Large-Scale Structure

in 3 hours

Avishai Dekel

SLAC 98

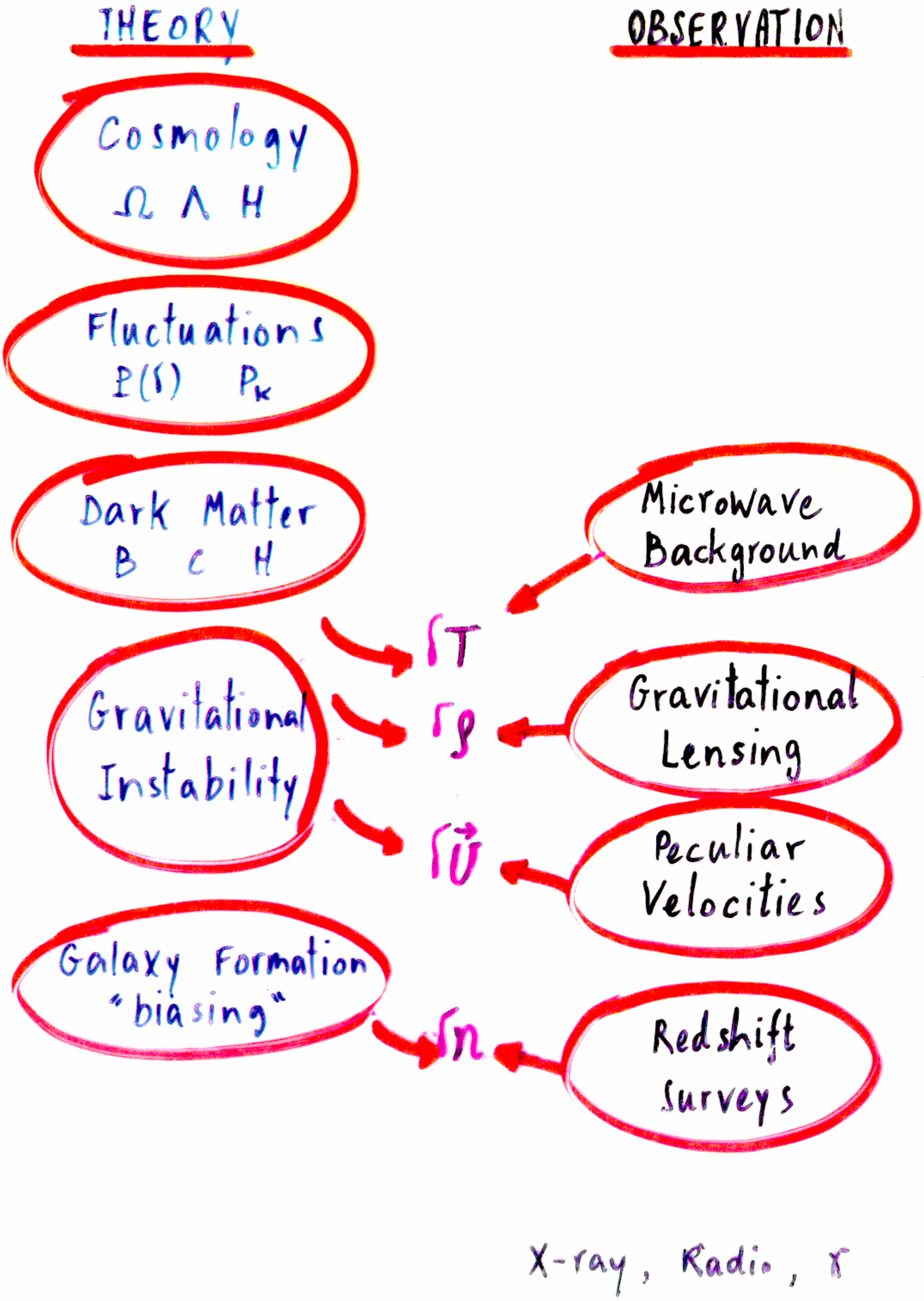
- Cosmology: H_0, t_0, Ω, Λ
- Dark Matter: Baryonic, Cold, Hot
- Fluctuations: $\underline{P}(r)$, ... , $P(k)$
- Evolution: Gravitational Instability
  Thermal History
- Galaxy Formation

Texts on Physical Cosmology

- Peebles
- Padmanaban
- Kolb & Turner
- Coles & Luccin

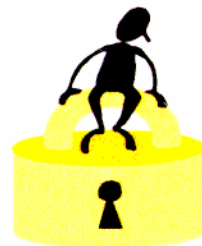
} Graduate text books ~ 1993

- Silk — Not technical
- Dekel & Ostriker 98 in press (CUP)


THEORY
OBSERVATION
Cosmology
Ω Λ H
Fluctuations
P(δ) P_k
Dark Matter
B C H
Gravitational Instability
Galaxy Formation
"biasing"
Microwave Background
Gravitational Lensing
Peculiar Velocities
Redshift Surveys
δT
δρ
δv
δn
X-ray, Radio, γ

Is the Universe

- **Open or Closed?**



- **Bound or Unbound?**

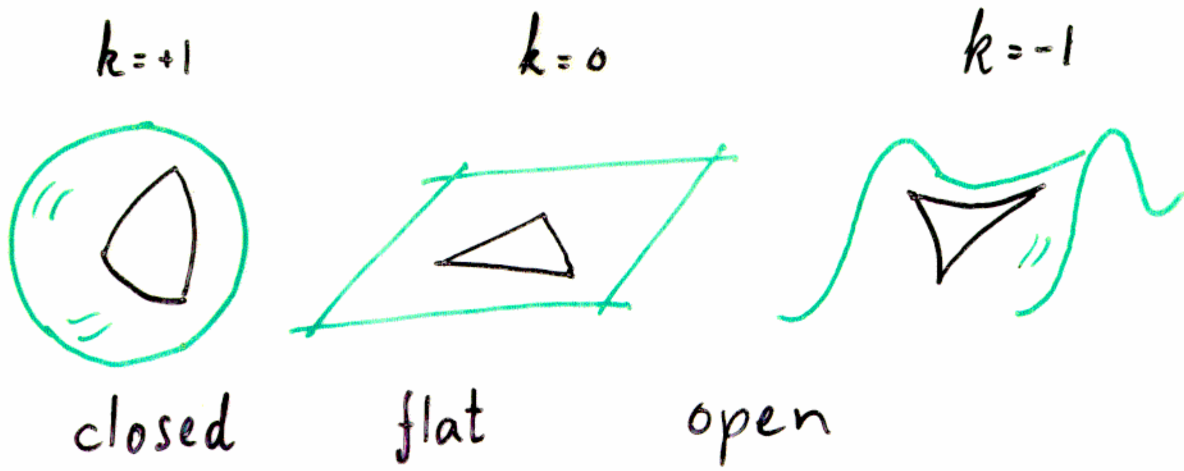
- **Decelerating or Accelerating?**

Avishai Dekel

COSMOLOGY

Homogeneity → R-W metric

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + S_k(r)\, dw^2\right]$$



- Hubble expansion: $H \equiv \dot{a}/a$
- Deceleration: $q \equiv -\ddot{a}a/\dot{a}^2$

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Robertson-Walker + Gravity (GR) →

$$\dot{a}^2 = \frac{\rho_0}{a} - k + \Lambda a^2$$ Friedman (matter)

or: $$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

$$\Omega_m \equiv \frac{\rho_m}{(3H^2/8\pi G)} \qquad \Omega_k \equiv -\frac{kc^2}{a^2H^2} \qquad \Omega_\Lambda \equiv \frac{\Lambda c^2}{3H^2}$$

$$q = \frac{1}{2}\Omega_m - \Omega_\Lambda$$

decelerate/acc.

$$\Omega_{tot} \equiv \Omega_m + \Omega_\Lambda$$

open/closed

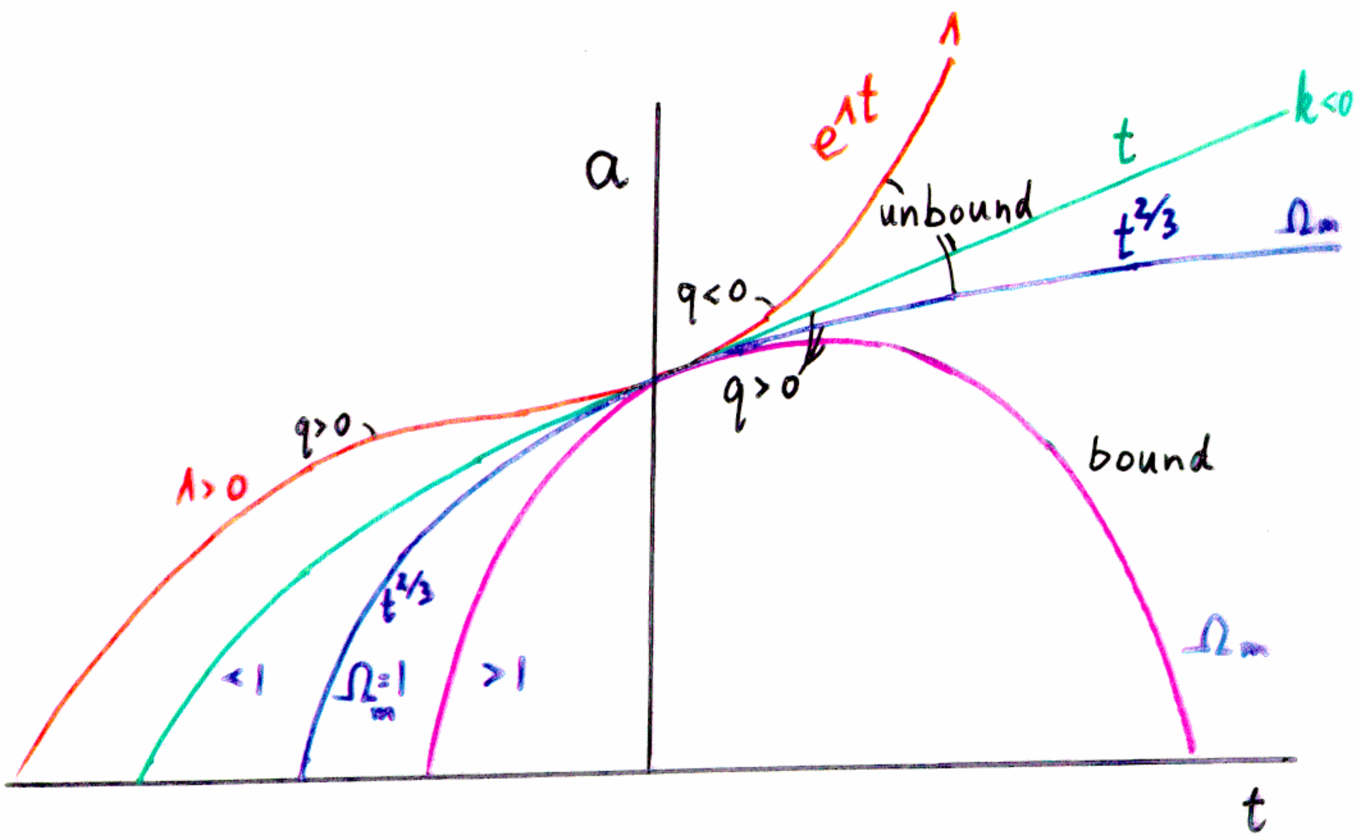


Table 1.2. *Theoretical Framework: GR Cosmology*

GR:	MATTER TELLS SPACE HOW TO CURVE,	CURVED SPACE TELLS MATTER HOW TO MOVE.

$$(E) \quad R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = -8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

COBE - Copernicus Th: If all observers measure nearly isotropic CBR, then universe is locally nearly homogeneous and isotropic – i.e., nearly FRW.

$$\textbf{FRW E(00)} \quad \frac{\dot a^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\textbf{FRW E}(ii) \quad \frac{2\ddot a}{a} + \frac{\dot a^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$$

$$H_0 \equiv 100h \,\mathrm{km\,s^{-1}Mpc^{-2}} \equiv 50h_{50} \,\mathrm{km\,s^{-1}Mpc^{-2}}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H_0 \equiv \tfrac{\dot a_0}{a_0},\ a_0 \equiv 1,\ \Omega_0 \equiv \tfrac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \tfrac{\Lambda}{3H_0^2},$$

$$\rho_c \equiv \tfrac{3H_0^2}{8\pi G} = 0.70 \times 10^{11} h_{50}^2 M_\odot \mathrm{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot a}{a} = -\frac{8\pi}{3} G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot a}{a}\frac{a^2}{\dot a^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{\delta a}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{\delta a}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78 h^{-1} \mathrm{G\,yr} \qquad f(1,0) = \tfrac{2}{3},\ f(0,0) = 1,\ f(0,1) = \infty$$

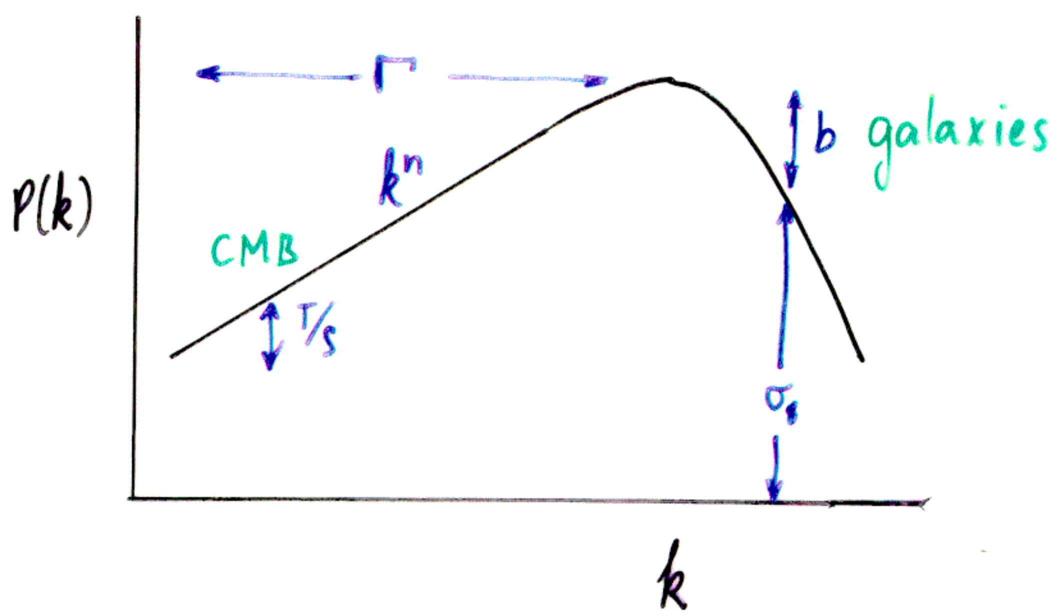
$$[E(00)a^3]' \text{ vs. } E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2 \ (\text{“continuity”})$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

Examples:
$p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)
$p = \frac{\rho}{3},\ k = 0 \Rightarrow \rho \propto a^{-4}$

Cosmological Parameters

- $\Omega_m \quad \Omega_\Lambda \quad H_0$ — bound/unb.
  - $\Omega_{tot} = \Omega_m + \Omega_\Lambda \qquad \Omega_k = 1 - \Omega_{tot}$ — open/close
  - $q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda$ — deceleration
  - $t_0 H_0 = f(\Omega_m, \Omega_\Lambda)$
- $\Omega_m = \Omega_b + \Omega_c + \Omega_\nu + \ldots$
- Fluctuations: n, T/S, Γ, σ_8, b, ...



