Gravitational Instability

Fluctuations : $\delta,\ \overset{av}{v},\ \overset{a^2\phi}{\phi}$ $\quad \overset{ax}{(x,t)}$

<u>Continuity:</u> $\dot{\delta} + \nabla\cdot v + \nabla\cdot(v\delta) = 0$

<u>Euler:</u> $\dot{v} + 2Hv + (v\cdot\nabla)v = -\nabla\phi$

<u>Poisson:</u> $\nabla^2\phi = \frac{3}{2}H^2\Omega\,\delta$

- Linear approx:

  $\delta \propto D(t)$ $\quad \rightarrow \quad \delta = \begin{cases} \propto t^{2/3} & \Omega\sim 1 \\ \rightarrow \text{const.} & \Omega \ll 1 \end{cases}$

  $f(\Omega) \equiv \frac{\dot{D}}{HD} \simeq \Omega^{0.6}$

  $$\delta = -\frac{1}{H f(\Omega)}\nabla\cdot v$$

  $$\nabla\times v = 0 \quad \rightarrow \quad v = -\nabla\phi$$

- Qusi-linear approx: $\quad$ 2^{nd}-order, Zel'dovich

  $$x(q,t) = q + D(t)\,\psi(q) = q + \frac{1}{f(\Omega)}v$$

Initial Fluctuations

$\delta(\vec{x}) \xrightarrow{FT} \tilde{\delta}(\vec{k})$ Inflation

- Power spectrum: $P(k) \equiv \langle |\tilde{\delta}(\vec{k})|^2 \rangle \propto k^n$
  - $\rightarrow \xi(r) \propto r^{-(n+3)}$
  - $\rightarrow \delta M / M \propto M^{-(n+3)/6}$

  $n=1$ Harrison-Zeldovich

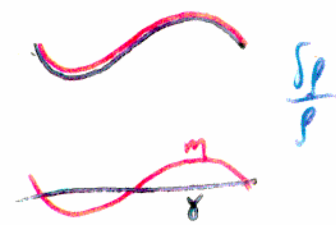
- Probability Distribution $P(\delta)$, $P(\delta_1, \delta_2)$, ...

  Gaussian $\propto e^{-\delta^2/2\sigma^2} \longleftrightarrow$ random phases

- Matter/Radiation

  Adiabatic: $\left(\frac{\delta\rho}{\rho}\right)_m = \left(\frac{\delta\rho}{\rho}\right)_\gamma$

  Isocurvature: $\delta\rho_m = -\delta\rho_\gamma$



Causality → Inflation

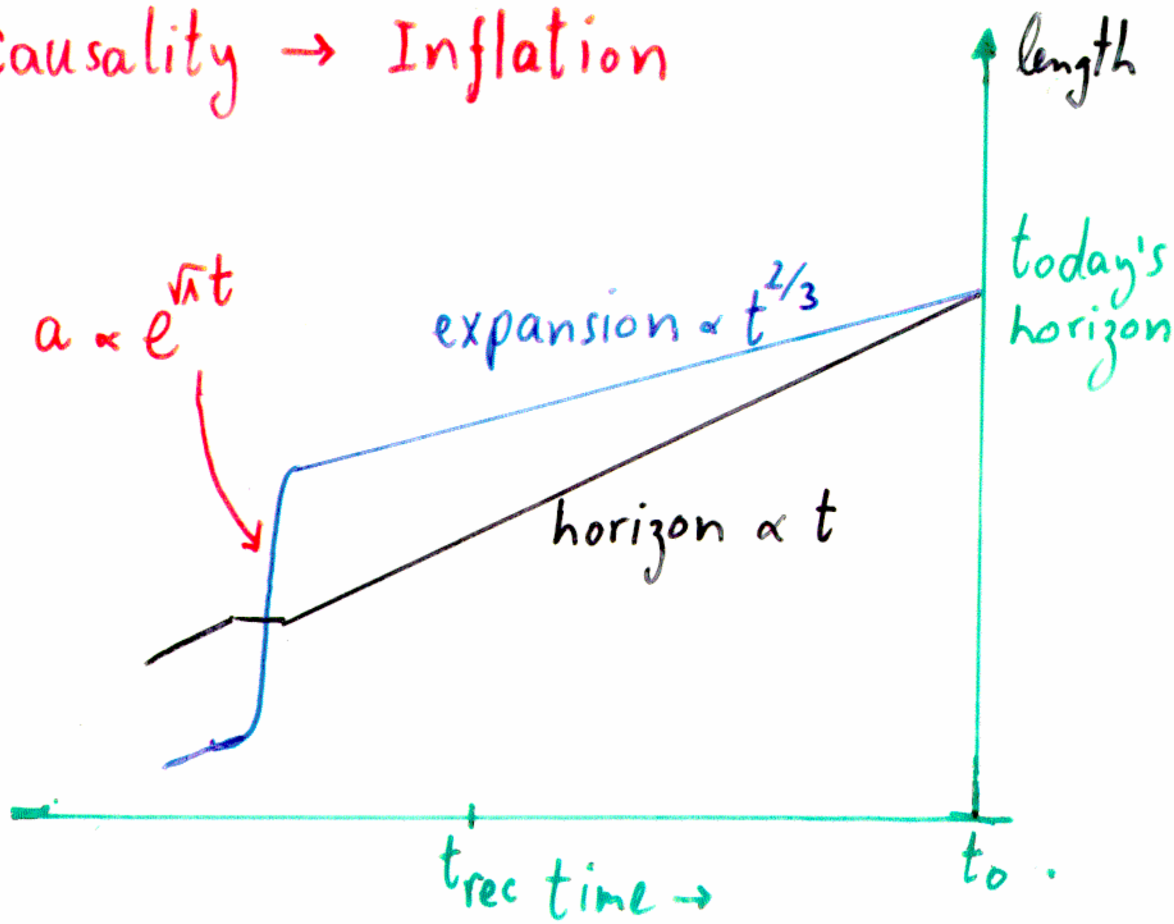


Table 1.6. *Inflation Summary*

PROBLEM SOLVED

• Horizon	Homogeneity, Isotropy, Uniform T
• Flatness/Age	Expansion and gravity balance
• "Dragons"	Monopoles, doman walls,... banished
• Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, $\Rightarrow$ de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda}t}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant ("false vacuum" energy) associated with hypothetical scalar field ("inflaton").

	FORCES OF NATURE	**Spin**
Known	Gravity	2
Known	Strong, weak, and electromagnetic	1
Goal of LHC	Mass (Higgs Boson)	0
Early universe	Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum ("re-"heating).

Inflation:

Klein-Gordon, expanding:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi)}{\partial \phi}$$

potential

inflaton field

slow roll → $H \equiv \frac{\dot{a}}{a} \sim$ const. → $a \propto e^{Ht}$

Inflation

If at 10^{14} Gev → $t \sim H^{-1} \sim 10^{-34}$ s

→ e^{66}, 10^{-32} s

Fluctuations

$\delta\phi \sim H/2\pi$ quantum fluct. (Hawking T)

$\Delta t \sim \delta\phi/\dot{\phi}$ end of inflation for different regions

$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \sim \frac{\Delta t}{t_H} = H\Delta t \sim$ const. (slow roll)

→ Scale-invariant spectrum: $n=1$, $P \propto k$

Table 1.7. *Linde's Classification of Inflation Models*

- **How Inflation Begins**

  Old Inflation — $T_{\rm initial}$ high, $\phi_{\rm in} \approx 0$ is false vacuum until phase transition
  Ends by bubble creation; Reheat by bubble collisions

  New Inflation — Slow roll down $V(\phi)$, no phase transition

  Chaotic Inflation — Similar to New Inflation, but $\phi_{\rm in}$ essentially arbitrary:
  any region with $\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i \phi)^2 \lesssim V(\phi)$ inflates

  Extended Inflation — Like Old Inflation, but slower (e.g., power $a \propto t^p$),
  so phase transition can finish

- **Potential $V(\phi)$ During Inflation**

  Chaotic typically $V(\phi) = \Lambda\phi^n$, can also use $V = V_0 e^{\alpha\phi}$, etc.
  $\Rightarrow a \propto t^p,\ p = 16\pi/\alpha^2 \gg 1$

- **How Inflation Ends**

  First-order phase transition — e.g., Old or Extended inflation
  Faster rolling $\rightarrow$ oscillation — e.g., Chaotic $V(\phi)^2\Lambda\phi^n$
  "Waterfall" — rapid roll of σ triggered by slow roll of ϕ

