New Sub-Millimeter Dimensions,

The Hierarchy Problem,

and

Quantum Gravity at a TeV.

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The Hierarchy Problem

- There are two seemingly fundamental energy scales in nature:
  - Electroweak scale $m_{EW} \sim 10^3$ GeV
  - Planck scale $M_{pl} \sim G_N^{-1/2} \sim 10^{18}$ GeV.

- Hierarchy problem: why is $m_{EW} \ll M_{Pl}$, and why is it radiatively stable?

- Central motivation for extensions of SM, with two major frameworks: low-energy SUSY, technicolor.

- But the problem only exists if we believe that both m_{EW}^{-1}, M_{Pl}^{-1} correspond to physical short-distance scales

- That m_{EW}^{-1} is a physical scale is an experimental certainty, since electroweak interactions have been probed @ distances approaching m_{EW}^{-1}.

- But why do we belive M_{Pl} is fundamental?
  - Gravity becomes strong (comparable to gauge interactions) @ energies $\sim M_{Pl}$:

$$\sigma(E) \sim 1/E^2$$

$$\sigma(E) \sim G_N^2 E^2 \sim E^2/M_{Pl}^4$$

  - Quantum gravitational effects become important @ energies $\sim M_{Pl}$: point particle of mass m has

$$R_{compton} \sim 1/m \quad , \quad R_{event\text{-}horizon} \sim G_N m .$$

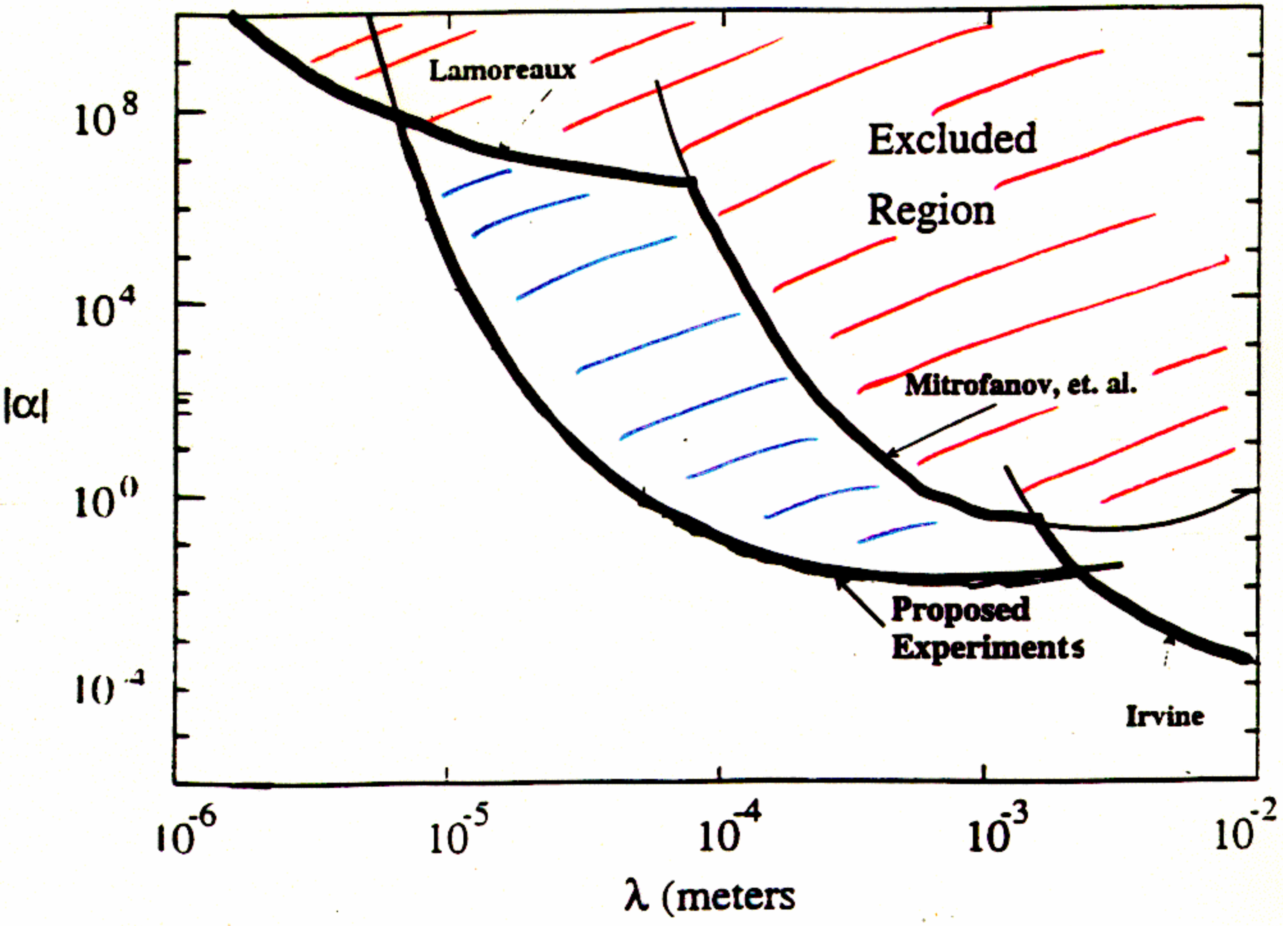
Quant. Gravity irrel.