A. Dekel, SSI 1998

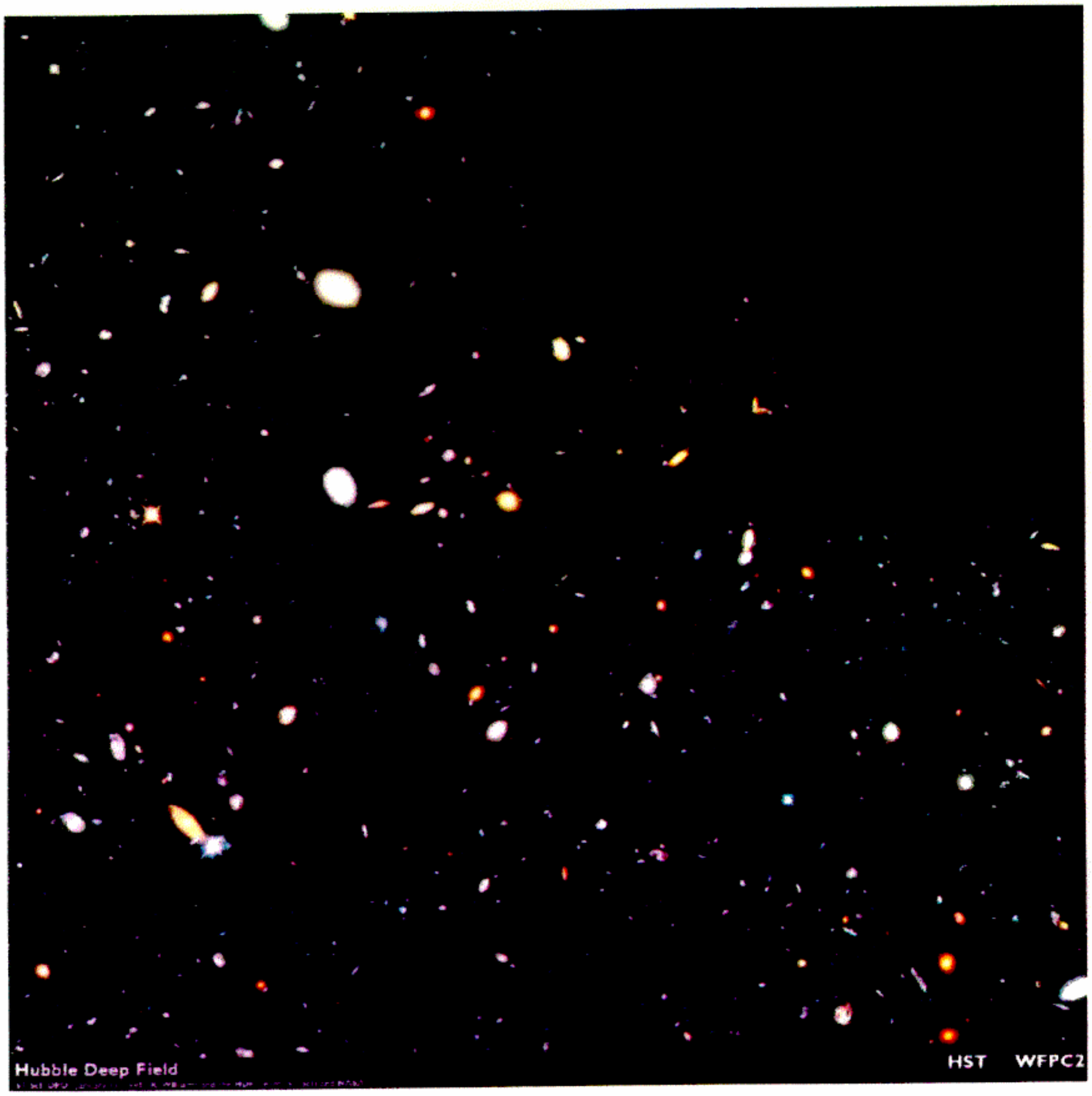
Table 1: Estimates of Ω_m

Global Measures	Inflation, Occam	$\Omega_m + \Omega_\Lambda = 1$ $(\Omega_m = 1, \Omega_\Lambda = 0)$	H
	• Lum. distance SNIa	$-0.3 < \Omega_m - \Omega_\Lambda < 1.5$ (90%)	L
		~~Flat $\Omega_m > 0.49$ (95%)~~	~~H~~
	• Lens Counts	Flat $\Omega_m > 0.34$ (95%)	H
	• CMB Peak	$\Omega_m + \Omega_\Lambda < 1.5$ (95%)	
		$\Omega_m + \Omega_\Lambda > 0.3$	H
	• $H_0 t_0$	$\Omega_m - 0.7\Omega_\Lambda < 2$ (likely ~ 1)	H
Virialized Objects	• $(M/L)\mathcal{L}$	$\Omega_m \sim 0.25$ $(0.1 - 1.0)$	L
	• Baryon fraction	$\Omega_m h_{65}^{1/2} \sim 0.3 - 0.5$ (low–high Ω_b)	L
	Cosmic Virial Th.	Point mass $\Omega_m \sim 0.2$ (halos $\rightarrow 1$)	L
	Local Group	Point mass $\Omega_m \sim 0.15$ (halos $\rightarrow 0.7$)	L
Large-Scale; Flows	• Peculiar velocities	$\Omega_m > 0.3$ (95%)	H
		${\Omega_m}^{0.6}\sigma_8^a = 0.8 \pm 0.2$ $(\beta_I^b \simeq 1.05^c)$	
	Redshift Distortions	$\beta_I \sim 0.5 - 1.1$	H
	• Velocity vs Density	$\beta_I \sim 0.5 - 1.0$ (scale dependent)	H
		$\beta_O \sim 0.4 - 0.8$	
	Cluster Abundance	${\Omega_m}^{0.6}\sigma_8 \simeq 0.5 - 0.6$ $(\beta_I \simeq 0.7 - 0.8^c)$	H
Fluct. Growth	Cluster Morphology	$\Omega_m > 0.2$ (?)	
	Galaxy Formation	(?)	
	$P_k(\rho)$ vs C_l	CDM $n = 1$ $b = 1$: $\Omega_m h_{65} \sim 0.3$	L
	• $P_k(v)$ vs C_l	CDM flat: $\Omega_m h_{65} n^2 \simeq 0.7 \pm 0.1$	H

[a] σ_8 is the *rms* mass density fluctuation in a top-hat sphere of radius $8\,h^{-1}$Mpc.
[b] $\beta \equiv \Omega^{0.6}/b$, b_I for IRAS galaxies, b_O for optical galaxies.
[c] $b_O/b_I \simeq 1.3$, $b_O \simeq 1/\sigma_8$.

Mass-Density Budget

- $\Omega_{luminous} \sim 0.01$ — stars, gas
- $\Omega_{galaxy\ halos} \sim 0.1$ — MACHOs ?
- $\Omega_{clusters} \sim 0.2$
- $\Omega_{large\text{-}scale} \overset{?}{\sim} 1$ — Non baryonic
- $\Omega_{baryons} \sim 0.05 - 0.1$ — Nucleosynthesis D, He



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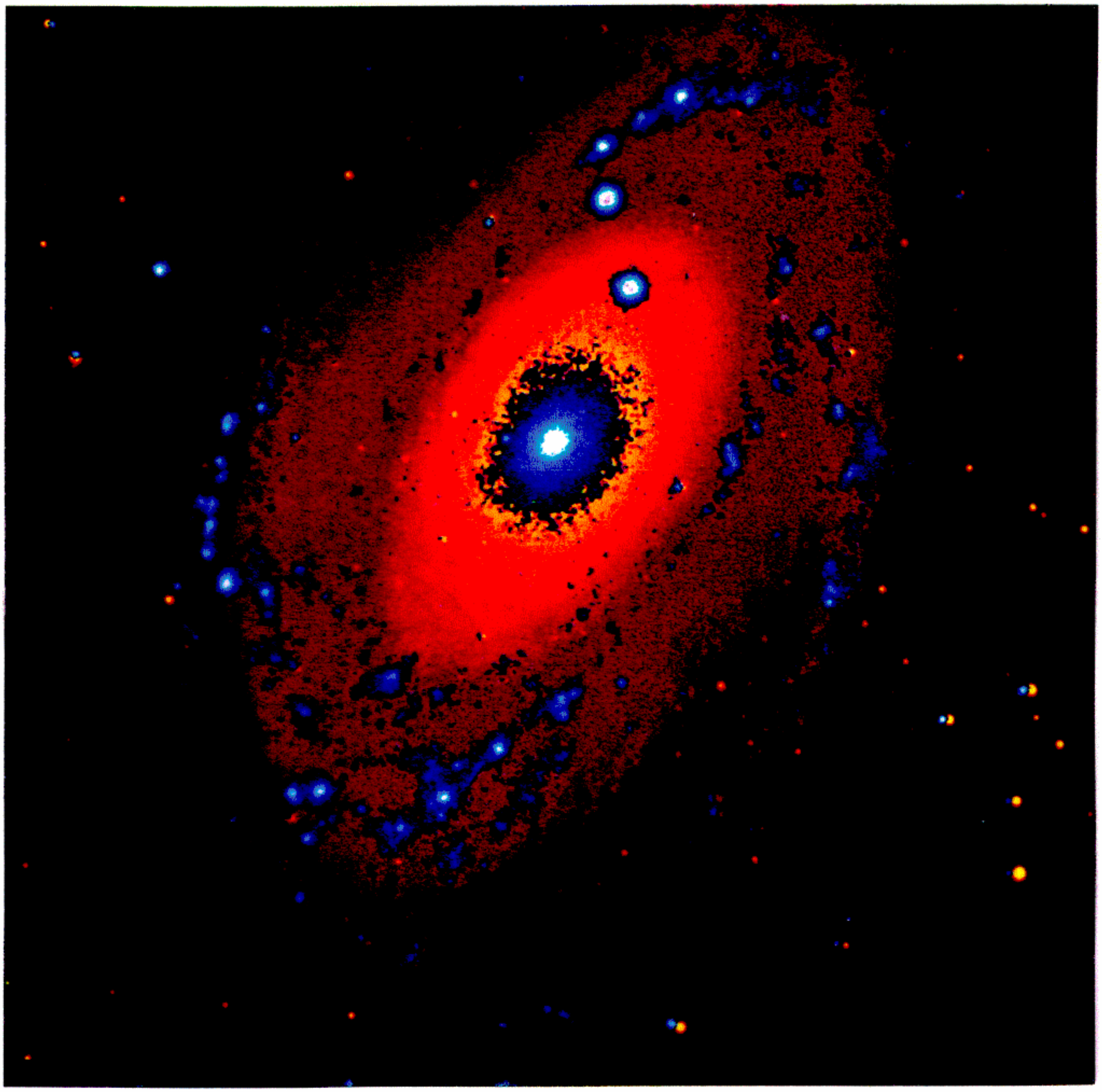
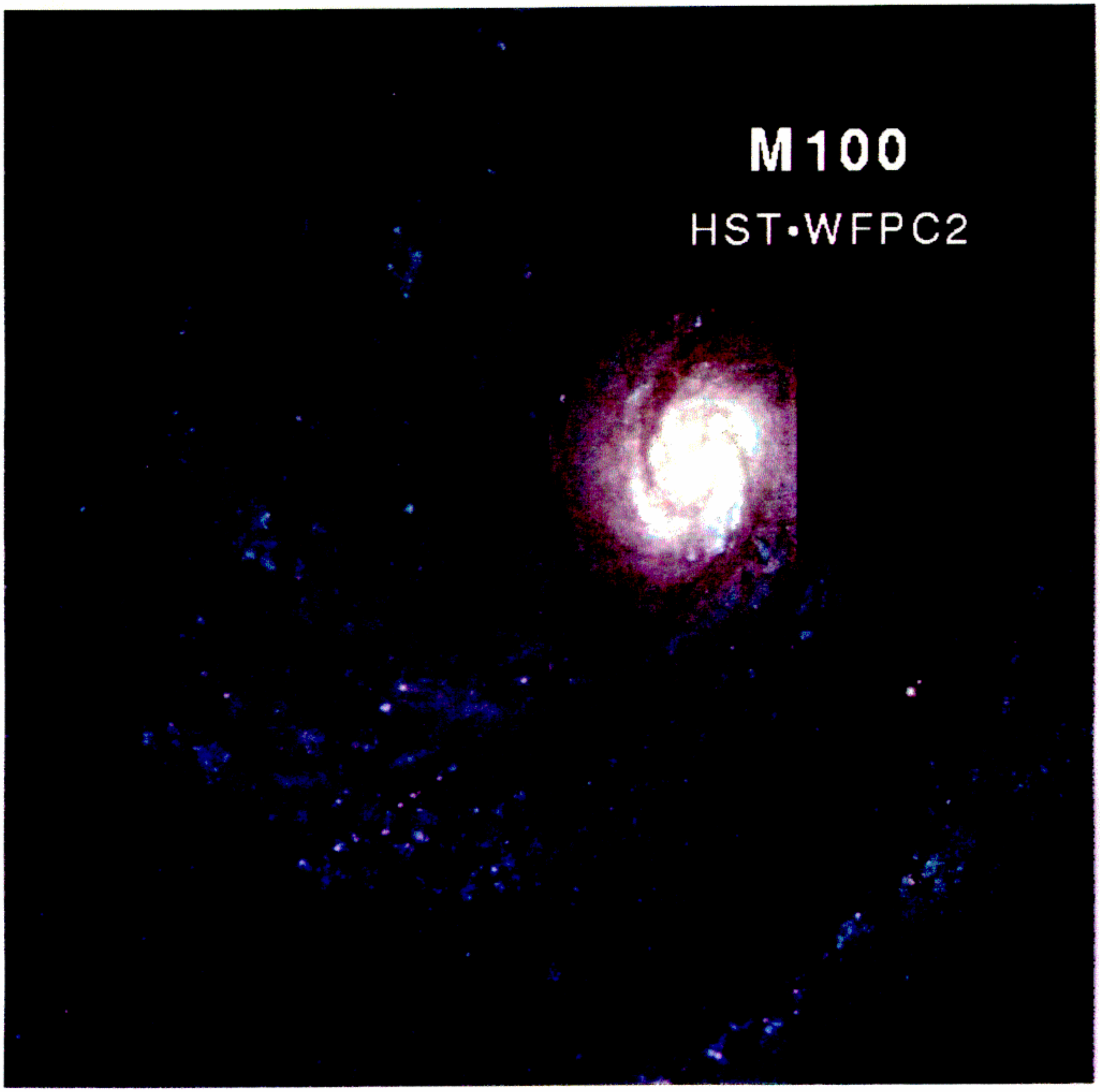


