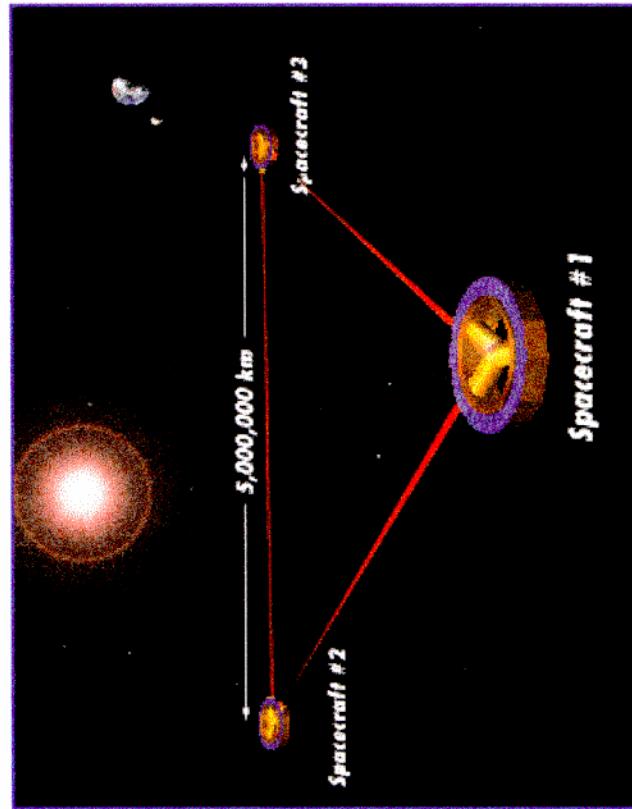
- ◆ Assumptions

- Uniform rate density
 - » better to track star formation
 - » $\dot{n}_0 = 8 \times 10^{-8} \text{ Mpc}^{-3} \text{ yr}^{-1}$
- Cut-off at quasar distribution peak
- $\Omega=1, \Lambda=0$ (H_0 cancels)



LiSA

- ◆ Laser Interferometer
- Space Antenna
 - $10^{-4} < f / \text{Hz} < 10^{-1}$
 - ESA, maybe NASA, project
- ◆ Galactic binaries
 - Confusion limited



CWDB Census

- ◆ CWDBs

- follow common envelope binary evolution phase
- evolve by gravitational radiation reaction
- confusion limited source: dn/df observed
- ◆ Continuity equation
$$-\frac{dn_+}{df} - \frac{dn_-}{df} = \frac{d}{df} \left[\frac{dn}{df} \frac{df}{dt} \right]$$
- observing spectrum tells us birth rate dn_+/df

Summary

- ◆ Periodic sources
 - all involve rapidly rotating neutron stars
 - require mechanism to generate, maintain asymmetry
- ◆ Detection challenges
 - frequency modulated signal
 - weak signal requires long (month to year) integration times
- ◆ Unknowns
 - Mechanisms for producing asymmetries
 - QPOs promising; R-modes less so