Quant. Gravity Imp.

Is M_{Pl} Fundamental?

Experimental Status of Gravitational-Strength Forces in the Sub-Centimeter Regime Figure 1

(Price et. al. '98)



- Price, Long, Chan et. al.
- Kapitulnik, Kenny et. al.

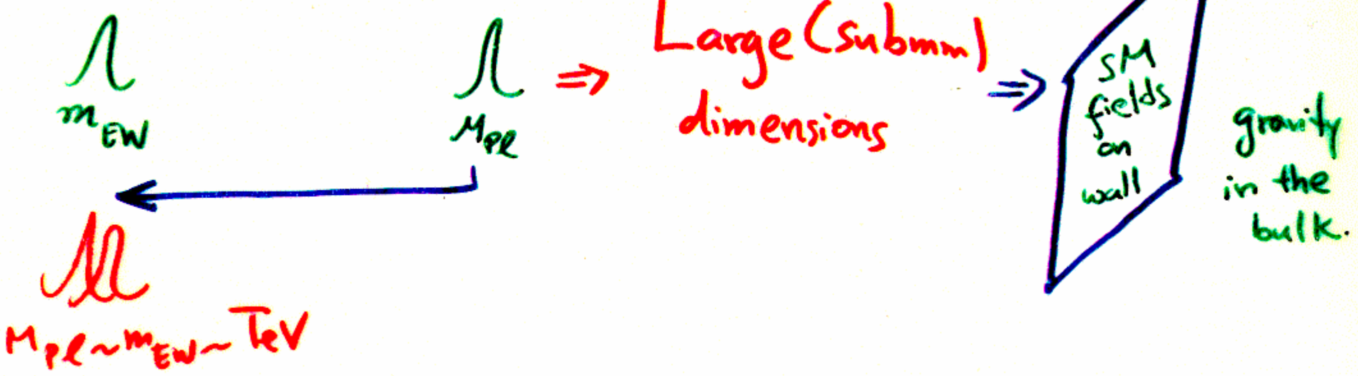
- In sharp contrast to electroweak interactions, gravity has not remotely been measured at distances approaching $M_{P\ell}^{-1} \sim 10^{-33}$ cm. Gravity has only been directly measured @ $\sim$ 1 cm.

- Interpretation of $M_{P\ell}$ as a fundamental energy scale is based on assumption that gravity is unmodified over 33 orders of magnitude, from where it is measured down to $M_{P\ell}^{-1}$.

- It is worthwhile to question this extrapolation

Outline

(1) Our Framework



(2) Is it alive?

- Lab bounds
- Astrophysics { Sun, Red Giants, SN 1987 A
- Cosmology.

(3) Experimental signatures

- sub-mm forces
- Quantum gravity at LHC, NLC.

(4) Model-building issues (proton decay, gauge coupling unification, . . .)

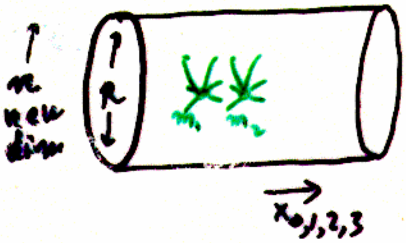
Our Scenario

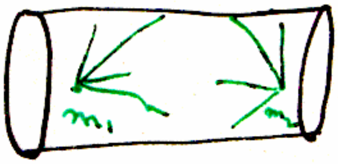
- Simplest "solution" to Hierarchy problem:

  gravity becomes strong near the electroweak scale, i.e. $M_{Pl\,\text{fund.}} \sim m_{EW}$.

- How can this be reconciled with observed weakness of gravity at distances $\gtrsim$ cm ?