M81 Sab



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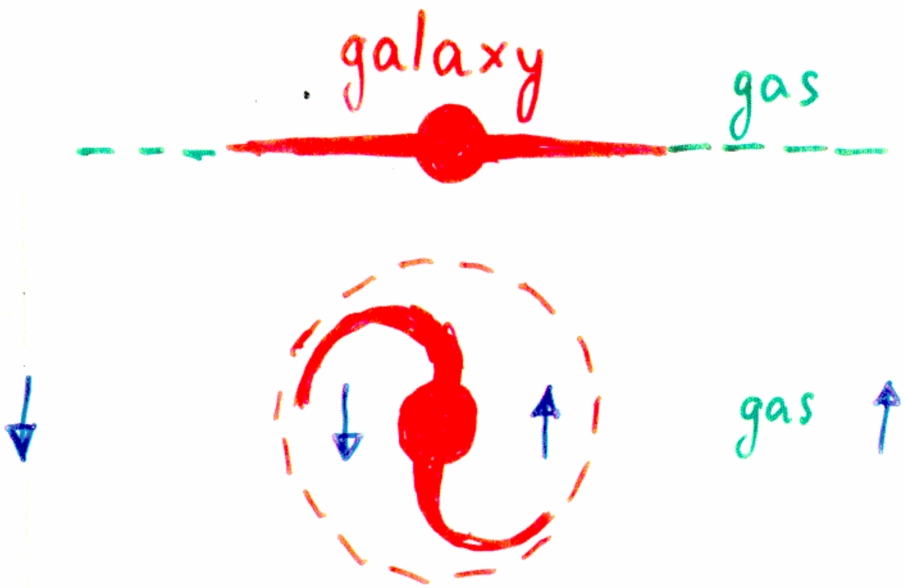
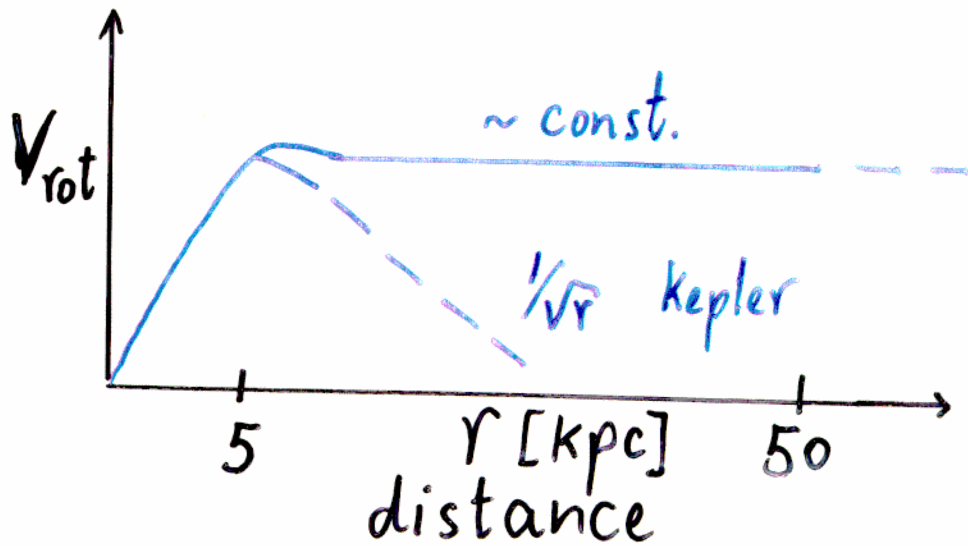


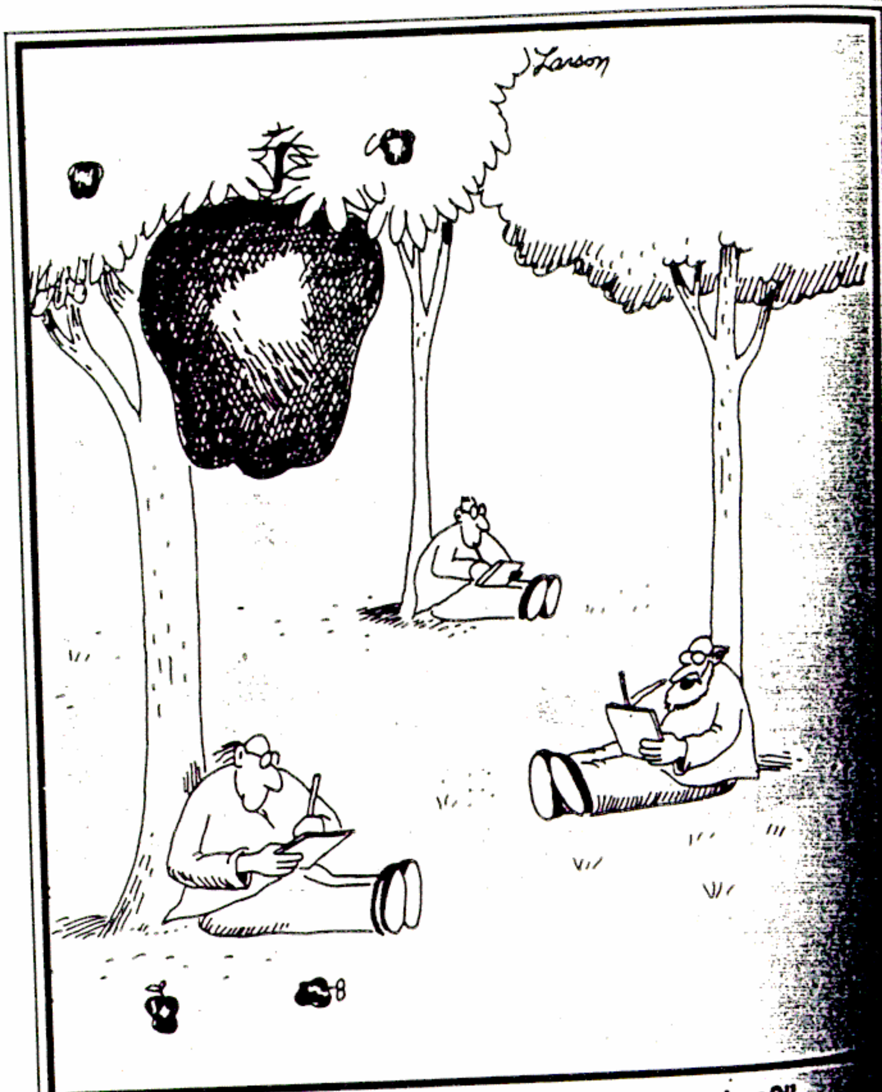


$$V^2 = \frac{GM(r)}{r} \longrightarrow M(r) \propto r$$

→ Dark, Massive Halos

→ $\Omega_m \geqslant 0.1$

NGC 4565



"Nothing yet. . . . How about you, Newton?"

People who don't see the
evidence for lots of gravity...

Big Bang Nucleosynthesis

Baryon Density Ω_b

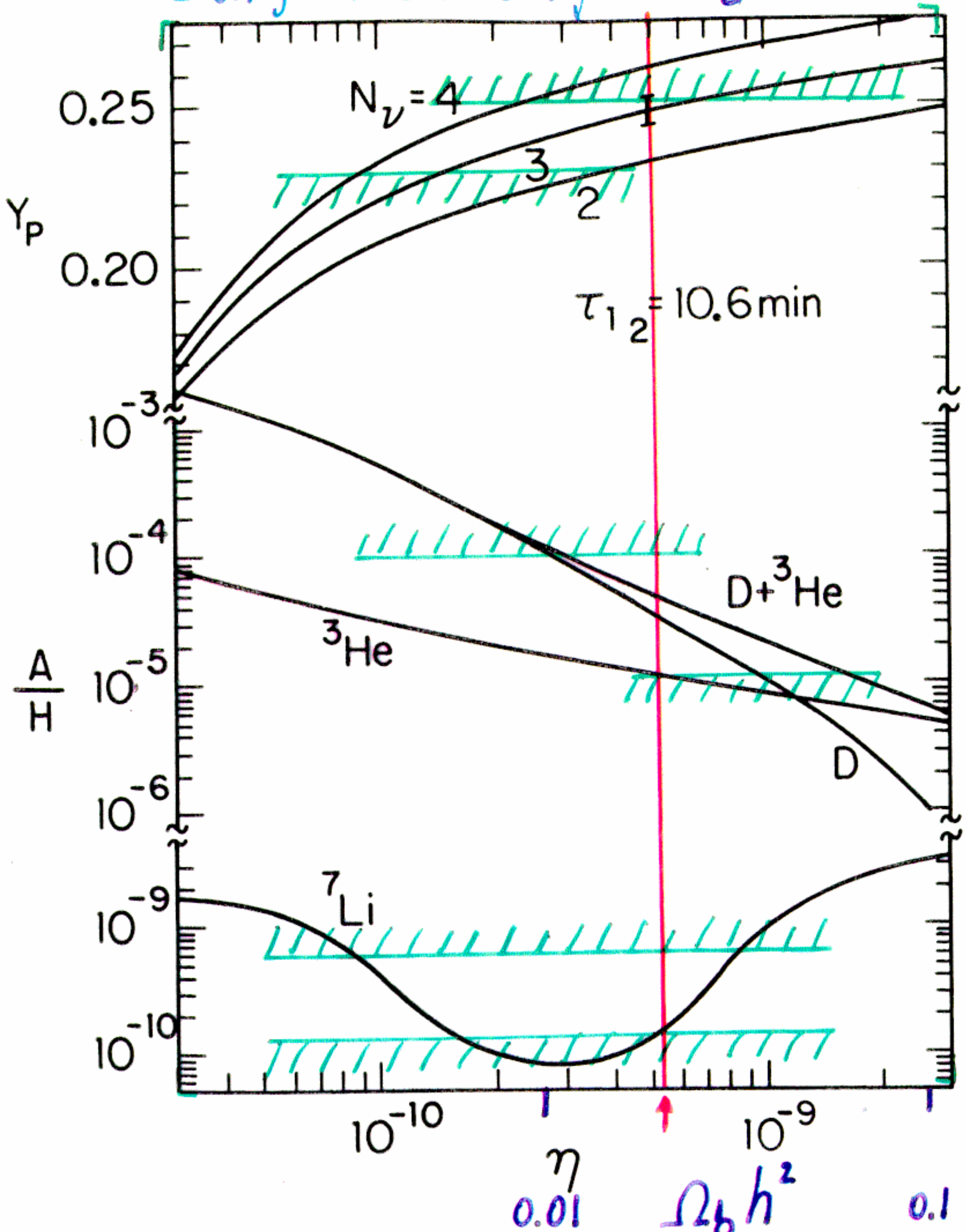
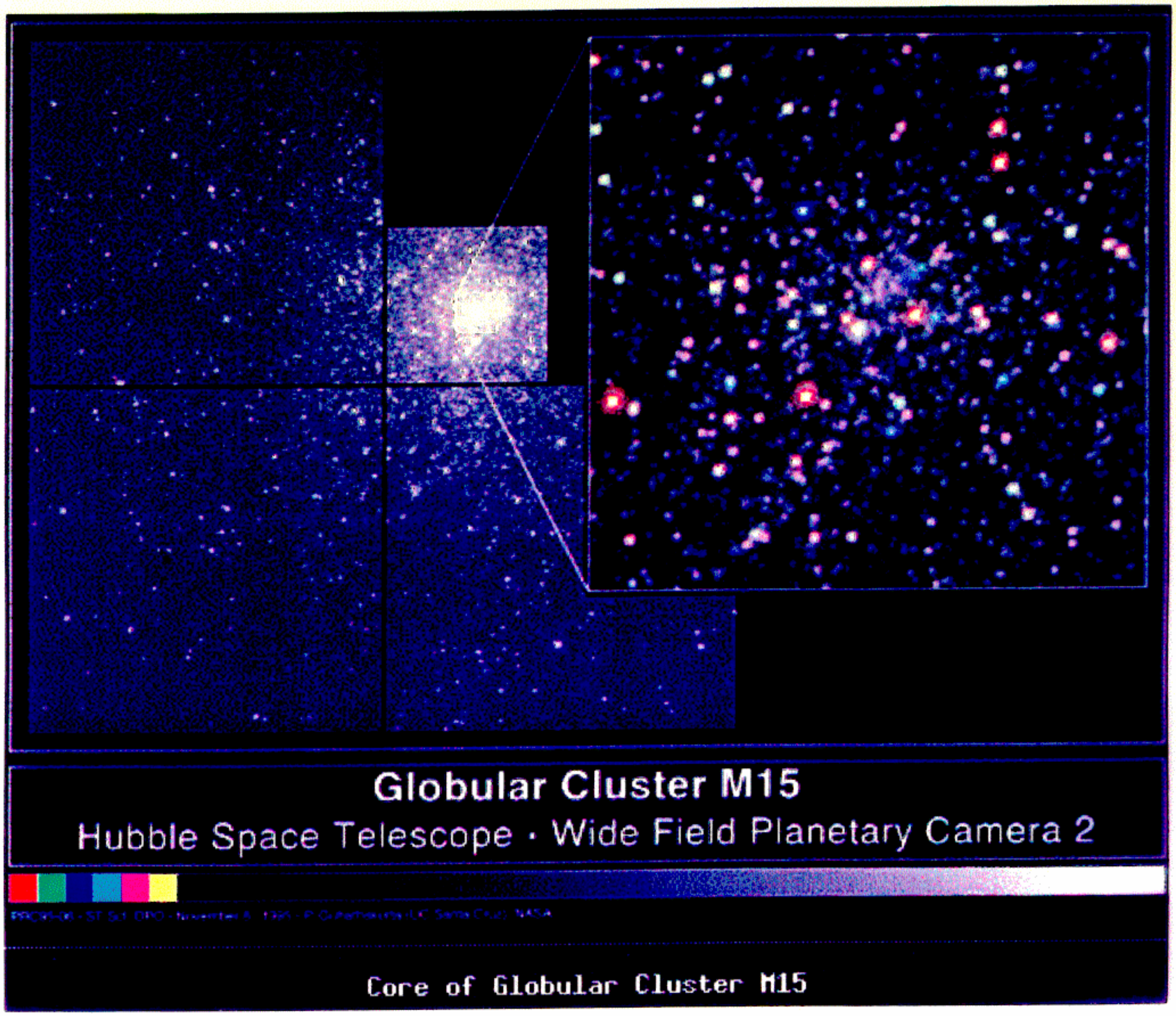
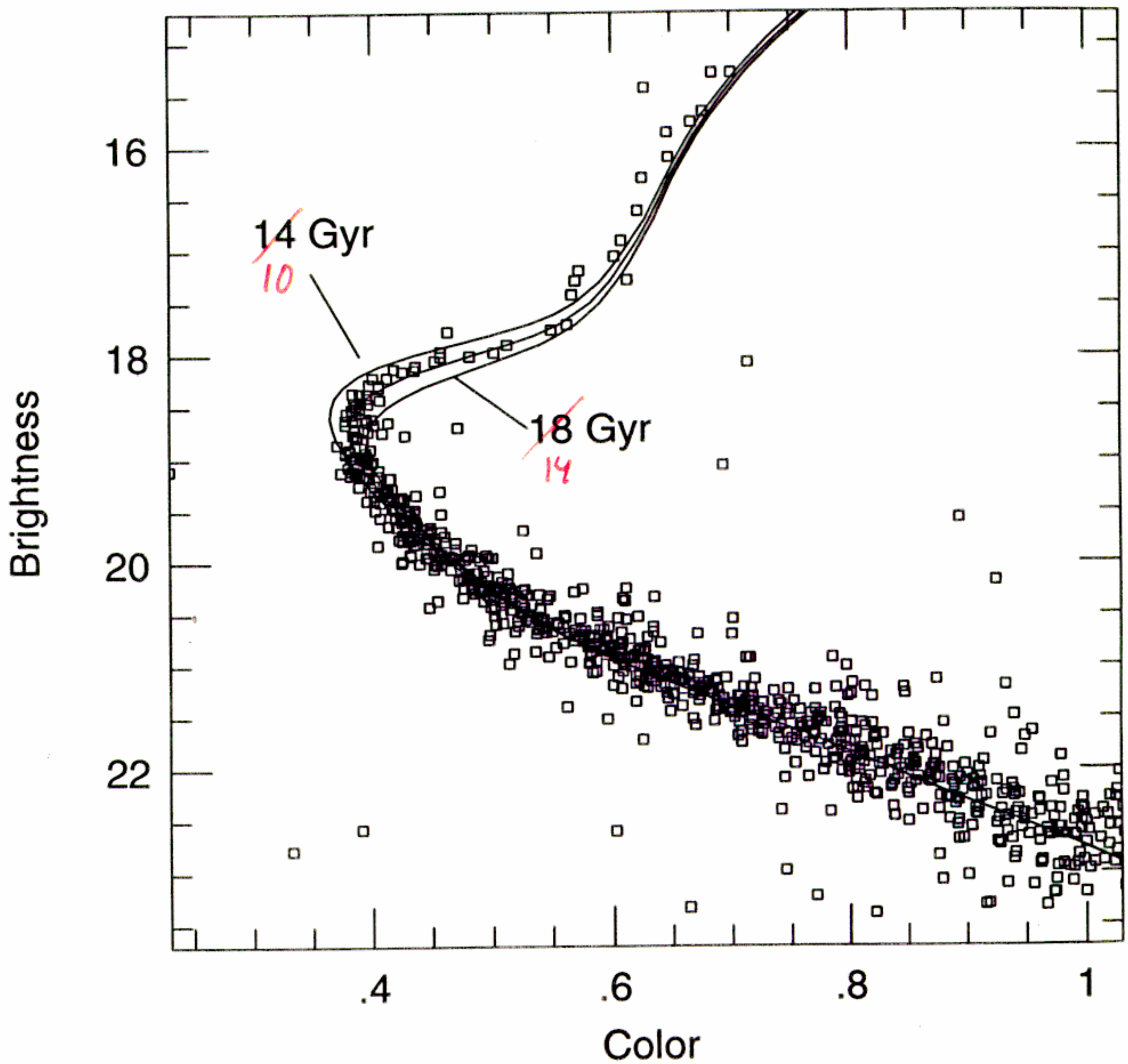