- **(Re)heating**

  Decay of inflatons
  "Preheating" by parametric resonance, then decay

- **Before Inflation?**

  Eternal Inflation? Can be caused by
  - Quantum $\delta\phi \sim H/2\pi >$ rolling $\Delta\phi = \dot{\phi}\Delta t = \dot{\phi}H^{-1} \approx V'/V$
  - Monopoles or other topological defects

Fluctuations - Random field δ

- Inflation: Gaussian? ✓

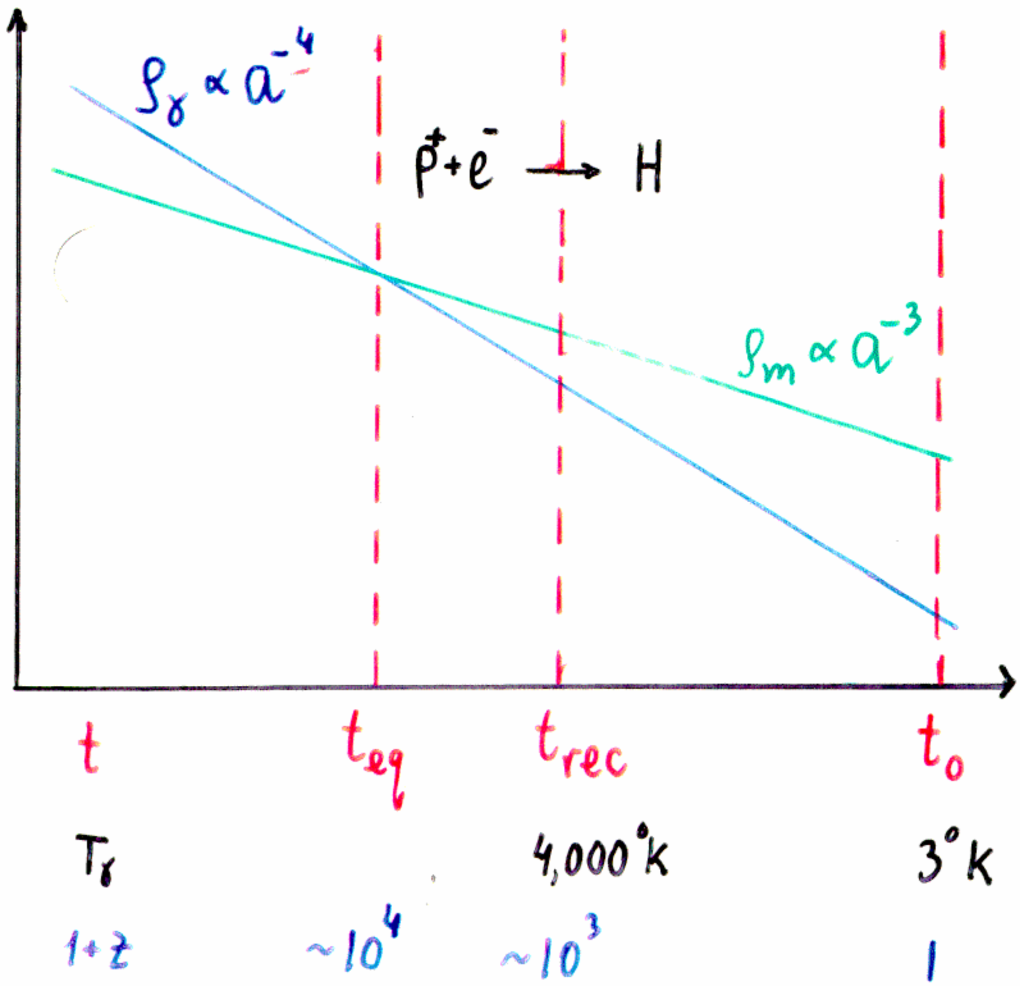
  $P_k \propto k^n$? $n \approx 1$ ✓

  Adiabatic / isocurvature?