Fig. 4.4: The predicted primordial abundances of the light elements as a function of η. The error bar indicates the change in Y_P for $\Delta\tau_{1/2} = \pm 0.2$ min.

$$\Omega_b h^2 = 0.02 \pm 0.005$$


Globular Cluster M15
Hubble Space Telescope · Wide Field Planetary Camera 2
Core of Globular Cluster M15



Hipparcos: distances up ~10%

→ $t_0 = 12 \pm 2$ Gyr

$$H_0 t_0 = \int_0^1 \frac{da}{a}\left(\frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda\right)^{-1/2} \equiv f(\Omega_m, \Omega_\Lambda)$$

Age Crisis ?

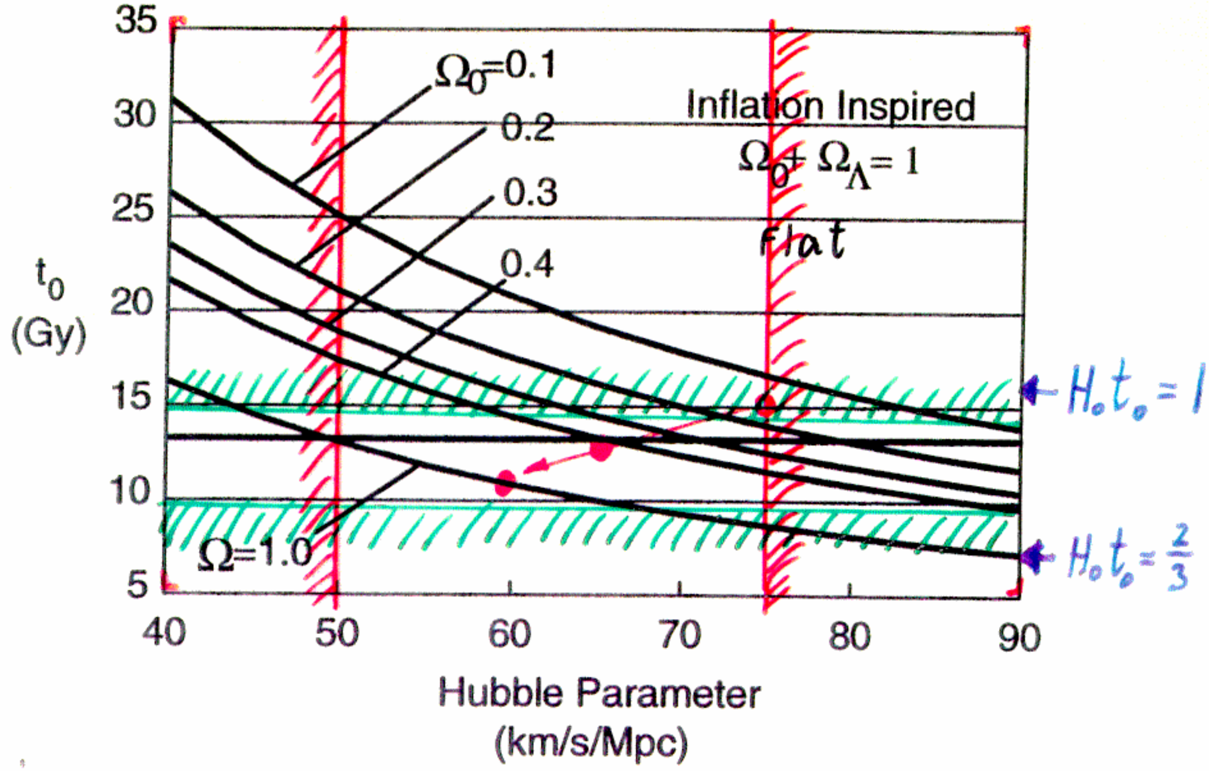


Fig. 1.2. Age of the universe t_0 as a function of Hubble parameter H_0 in inflation inspired models with $\Omega_0 + \Omega_\Lambda = 1$, for several values of the present-epoch cosmological density parameter Ω_0.

$$\Omega_m - 0.7\,\Omega_\Lambda \simeq 5.8\,(1 - 1.3\,h\,t_{10})$$

~~$H_0 \sim 75$~~ ~~$t_0 \sim 15$ Gyr~~ → $\Omega_m \ll 1$

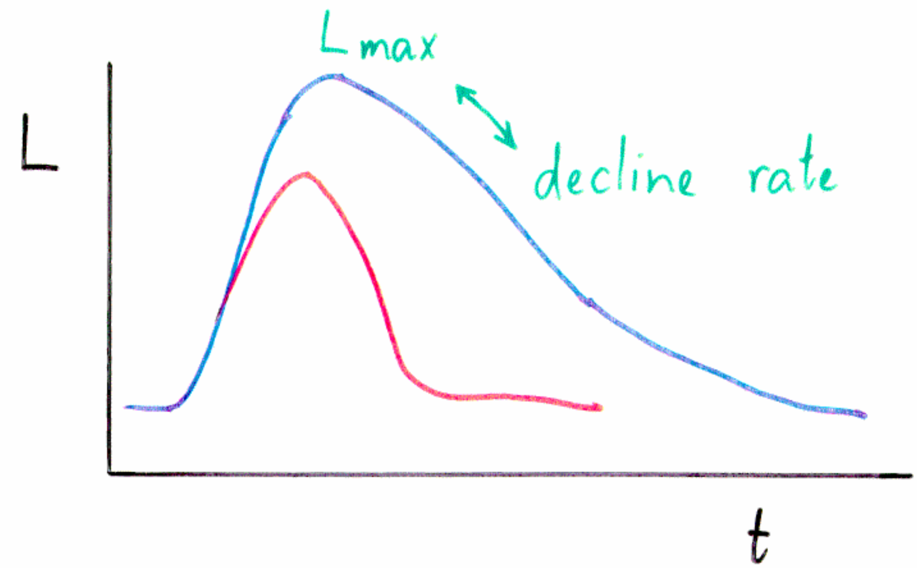
$H_0 \sim 60$ $t_0 \sim 11$ → $\Omega_m \sim 1$?

$H_0 \sim 65$ $t_0 \sim 12.5$ → $\Omega_m \sim 0.25$?

$H_0 = 65 \pm 15$

Geometry of Space-Time

- Supernovae Ia – "Standard Candle"



- Luminosity distance

intrinsic → L, apparent → ℓ:

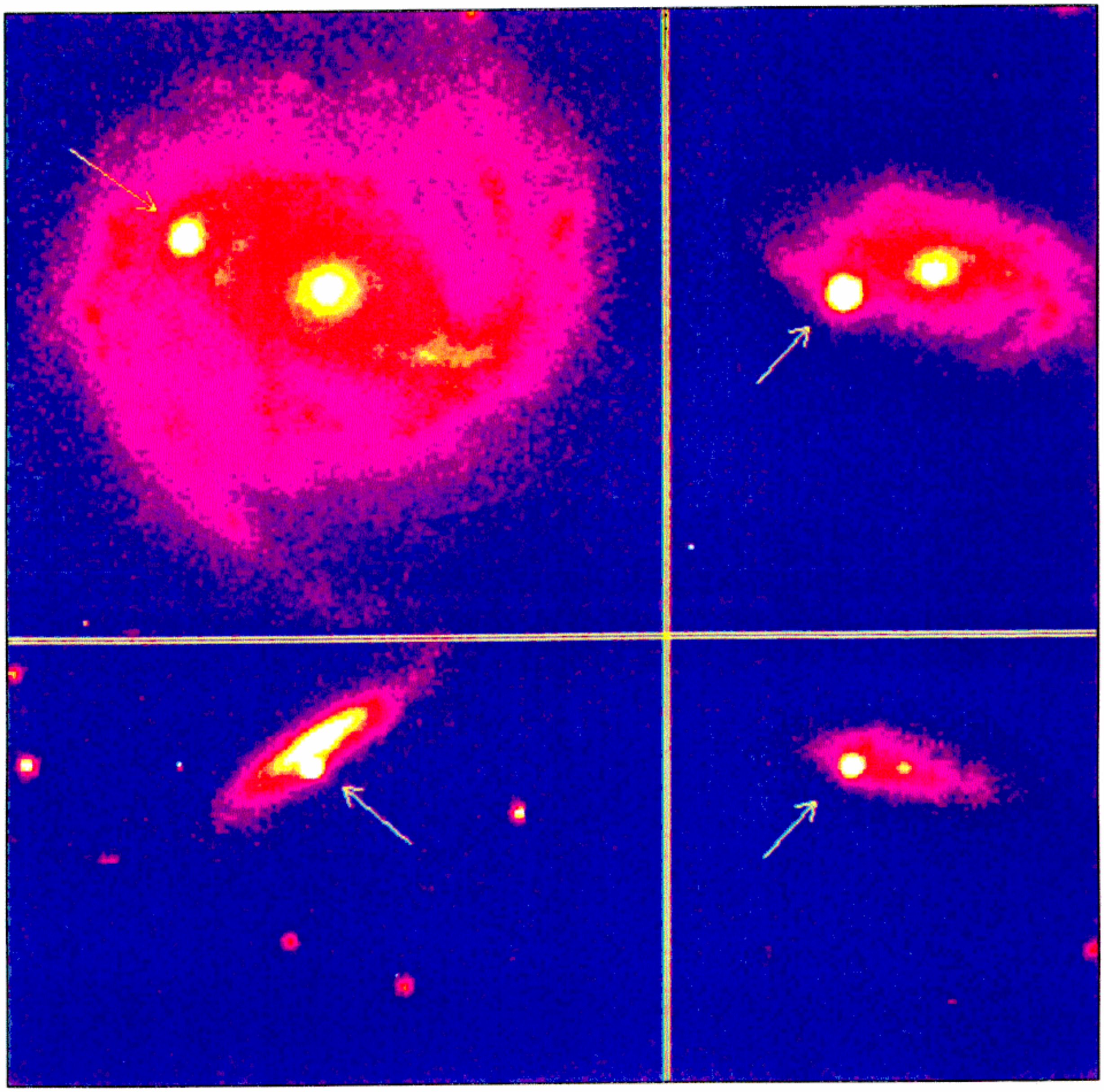
$$L/\ell \sim d_L^2(z,\ \Omega_m - \Omega_\Lambda)$$

at $z \sim 0.4-1$

$$d_L = \frac{c(1+z)}{H_0 |\Omega_K|^{1/2}} S_K\left[|\Omega_K|^{1/2} \int_0^z dz' \left[(1+z')^2(1+\Omega_m z') - z(2+z)\Omega_\Lambda\right]^{-1/2}\right]$$