- Topological defects: strings, Textures

- Explosions

Thermal History

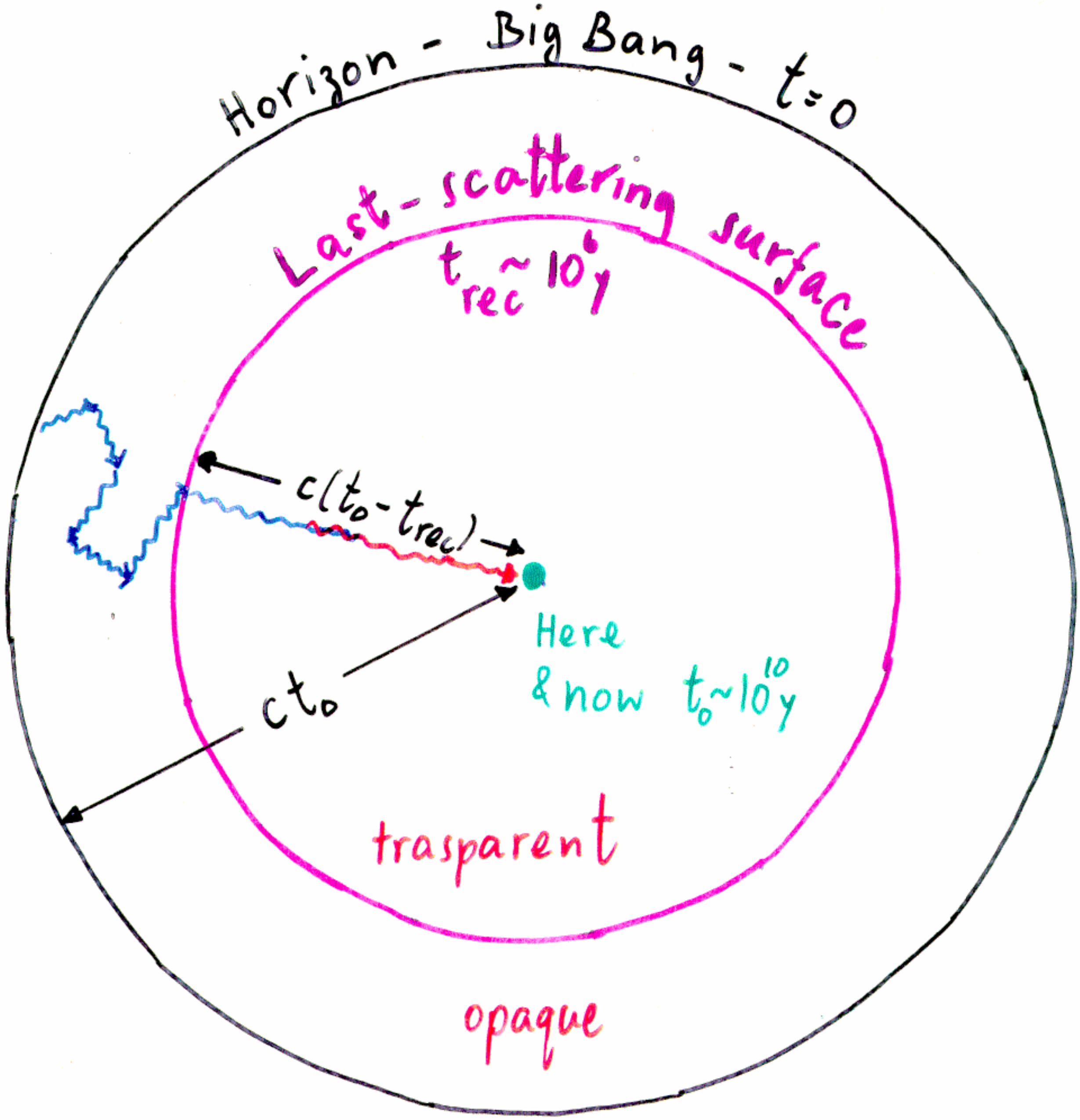


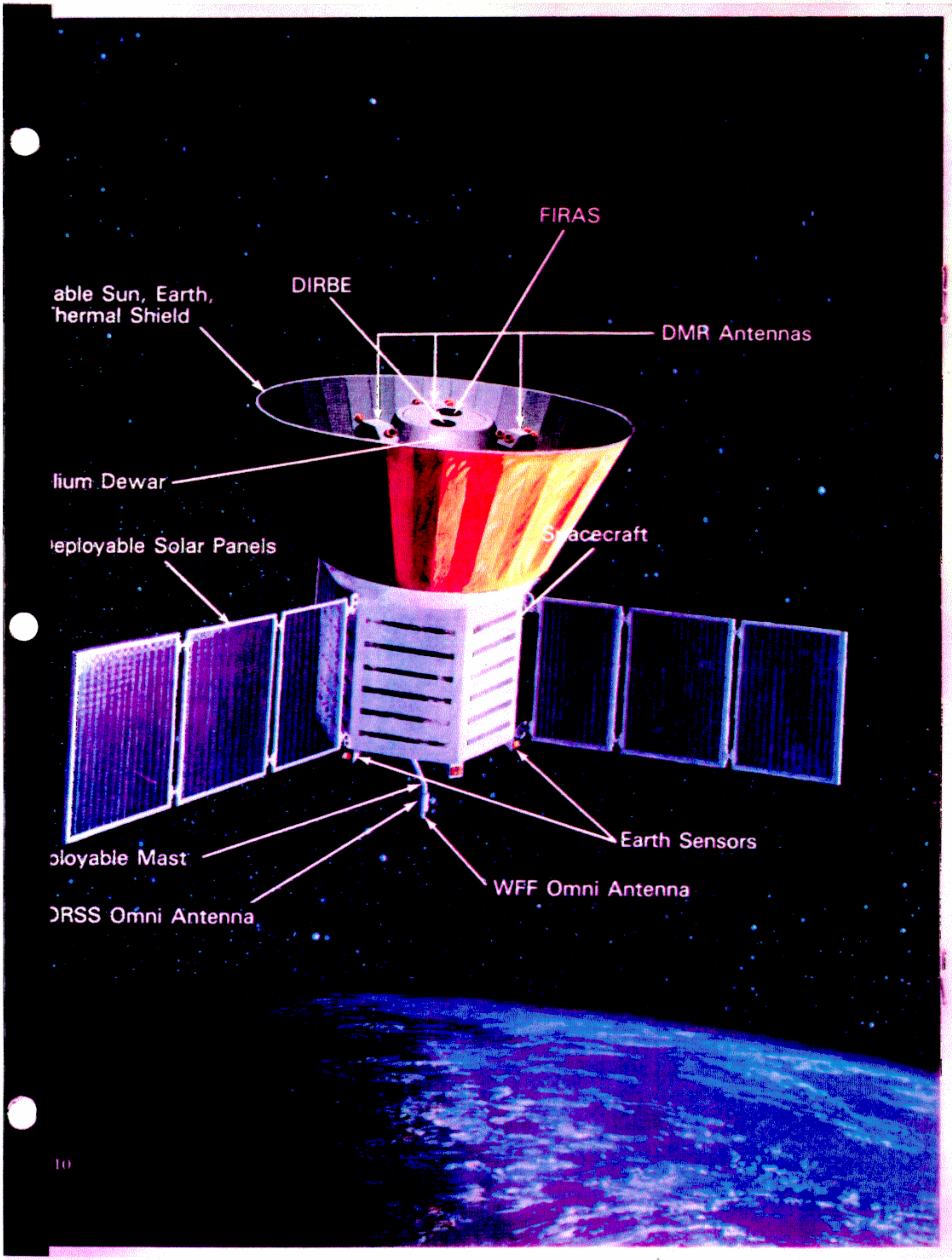
$a \propto t^{2/3}$ Expansion

$1+z \equiv \frac{\nu_e}{\nu_0} \propto a^{-1}$ Redshift

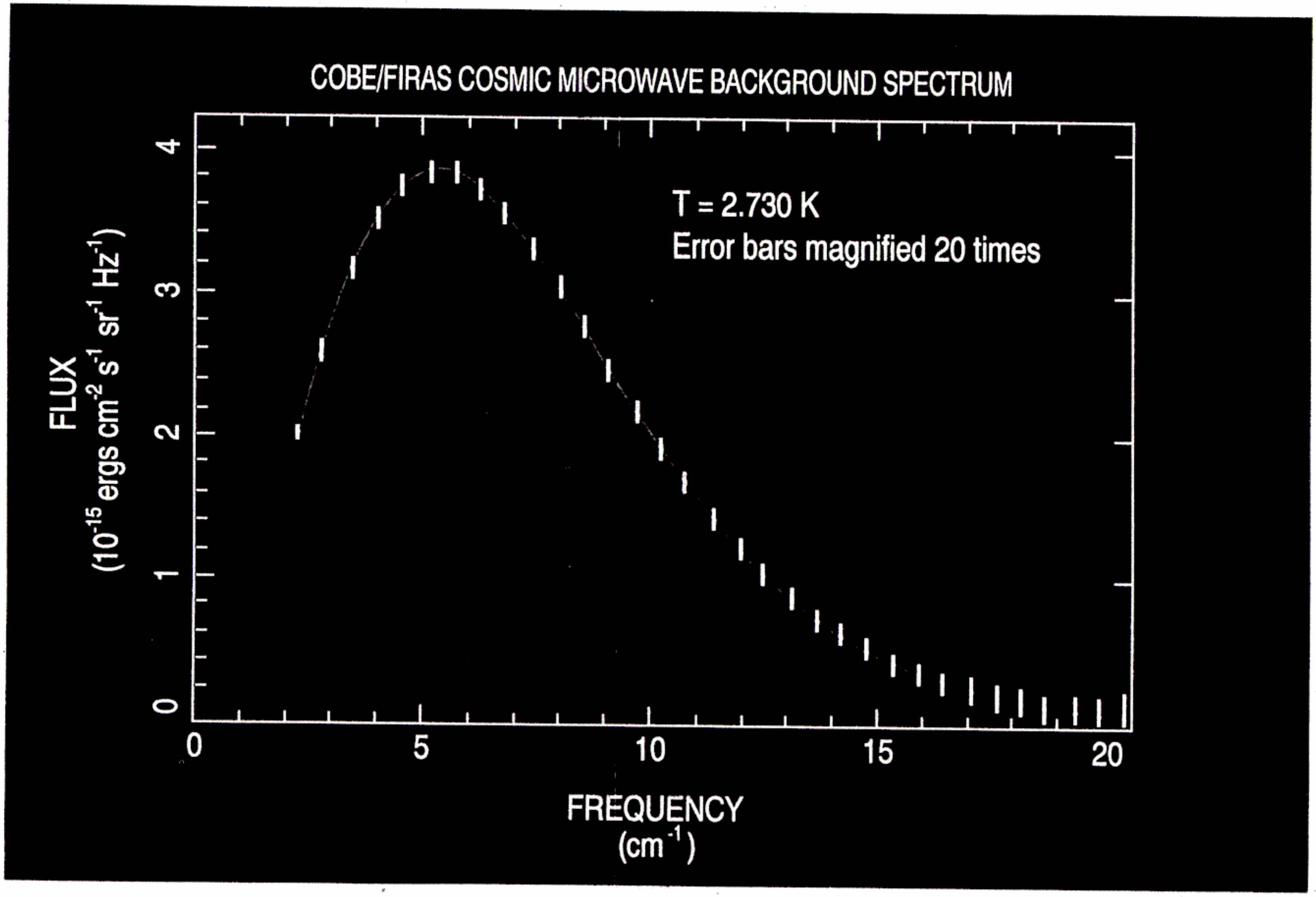
$T_\gamma \propto a^{-1}$ Black body

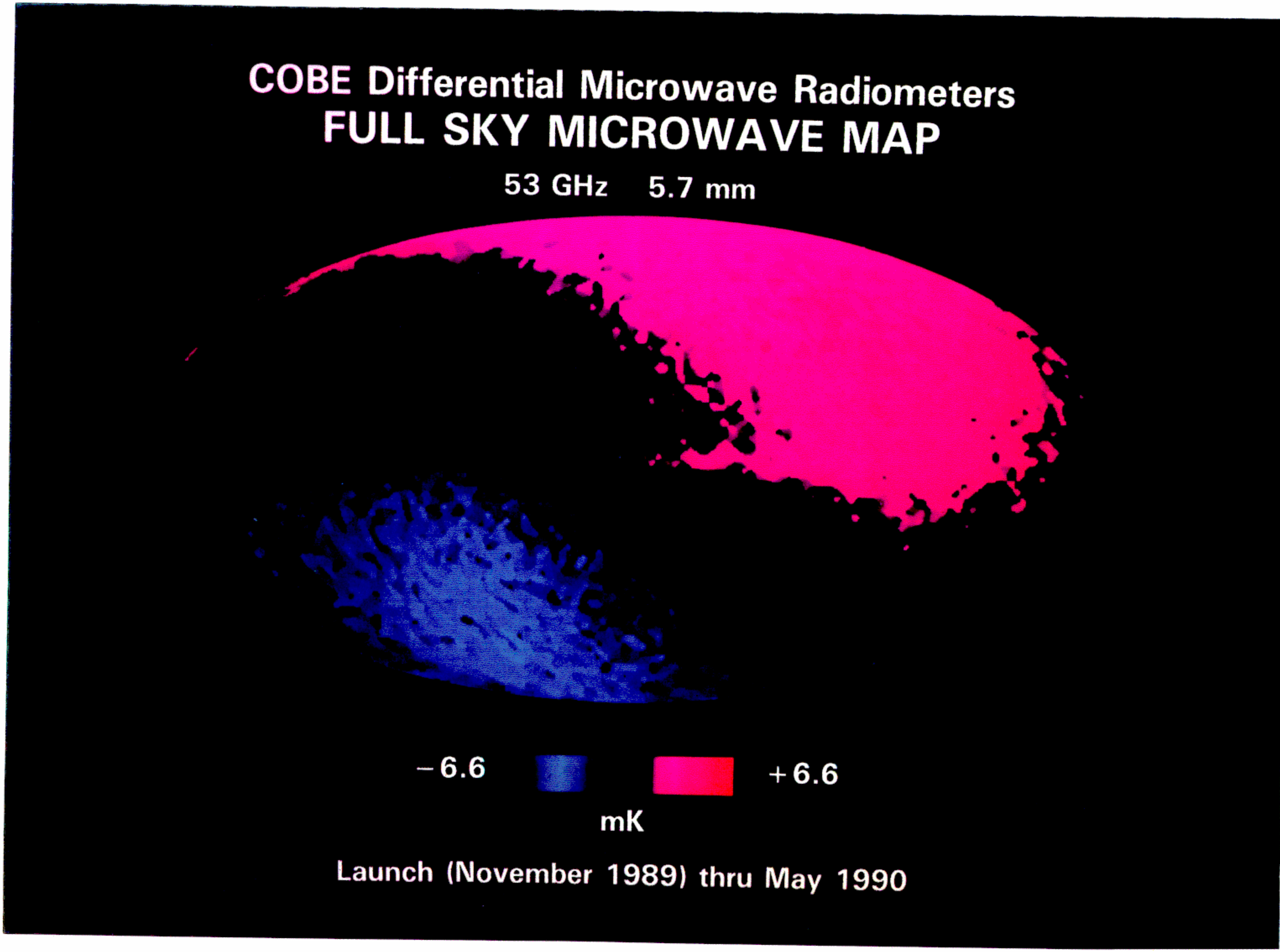
Cosmic Microwave Background



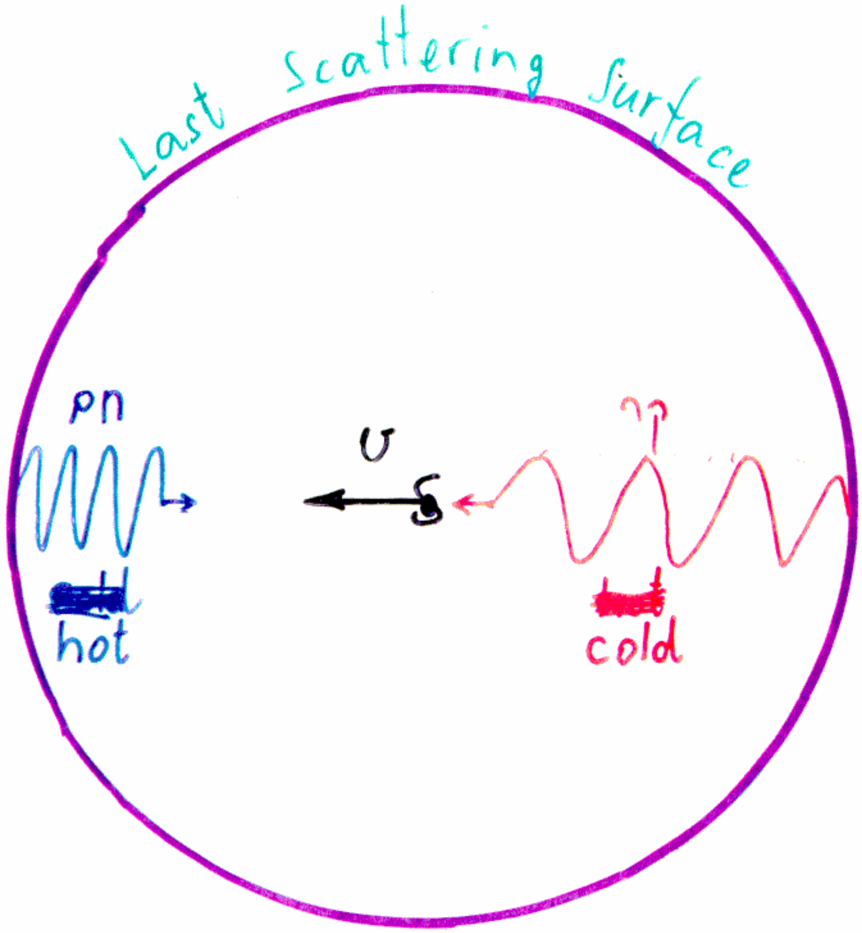


Far Infra Red Spectrometer   Differential Microwave Radiometer


COBE/FIRAS COSMIC MICROWAVE BACKGROUND SPECTRUM
T = 2.730 K
Error bars magnified 20 times
FLUX
(10^-15 ergs cm^-2 s^-1 sr^-1 Hz^-1)
0
1
2
3
4
0
5
10
15
20
FREQUENCY
(cm^-1)

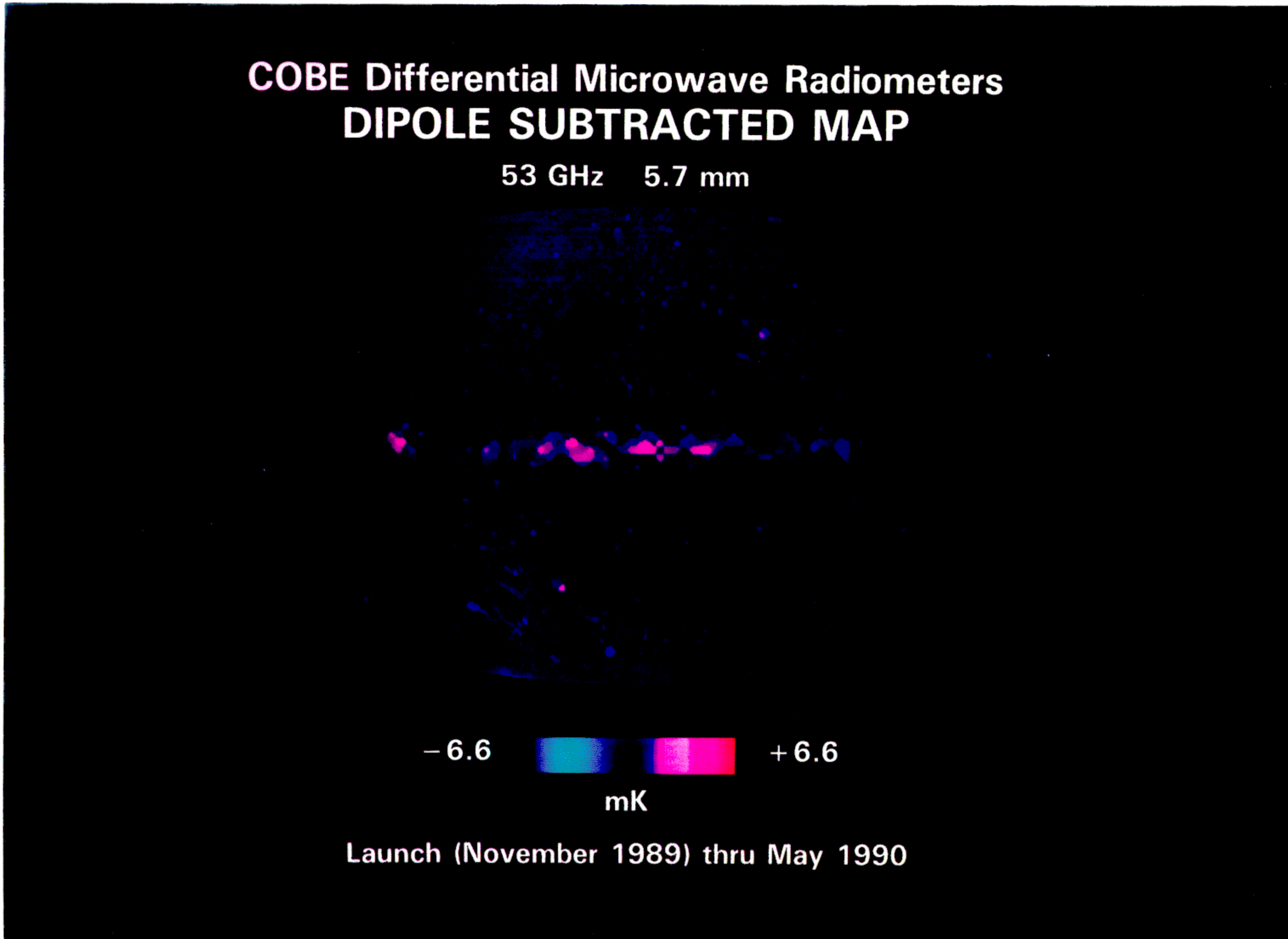

COBE Differential Microwave Radiometers
FULL SKY MICROWAVE MAP
53 GHz 5.7 mm
−6.6
+6.6
mK
Launch (November 1989) thru May 1990