- ~ Rate of change of $d/v \sim H$

Supernovae



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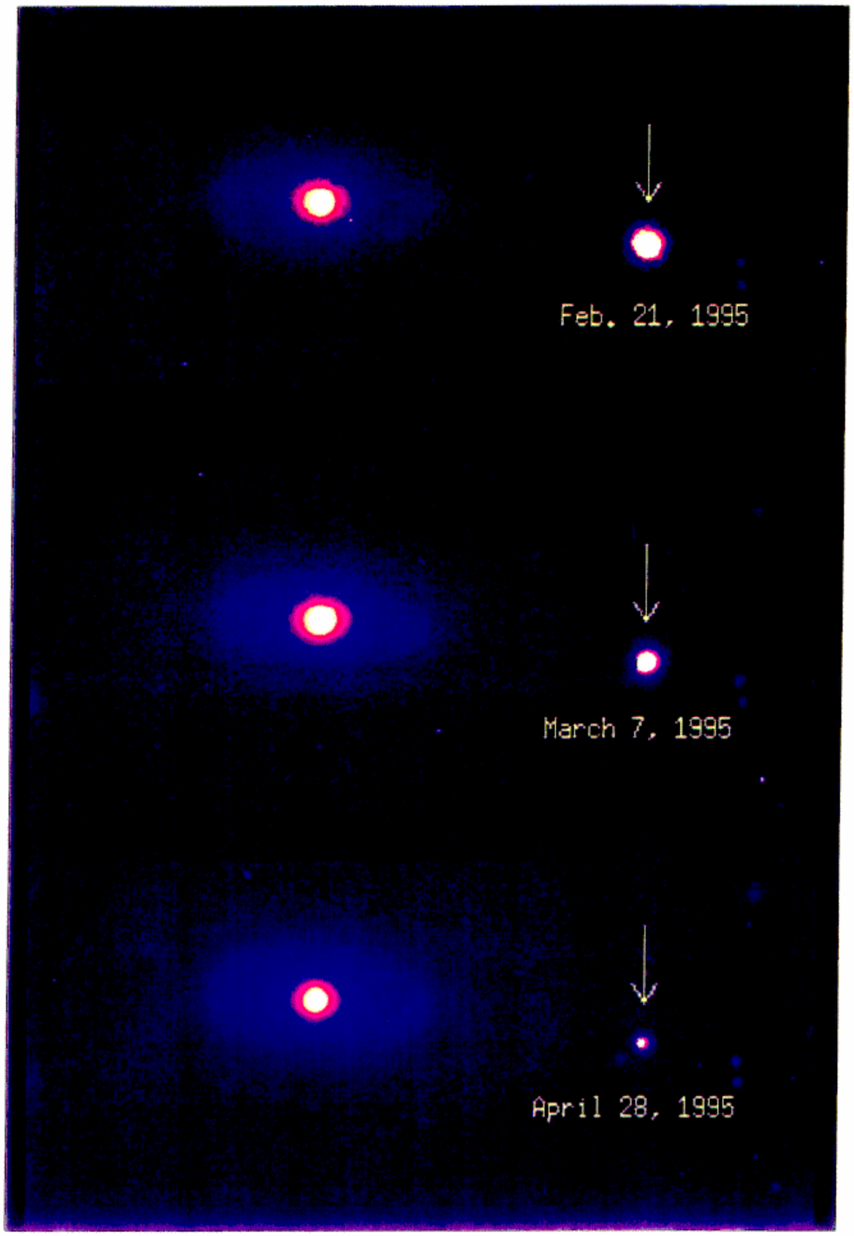

Feb. 21, 1995
March 7, 1995
April 28, 1995

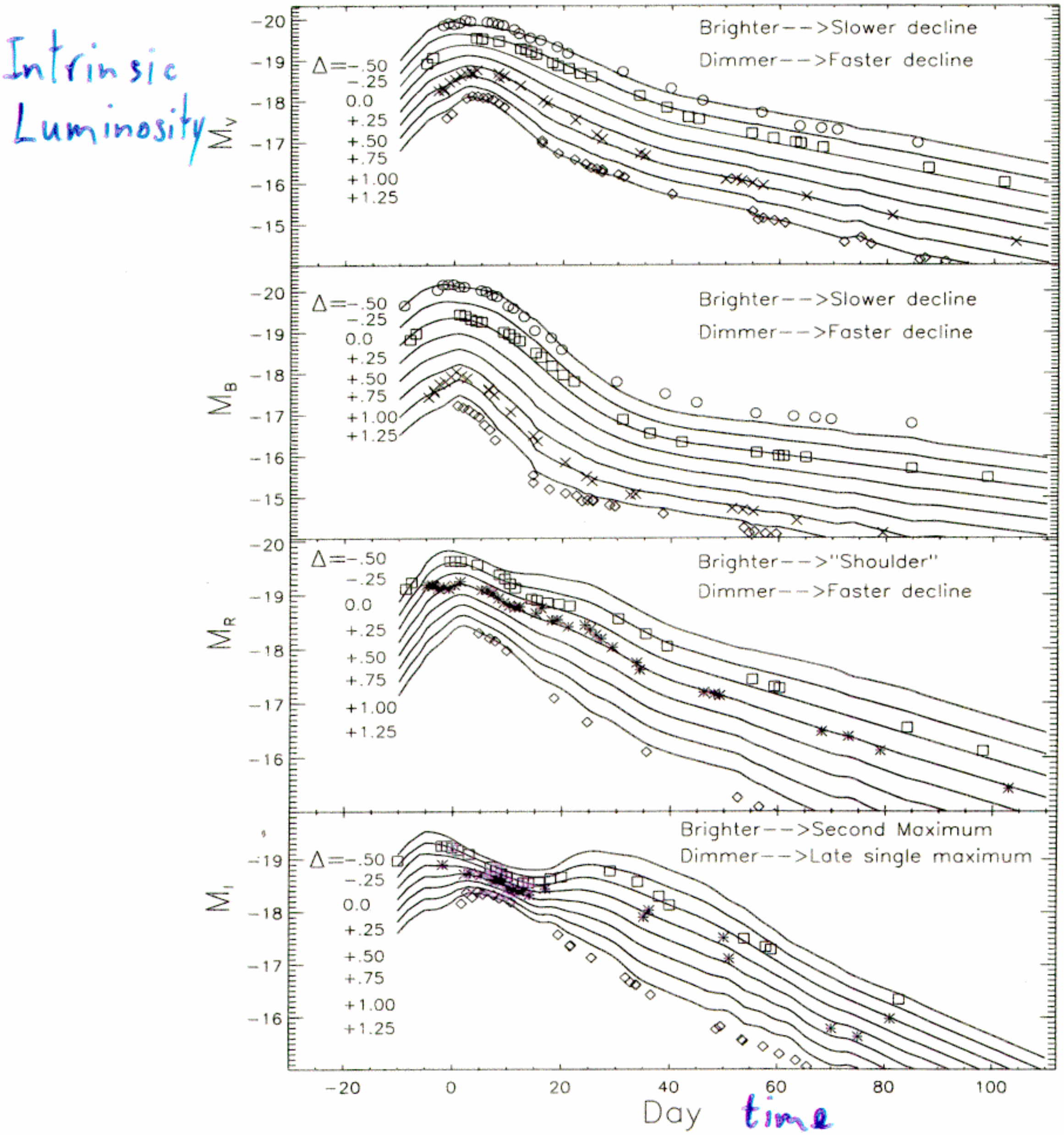


Figure 2: Empirical family of SN Ia BVRI light curves parameterized by luminosity. This family of light curves is derived in the same way as the families in figure 1 and shows the differences in photometric behavior for bright and dim SN Ia. Intrinsically dim SN Ia rise and fall faster in B and V than intrinsically bright SN Ia. For the R light curve, a "shoulder" occurs $\sim$ 25 days after B maximum in the bright SN Ia. This shoulder is weaker for dimmer SN Ia and is absent for the most underluminous ones. In the I band, the bright SN Ia have two maxima; one early ($\sim$ 5 days before B maximum) and one later ($\sim$ 30 days after B maximum). As the luminosity of the SN Ia decreases the first maximum occurs later and is broader while the second maximum is dimmer and occurs earlier. For the most underluminous SN Ia, the two maxima merge into one maximum which is broad and occurs $\sim$ 5 days after B maximum. Data shown as reconstructed, 91T=○, 94ae=□, 86G=X,91bg=◇, 92A=+, 80N=△

Local Hubble Diagram

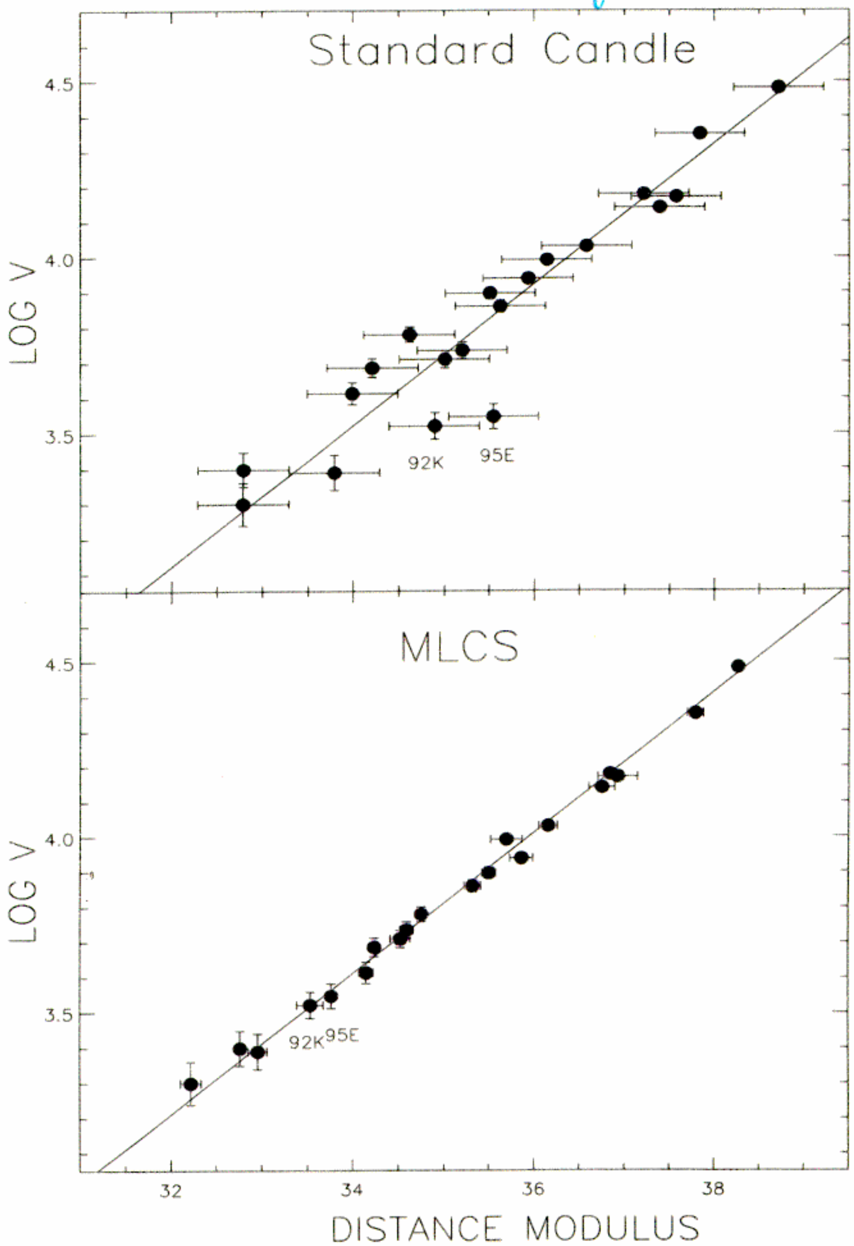


Figure 6: Hubble Diagrams for SN Ia with velocities in the COBE rest frame on the Cepheid distance scale (Sandage et al 1994, 1996). All velocity errors are 300 km s^{-1} reflecting a plausible estimate of random velocities with respect to the Hubble flow. (a) Distances estimated with a standard luminosity assumption and no correction for extinction. This method yields σ_v=0.52 and H_0=52 ± 8 (statistical) km s^{-1} Mpc^{-1}(b) Distances from the MLCS method which makes a correction for intrinsic luminosity variation and total extinction as determined from the light and color curve shapes. This method yields σ_v=0.12 and H_0=65 ± 3 (statistical) km s^{-1} Mpc^{-1}.