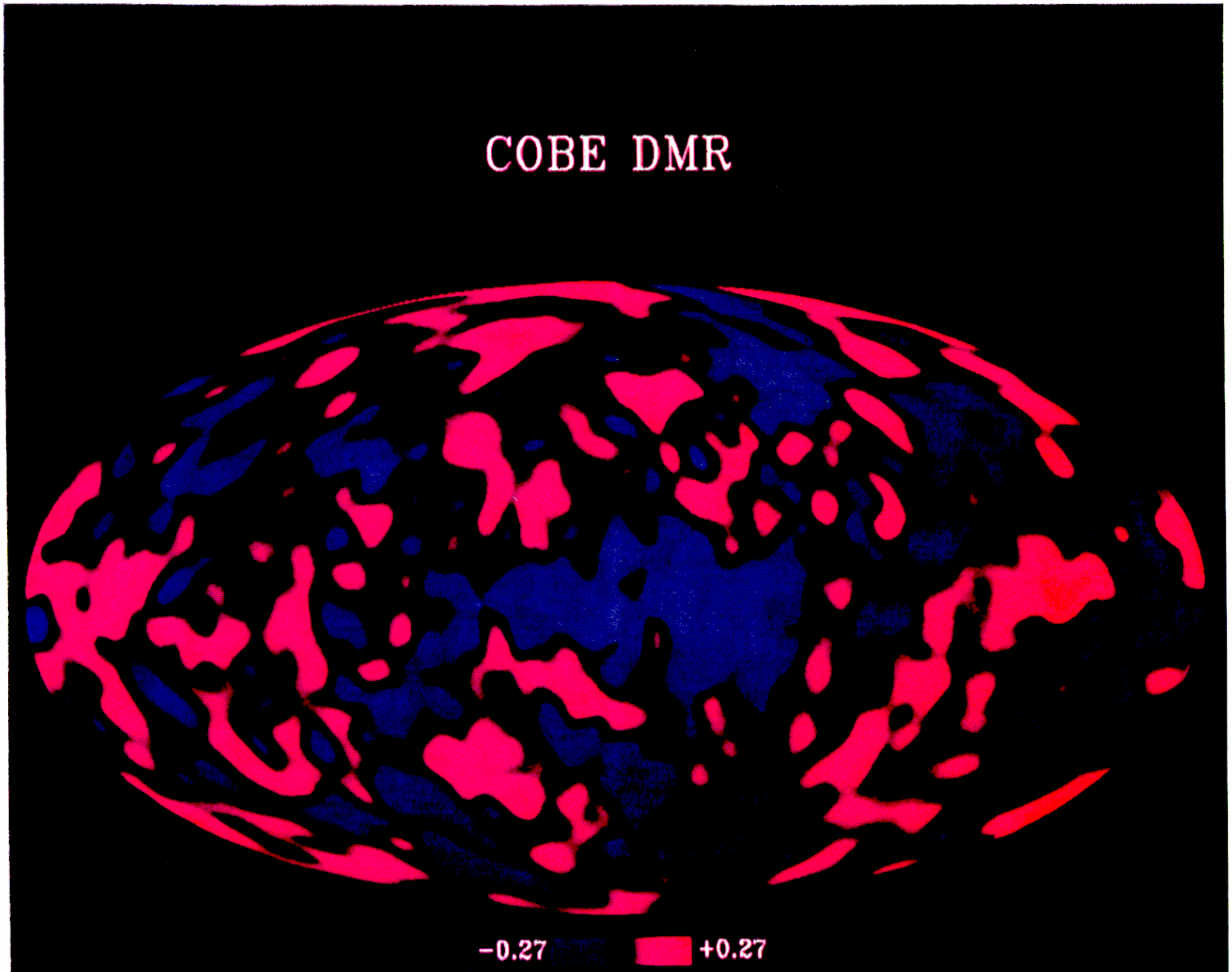
③ Dipole - Doppler



$$\frac{v}{c} \sim \frac{\delta \nu}{\nu} \sim \frac{\delta T}{T} \sim 2 \times 10^{-3} \longrightarrow v \sim 600 \ km \, s^{-1}$$

Local motion?

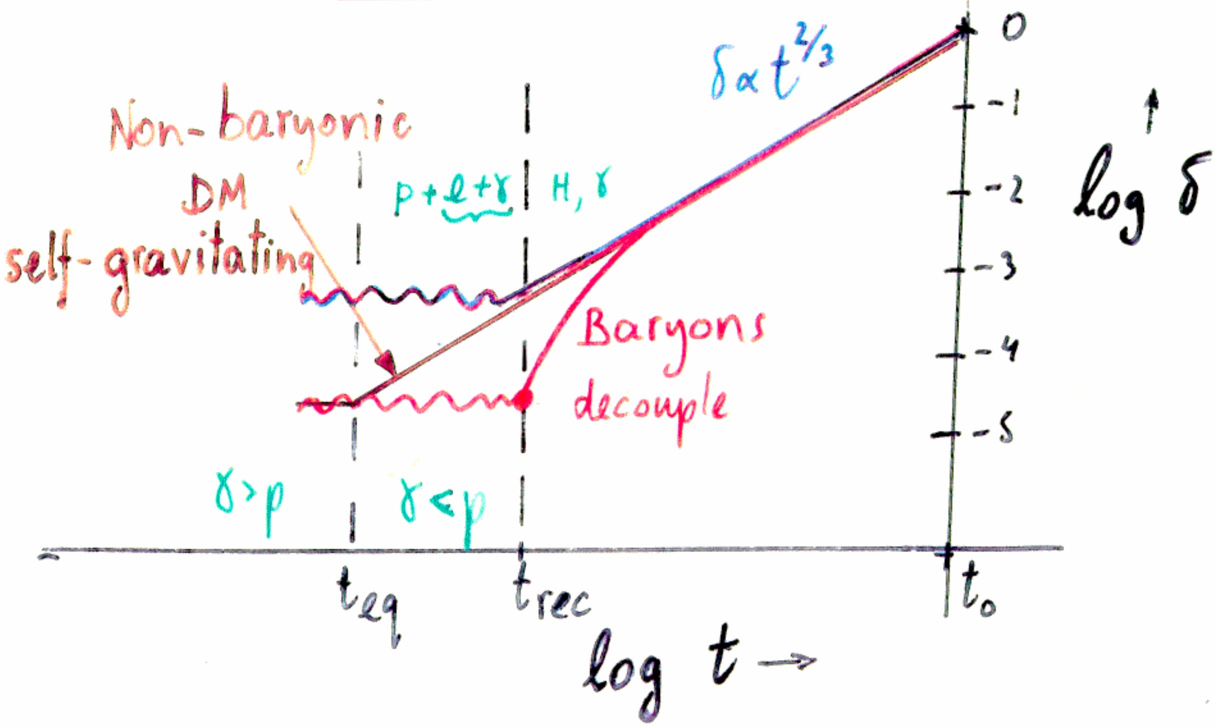

COBE Differential Microwave Radiometers
DIPOLE SUBTRACTED MAP
53 GHz 5.7 mm
−6.6
+6.6
mK
Launch (November 1989) thru May 1990



Górski $\pm 10^{-4}$ 10° 1992

AD

CMB Baryonic Matter!