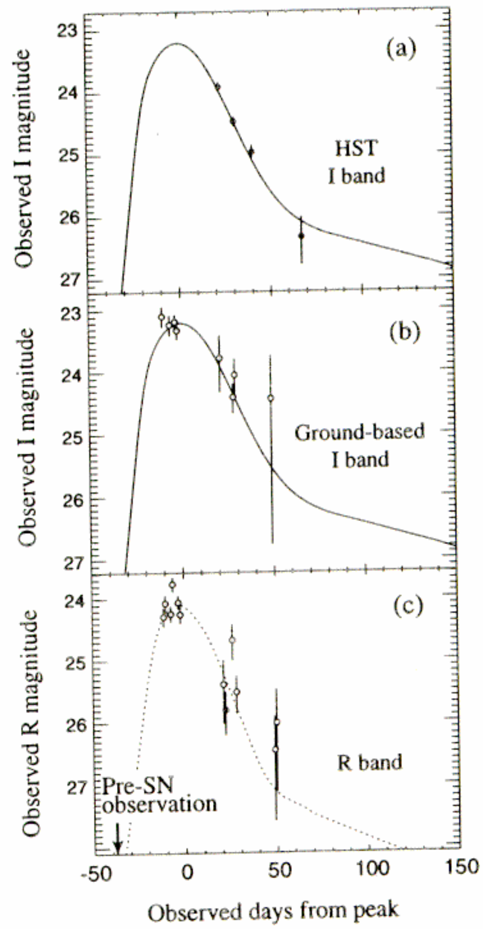
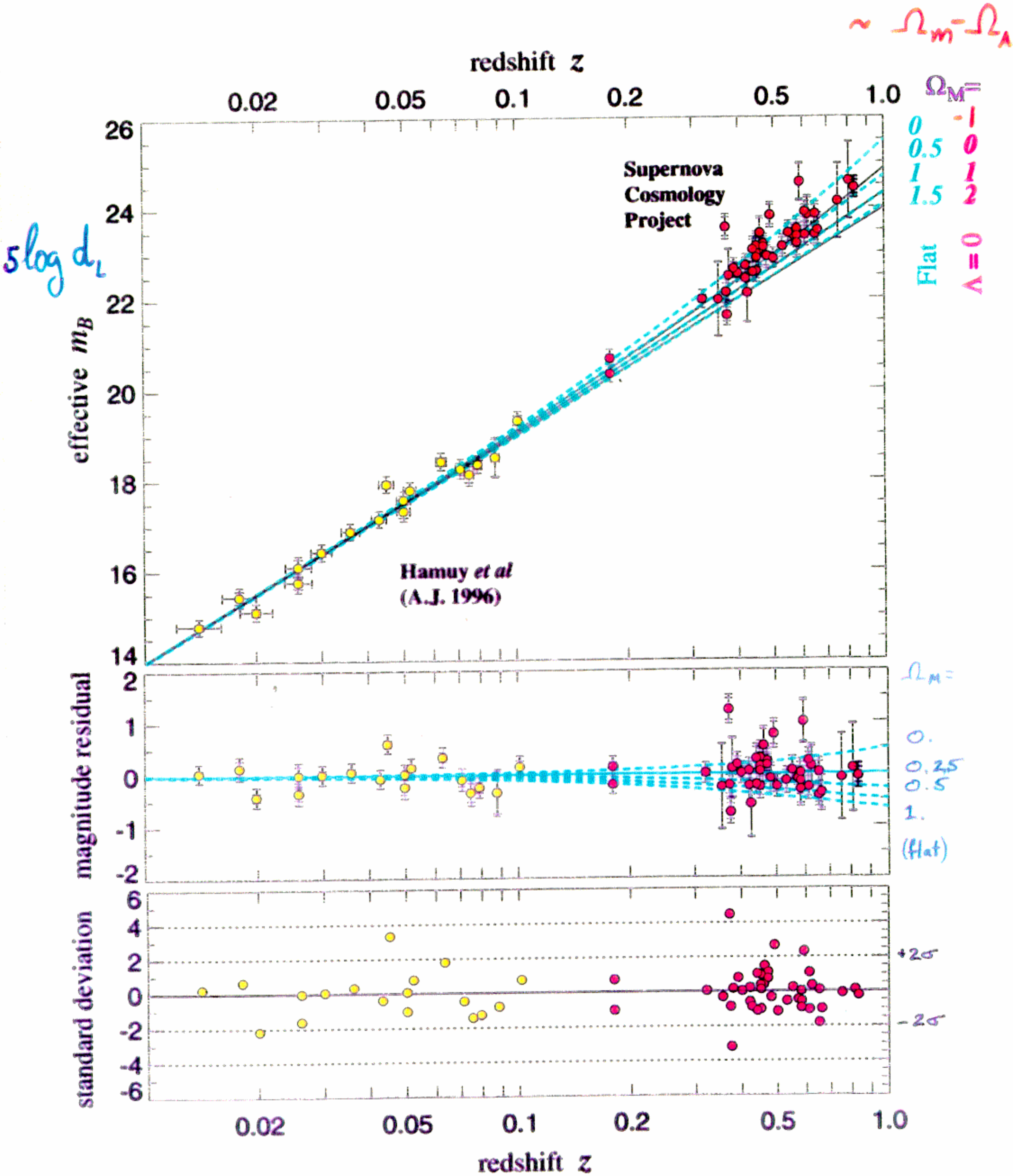
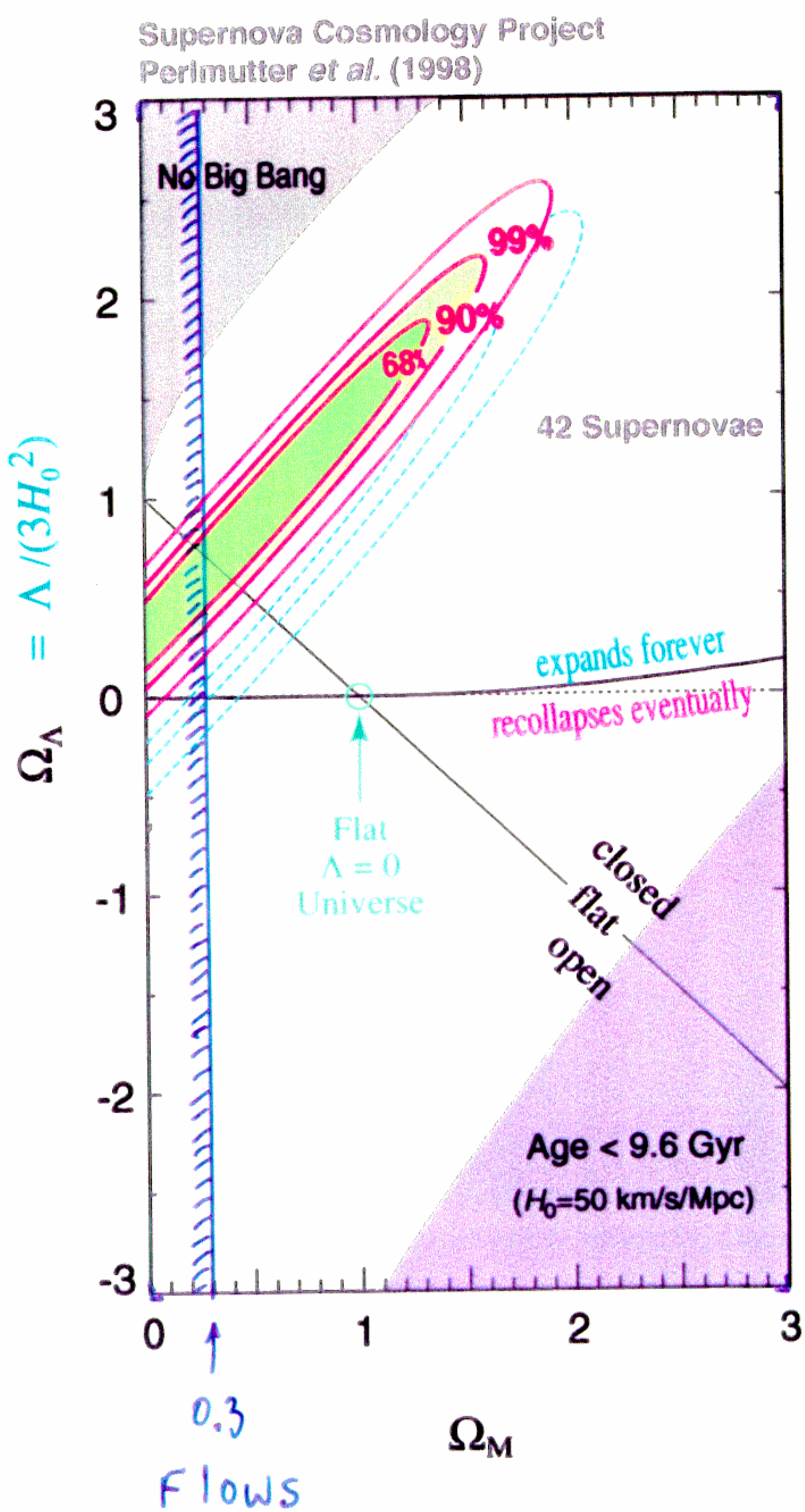


Figure 2: Photometry points for SN 1997ap (a) as observed by the HST in the F814W filter; (b) as observed with ground-based telescopes in the Harris I filter; and (c) as observed with the ground-based telescopes in the Harris R filter (open circles) and the HST in the F675W filter (filled circle); with all magnitudes corrected to the Cousins I or R systems[13]. The solid line shown in both (a) and (b) is the simultaneous best fit of the ground- and space-based data to the K-corrected, $(1+z)$ time-dilated Leibundgut B-band SN Ia template light curve[22], and the dotted line in (c) is the best fit to a K-corrected, time-dilated U band SN Ia template light curve. The ground-based data was reduced and calibrated following the techniques of ref 5, but with no host-galaxy light subtraction necessary. The HST data was calibrated and corrected for charge transfer inefficiency following the prescriptions of refs. 23 and 24. K-corrections were calculated as in ref 25, modified for the HST filter system. Correlated zeropoint errors are accounted for in the simultaneous fit of the lightcurve. The errors in the calibration, charge transfer inefficiency correction and K-corrections for the HST data are much smaller (~4% total) than the contributions from the photon noise. No corrections were applied to the HST data for a possible ~4% error in the zeropoints (P. Stetson, private communication) or for non-linearities in the WFPC2 response[26], which might bring the faintest of the HST points into tighter correspondence with the best fit lightcurve in (a) and (c). Note that the individual fits to the data in (a) and (b) agree within their error bars, providing a first-order cross check of the HST calibration.



Perlmutter, *et al.* (1998)

Riess et al 98


Supernova Cosmology Project
Perlmutter et al. (1998)
No Big Bang
99%
90%
68%
42 Supernovae
expands forever
recollapses eventually
Flat
Λ = 0
Universe
closed
flat
open
Age < 9.6 Gyr
(H0=50 km/s/Mpc)
ΩΛ = Λ/(3H0²)
ΩM
0.3
Flows
3
2
1
0
-1
-2
-3
0
1
2
3

Geometry of Space-Time

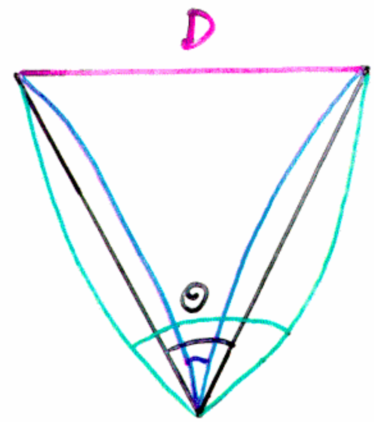
- Luminosity-distance of "standard candle"