$\delta_0 > 1 \rightarrow \delta_{rec} > 10^{-3}$

$\left(\frac{\delta T}{T}\right) \sim \frac{1}{3}\left(\frac{\delta \rho}{\rho}\right)_{rec} > 3 \times 10^{-4}$ (baryonic)

But $\left(\frac{\delta T}{T}\right)_{obs} \leq 3 \times 10^{-5}$!

- Non-B DM → Growth since t_{eq}

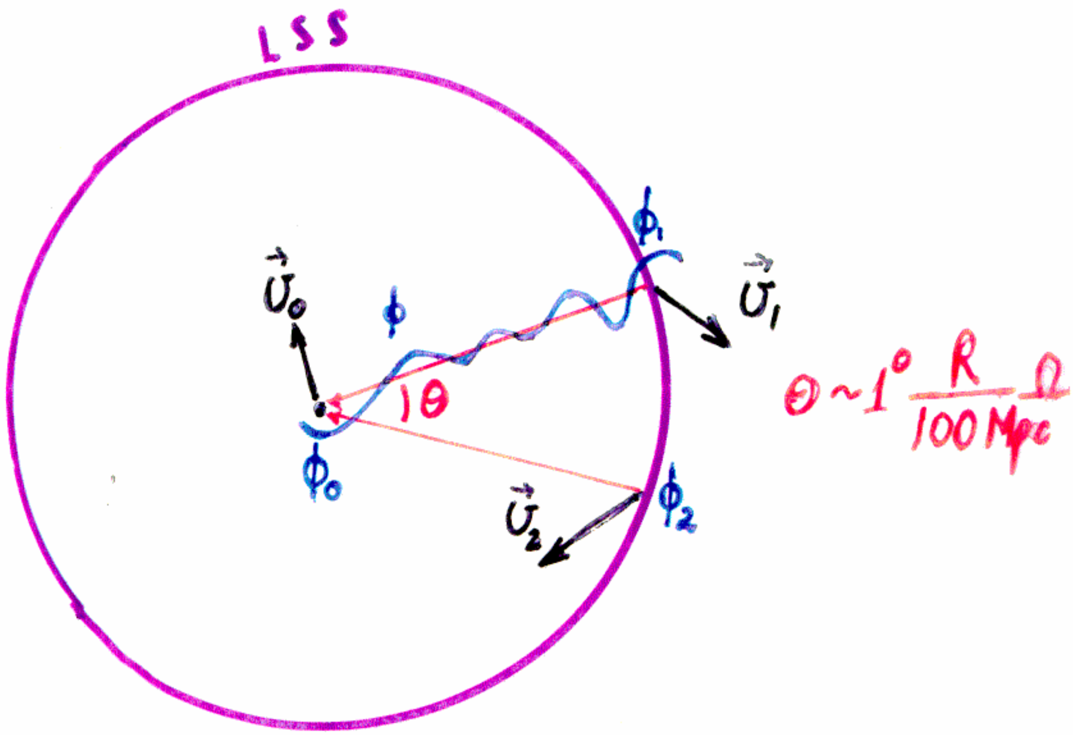
  → $\delta_B \sim 10^{-1} \delta_{DM}$

→ strong constraint : $\delta \sim 10^{-5}$

CMB T Fluctuations

$$\frac{\delta T}{T} = \frac{1}{3c^2}\delta\phi - \frac{1}{c}\hat{r}\cdot\delta\vec{v}$$

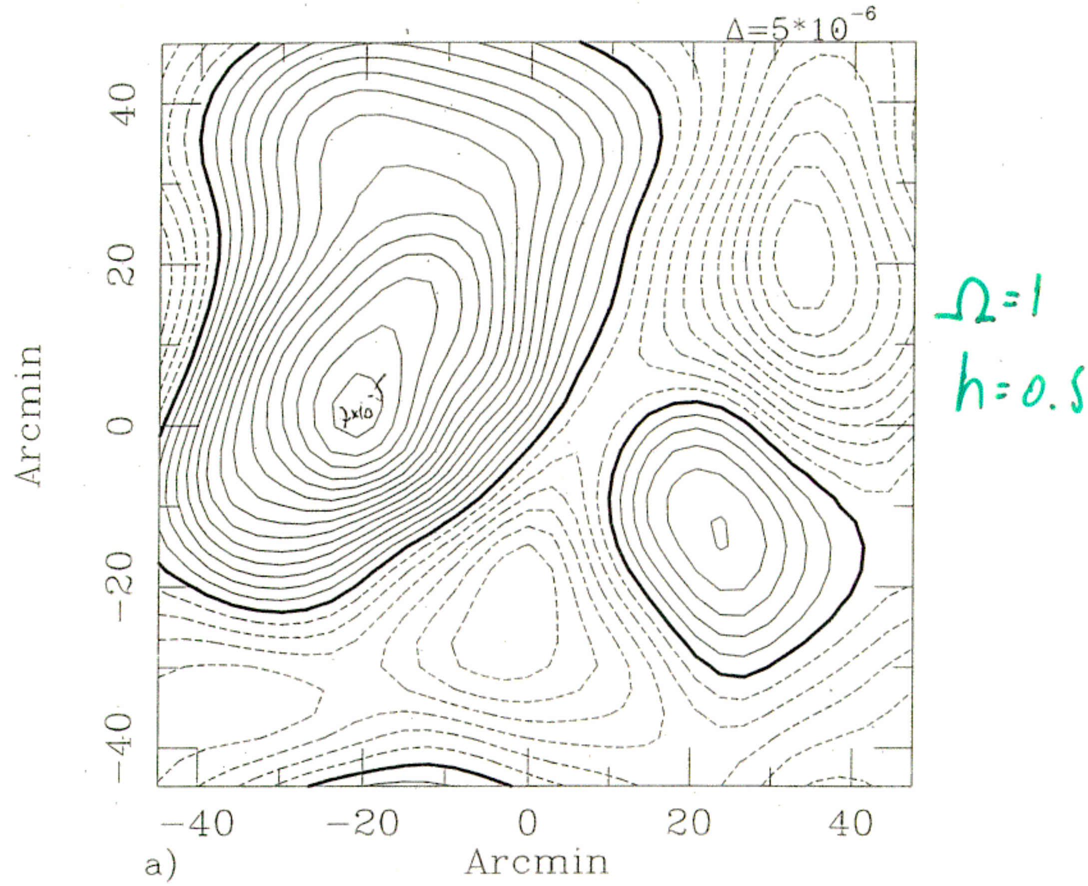
G-shift : Sachs-Wolfe $\gtrsim 1^\circ$ Doppler $\lesssim 1^\circ$



$$v \sim -\nabla\phi \rightarrow \Delta\phi \sim v\cdot\Delta x$$

$$\frac{\delta T}{T} \sim \frac{v\,\Delta x}{3c^2} \sim \frac{3\times10^7\times10^9}{3\times(3\times10^{10})^2} \sim 10^{-5}$$