$$\ell \sim L / d_L^2(z, \Omega_m - \Omega_\Lambda)$$

Supernova Ia
$z \sim 0.4$

Perlmutter et al 96 : Flat: $\Omega_m > 0.49$ 95%

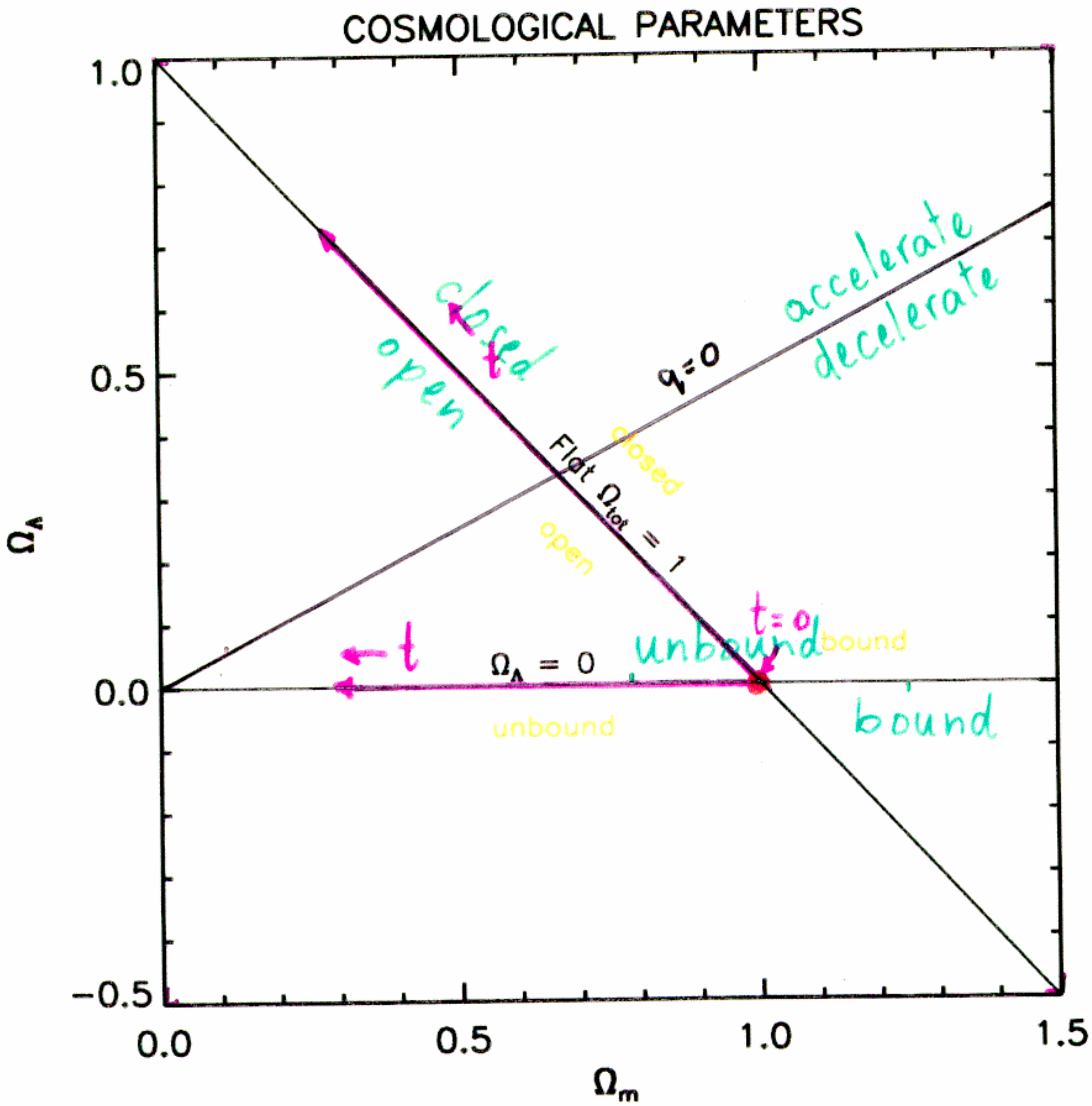
- N(z) of Lenses



$$\Theta \sim D / d_A(z, \Omega_m - \Omega_\Lambda)$$

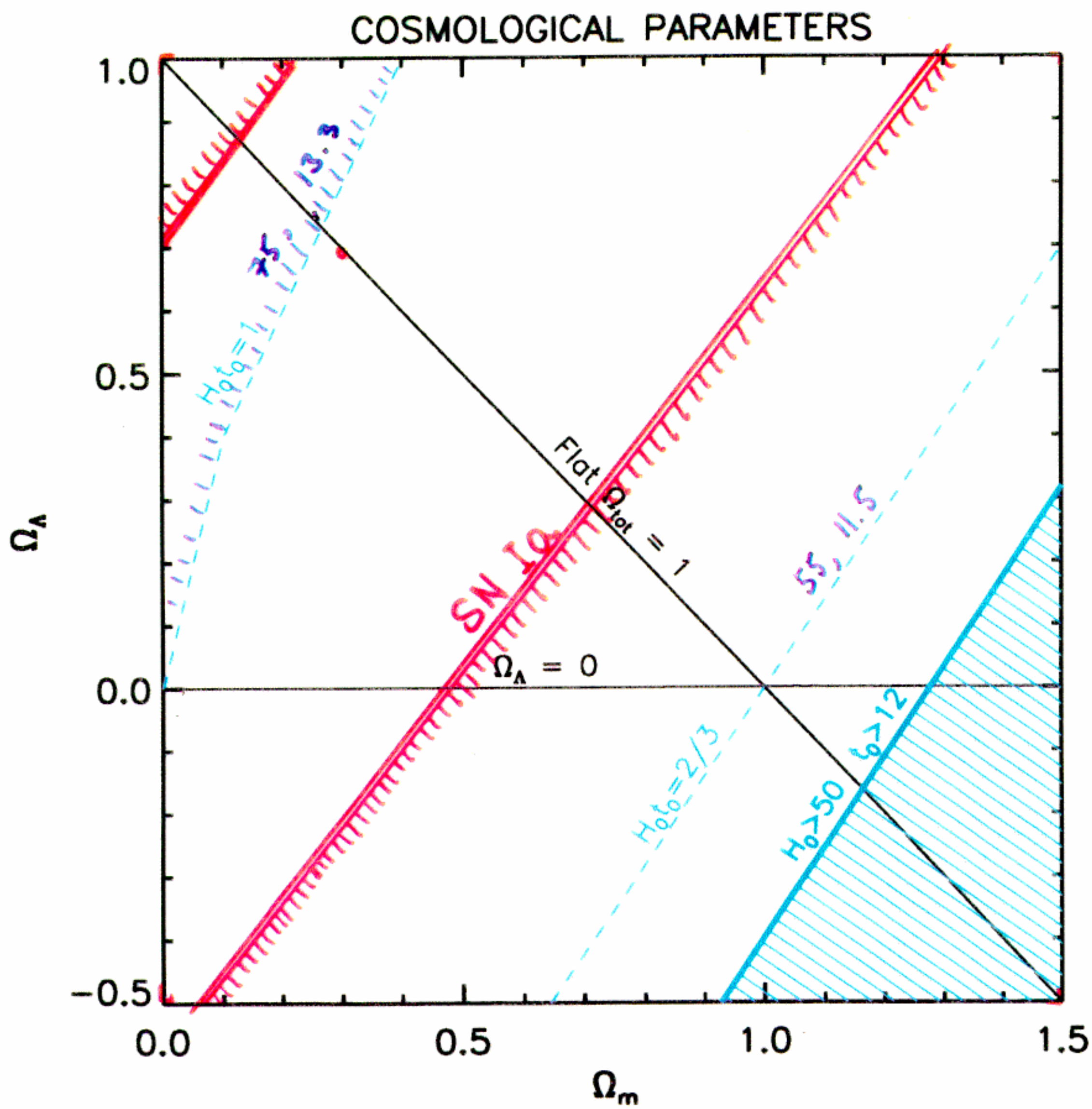
$$N_{lenses} \sim f(z_s, \Omega_m - \Omega_\Lambda)$$

Kochanek 96 : Flat: $\Omega_m > 0.34$ 95%

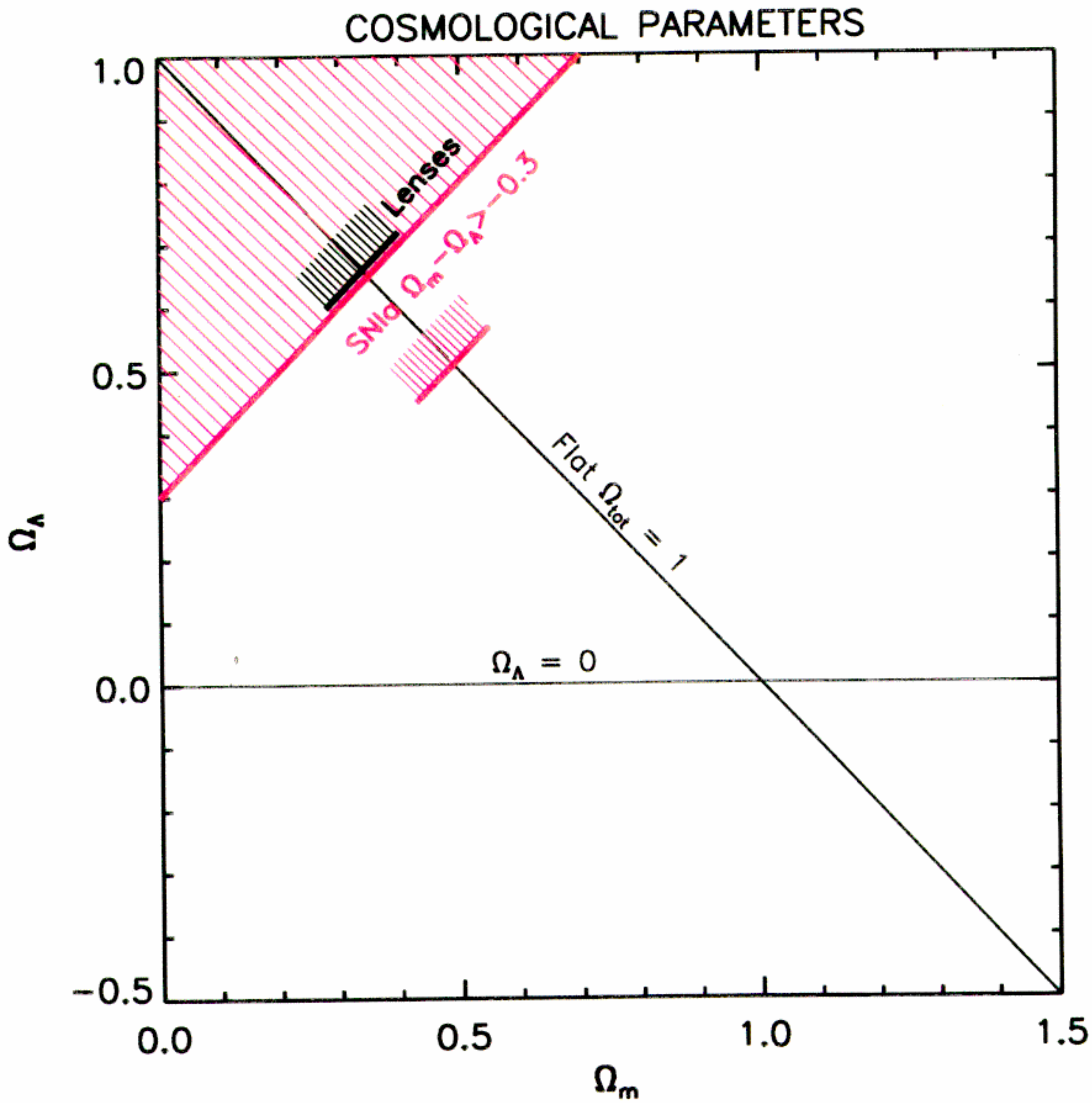


$\Omega_m=\rho_m/(3H_0^2/8\pi G)$ $\Omega_\Lambda=\Lambda c^2/3H_0^2$

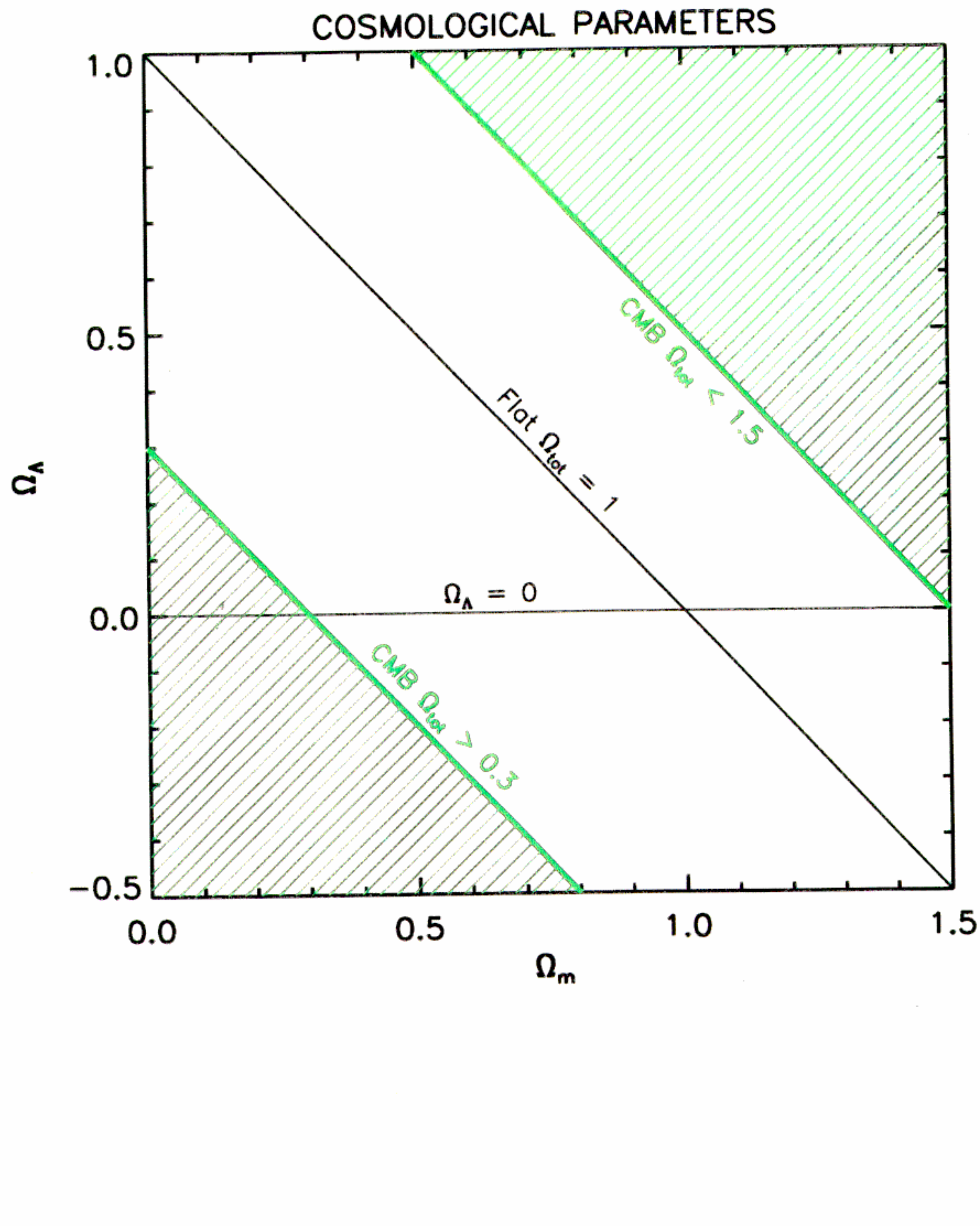
$\Omega_{tot}=\Omega_m+\Omega_\Lambda$ $\Omega_m-0.7\Omega_\Lambda \sim 5.8(1-1.3\ ht)$



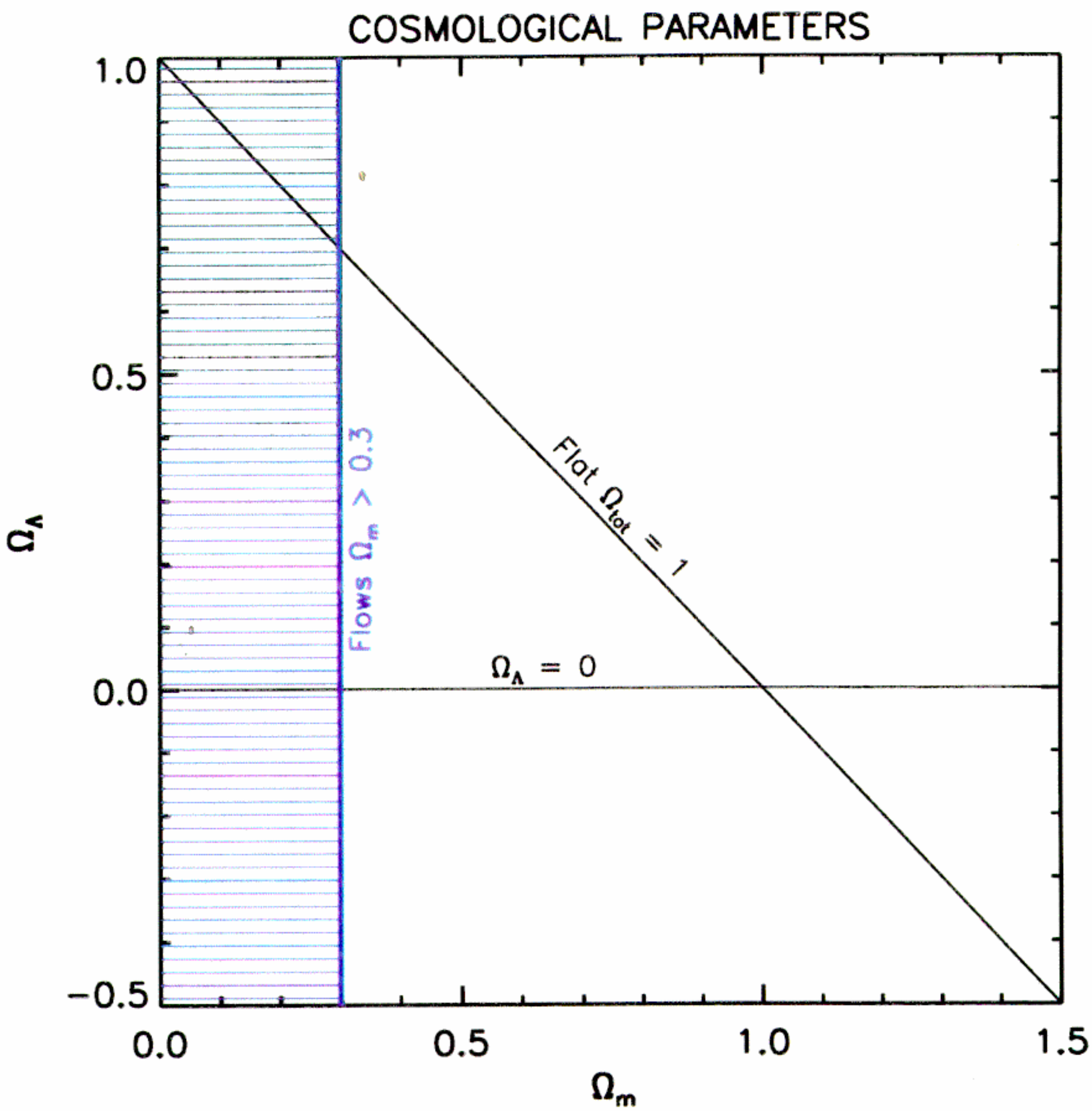
$$H_0 t_0 = \int_0^1 \frac{da}{a}\left(\frac{\Omega_m}{a^3} + \Omega_\Lambda - \frac{\Omega_k}{a^2}\right)^{-1/2} \equiv f(\Omega_m, \Omega_\Lambda)$$