$v \sim 300$ km/s



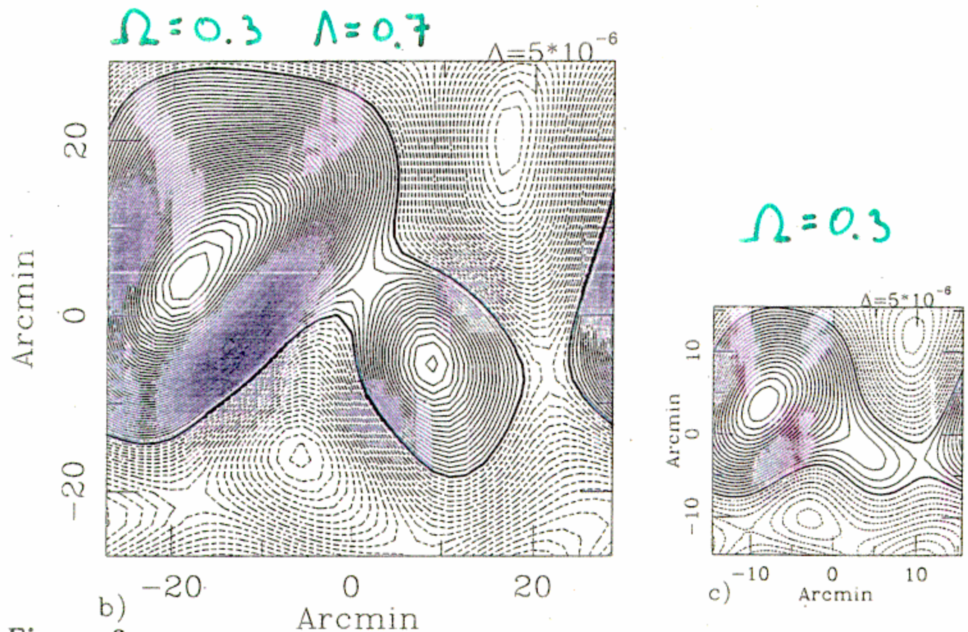


Figure 3

Zaroubi, Sugiyama, Silk, Hoffman, Dekel 96

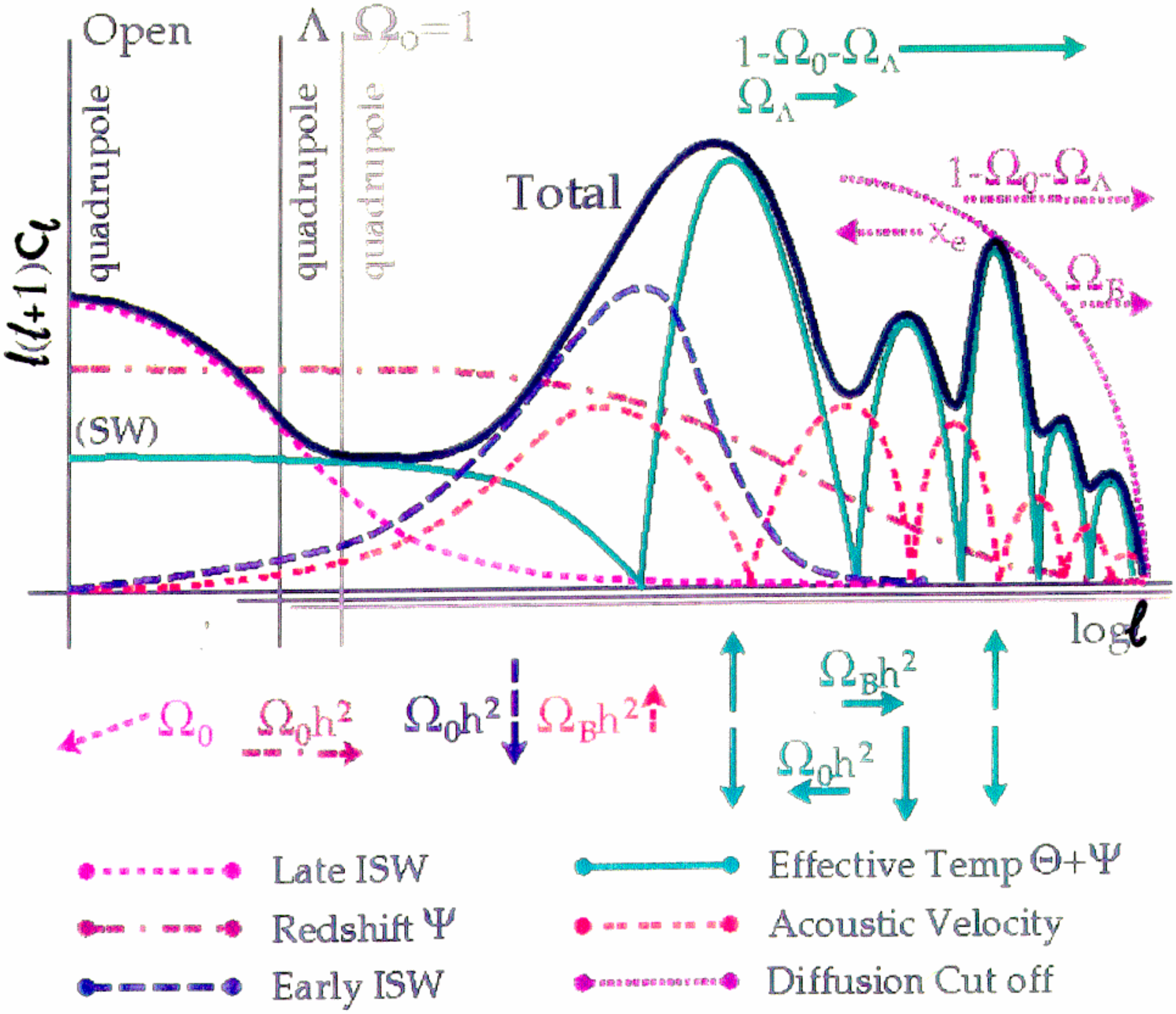
Bulk velocity ~ 100 Mpc

+

COBE ~ 1,000 Mpc

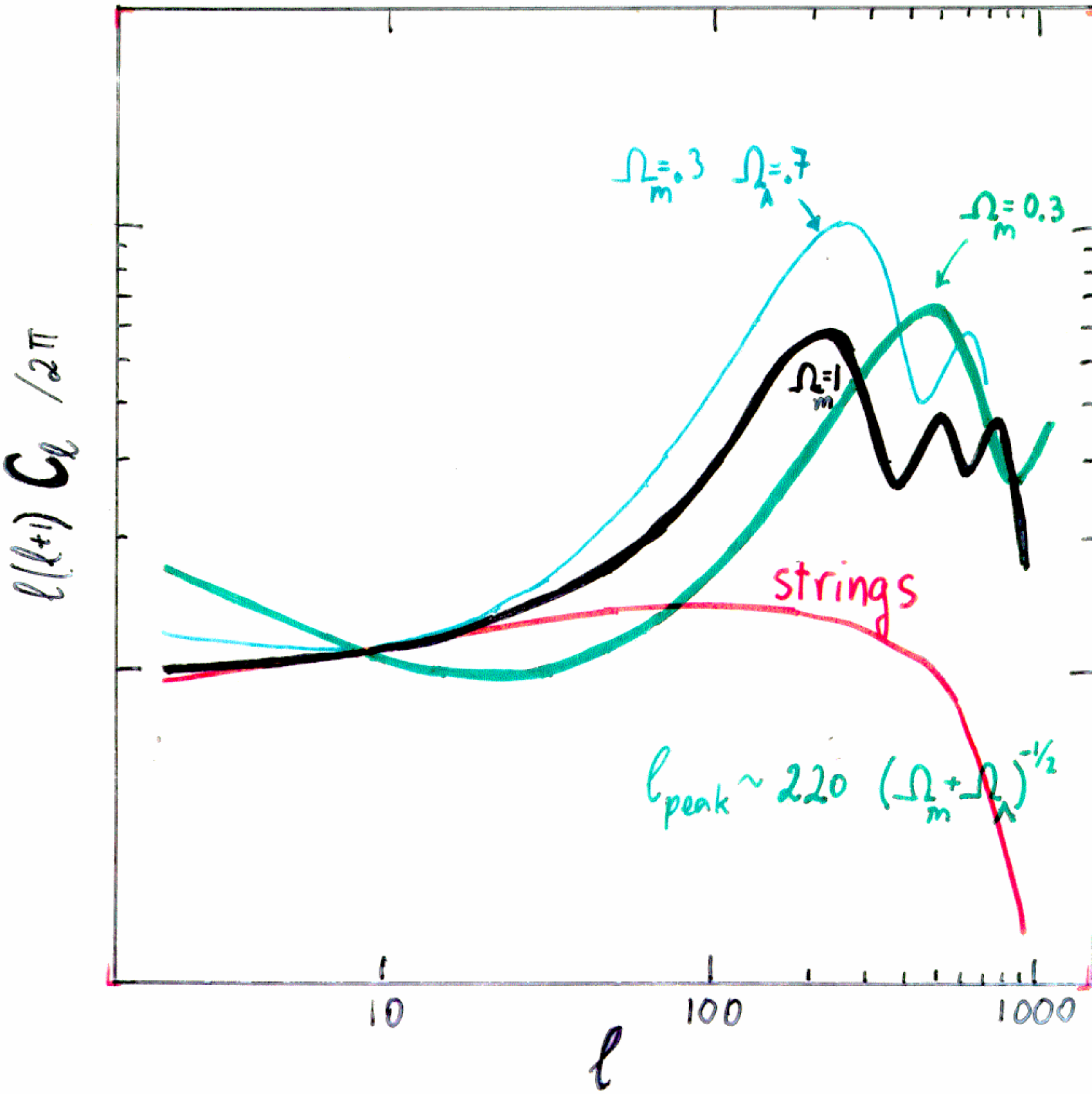
⇓

G.I. , $n \sim 1$, V_{pec} real

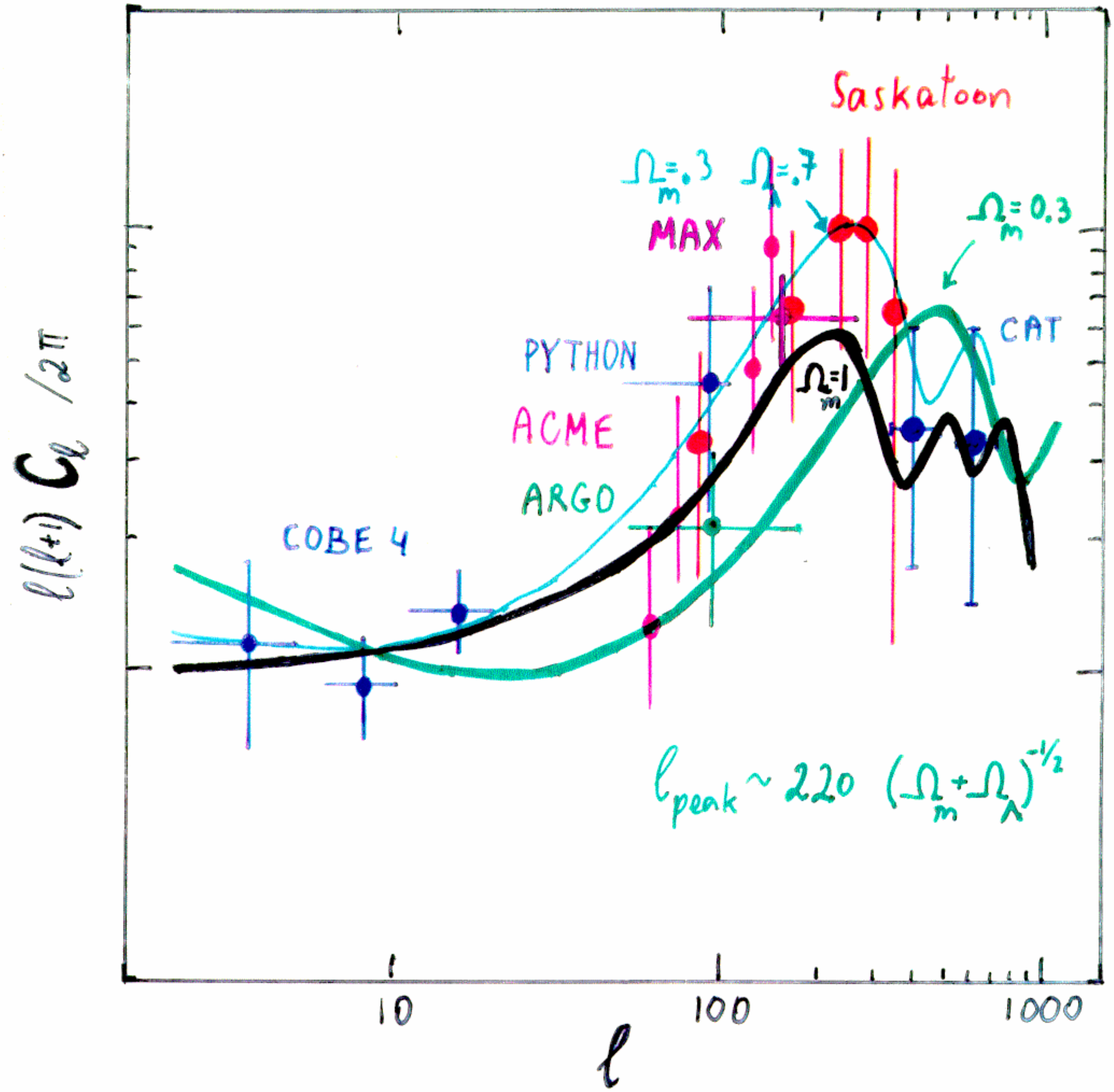


Hu, Sugiyama, & Silk (1995)

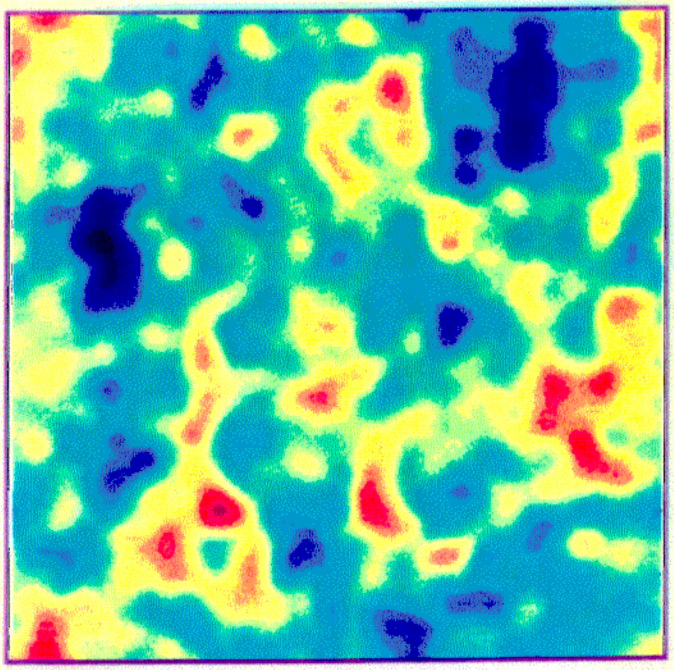
CMB Fluctuations



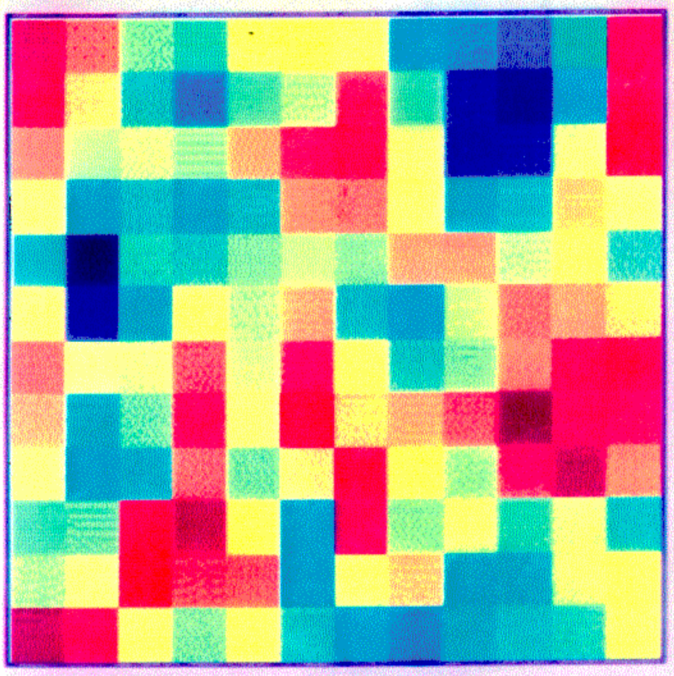
CMB Fluctuations



COBE 7°

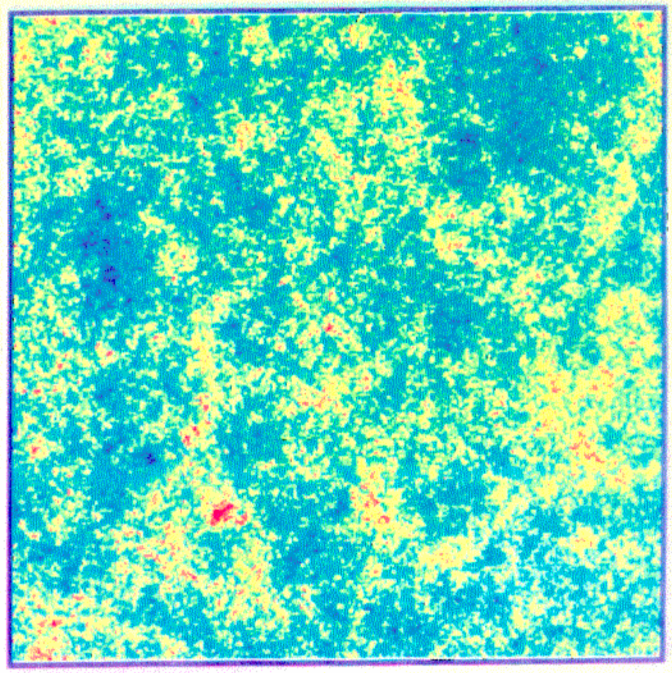


A 30x30 degree map of a standard CDM sky



A 30x30 degree map of a standard CDM sky
2.5 degree COBE pixels shown.

MAP, Planck 10'



A 30x30 degree map of a standard CDM sky