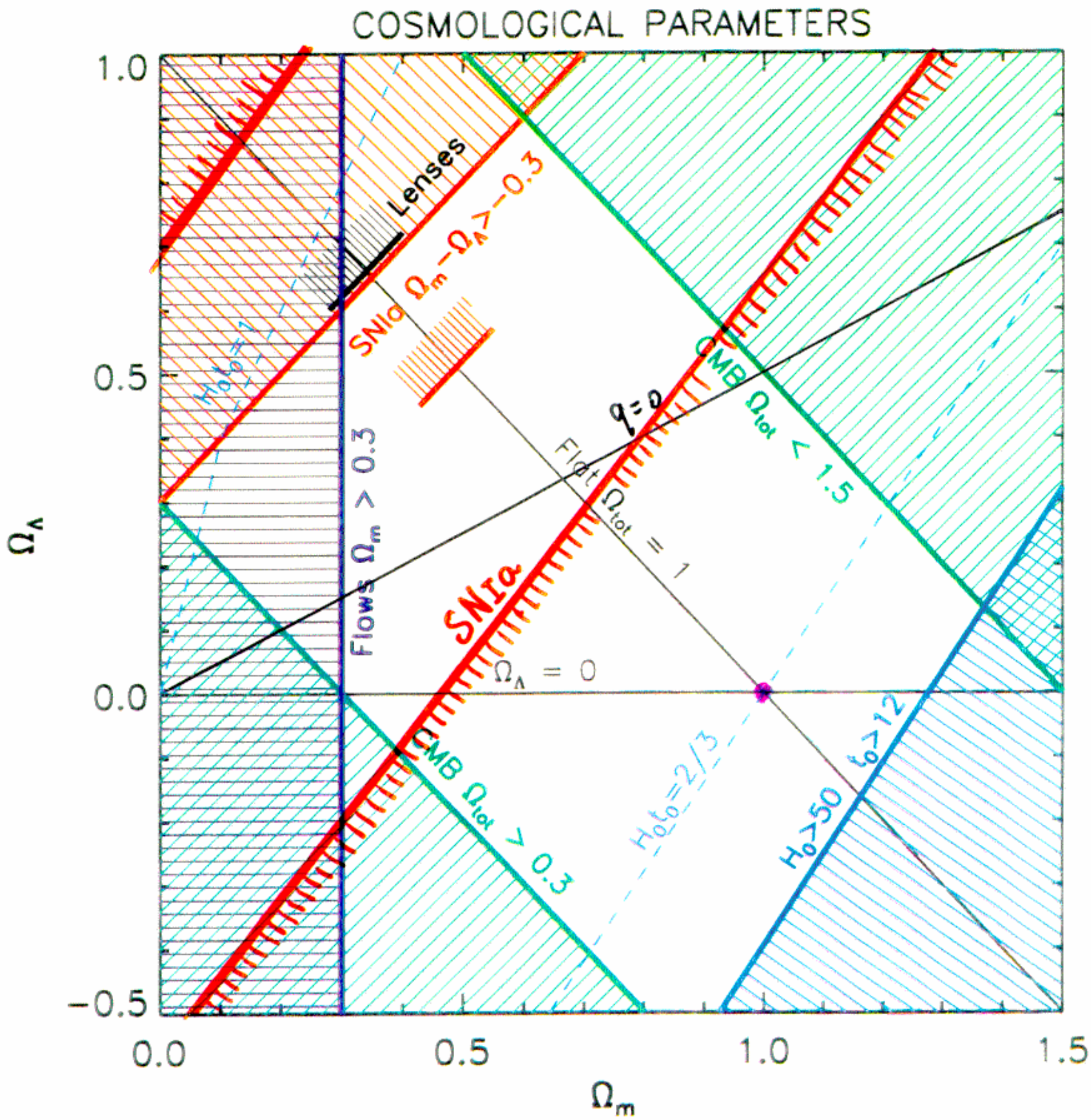


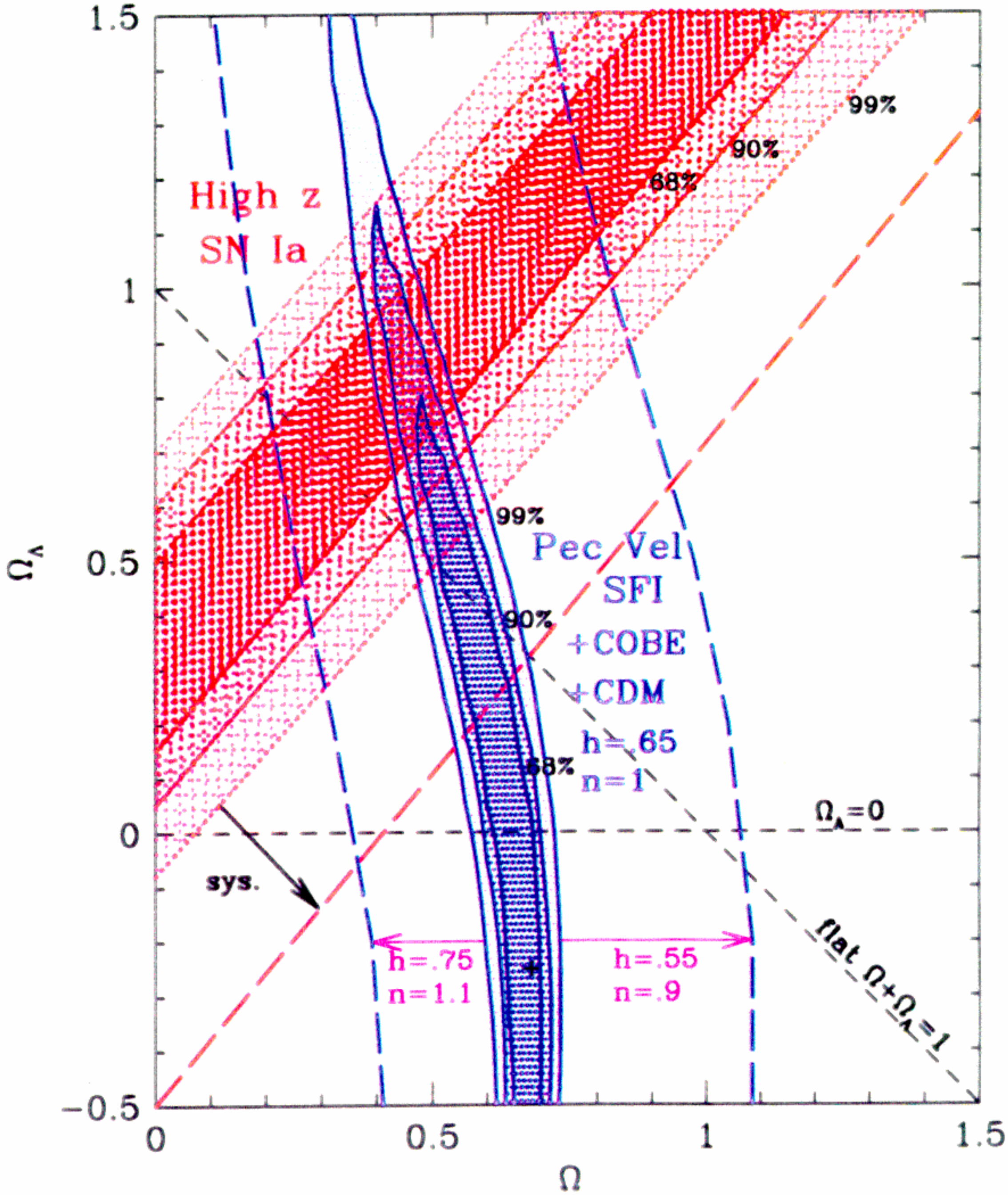
COSMOLOGICAL PARAMETERS
1.0
0.5
0.0
−0.5
0.0
0.5
1.0
1.5
Ω_Λ
Ω_m
Lenses
SNIa $\Omega_m - \Omega_\Lambda > -0.3$
Flat $\Omega_{tot} = 1$
$\Omega_\Lambda = 0$



COSMOLOGICAL PARAMETERS
1.0
0.5
0.0
−0.5
0.0
0.5
1.0
1.5
Ω_Λ
Ω_m
CMB Ω_{tot} < 1.5
Flat Ω_{tot} = 1
Ω_Λ = 0
CMB Ω_{tot} > 0.3

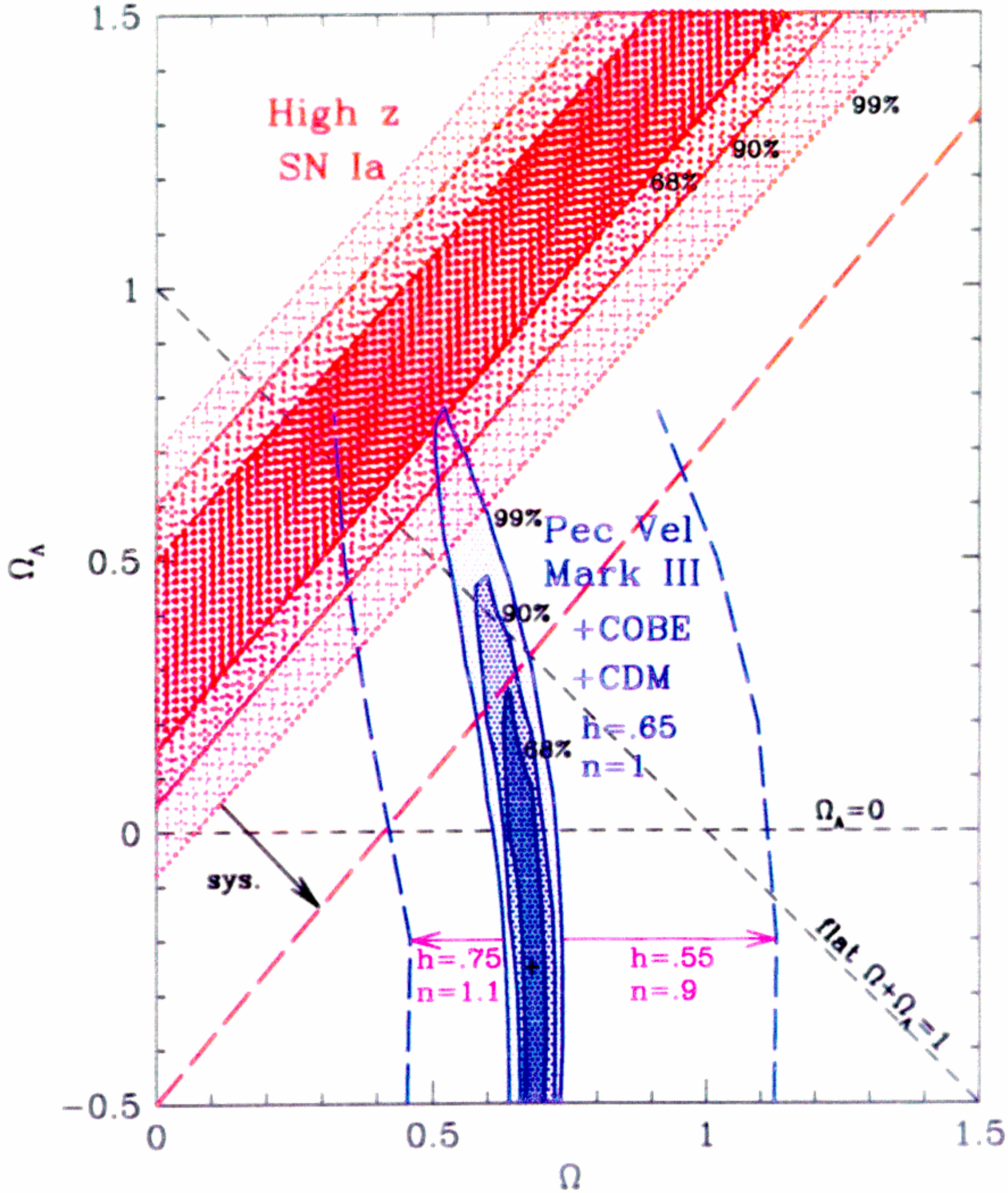

COSMOLOGICAL PARAMETERS
1.0
0.5
0.0
−0.5
0.0
0.5
1.0
1.5
Ω_Λ
Ω_m
Flows $\Omega_m > 0.3$
Flat $\Omega_{tot} = 1$
$\Omega_\Lambda = 0$


COSMOLOGICAL PARAMETERS
1.0
0.5
0.0
−0.5
0.0
0.5
1.0
1.5
Ω_Λ
Ω_m
Lenses
SNIa Ω_m −Ω_Λ > −0.3
Flows Ω_m > 0.3
q=0
CMB Ω_tot < 1.5
Flat Ω_tot = 1
SNIa
Ω_Λ = 0
CMB Ω_tot > 0.3
H_0t_0=2/3
H_0>50 t_0>12



High z
SN Ia
Pec Vel
SFI
+COBE
+CDM
h=.65
n=1
99%
90%
68%
sys.
h=.75
n=1.1
h=.55
n=.9
$\Omega_\Lambda=0$
flat $\Omega+\Omega_\Lambda=1$
Ω_Λ
Ω
1.5
1
0.5
0
−0.5

A. Dekel, SSI 1998



Cosmological Parameters

$H_0 = 65 \pm 10$ km s^{-1} Mpc^{-1}	SN Ia
$t_0 = 12 \pm 2$ Gyr	star clusters + Hipparcos
$\Omega_m = 0.3 - 1.0$	Flows, Global
$\Omega_\Lambda = 0.7 - 0.0$	Global
$\Omega_b = 0.05 - 0.1$	BBN + D
$\Omega_\nu < 0.3$	LSS, experiment
$n = 0.9 - 1.1$	CMB + LSS
$\sigma_8 \Omega^{0.6} = 0.5 - 0.8$	cluster abund, Flows
$\beta \equiv \Omega^{0.6}/b_I \sim 0.4 - 1.0$	Flows + IRAS

A ugly model that barely passes all current tests

- $H_0 = 65$ $t_0 = 12$
- $\Omega_{tot} = 1$ $\longrightarrow$ $\Omega_m = 0.5$ $\Omega_\Lambda = 0.5$

  $\Omega_m \downarrow$

  $\Omega_c = 0.3$

  $\Omega_\nu = 0.1$

  $\Omega_b = 0.1$
- $n = 0.95$ $\sigma_8 = 1$