  New dimensions large compared to m_{EW}:





$$V \sim G_{N(4+n)} \frac{m_1 m_2}{r^{n+1}} \quad (r \ll R) \qquad V \sim G_{N(4+n)} \frac{m_1 m_2}{R^n\, r} \quad (r \gg R)$$

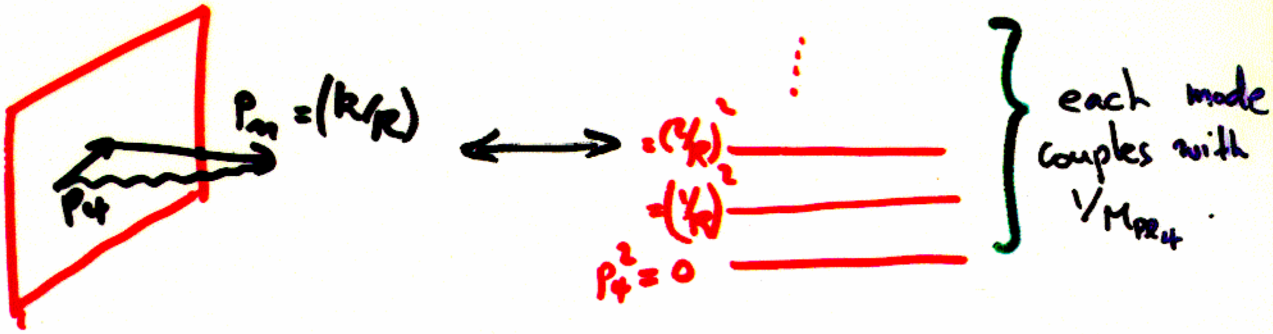
$$\Longrightarrow \boxed{M_{Pl_4}^2 = M_{Pl(4+n)}^{n+2} R^n} \quad \textbf{Gauss' Law}$$

- Alternately:

$$\int d^{4+n}x \sqrt{-g}\, M_{Pl(4+n)}^{n+2} R \longrightarrow \int d^4x \sqrt{-g} \left(M_{Pl(4+n)}^{n+2} V_n\right) R = \int d^4x \sqrt{-g}\, M_{Pl_4}^2 R$$

Kaluza-Klein

- Still another way: From 4-d view point, we have the graviton + its Kaluza-Klein excitations for each extra dimension with masses evenly spaced by $1/R$:



- $$V(r) = \sum_{KK} G_{N_4} m_1 m_2 \frac{e^{-m_{KK} r}}{r}.$$

For $r \gg R$, only the usual massless graviton is relevant and $V(r) = G_{N_4} \frac{m_1 m_2}{r}$.

For $r \ll R$, $\sim (R/r)^n$ KK excitations are unsuppressed, so $V(r) \sim G_{N_4} \frac{m_1 m_2}{r} \times (R/r)^n \sim \frac{G_{N(4+n)} m_1 m_2}{r^{n+1}}$.

$$\Rightarrow \boxed{M_{Pl_4}^2 = M_{Pl(4+n)}^{n+2} R^n}$$

- Since $M_{Pl_4}^2 = M_{Pl(4+n)}^{n+2} R^n$, Gauss

Can be enormous

$\sim$ TeV

large compared to TeV^{-1}.

Enormity of M_{Pl_4} ($\Longleftrightarrow$ weakness of gravity @ long distances) does not have to reflect a fundamental short-distance scale, it could merely reflect large new spatial dimensions.

- $$\boxed{R \sim 10^{\frac{30}{n} - 17}\,\text{cm} \times \left(\frac{1\,\text{TeV}}{M_{Pl(4+n)}}\right)^{1+\frac{2}{n}}}$$

$n=1$, $R \sim 10^{13}$ cm X

$n=2$, $R \sim$ mm!

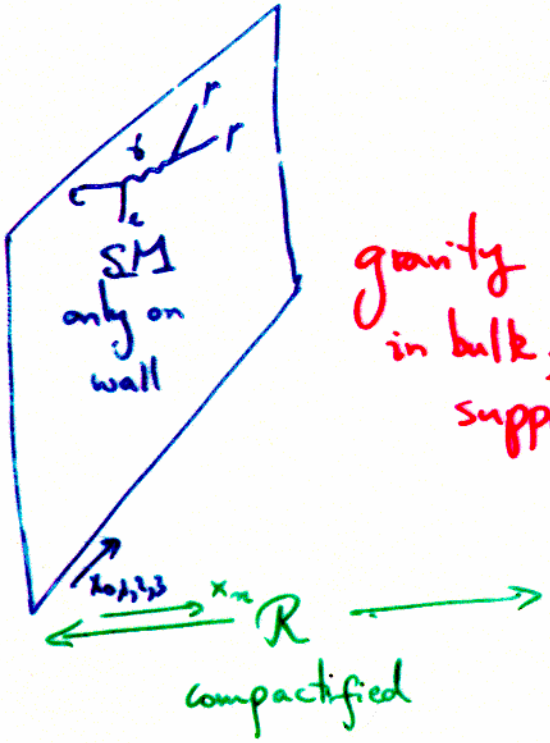
$\vdots$

$n=6$, $R \sim (10\,\text{MeV})^{-1}$

- Note as $n \to \infty$, $R \to M_{Pl(4+n)}^{-1}$
- Horava–Witten $10^3\,\text{GeV} \to 10^{16}\,\text{GeV}$ $R \sim (10^{14}\,\text{GeV})^{-1} \sim 10^{-27}$ cm

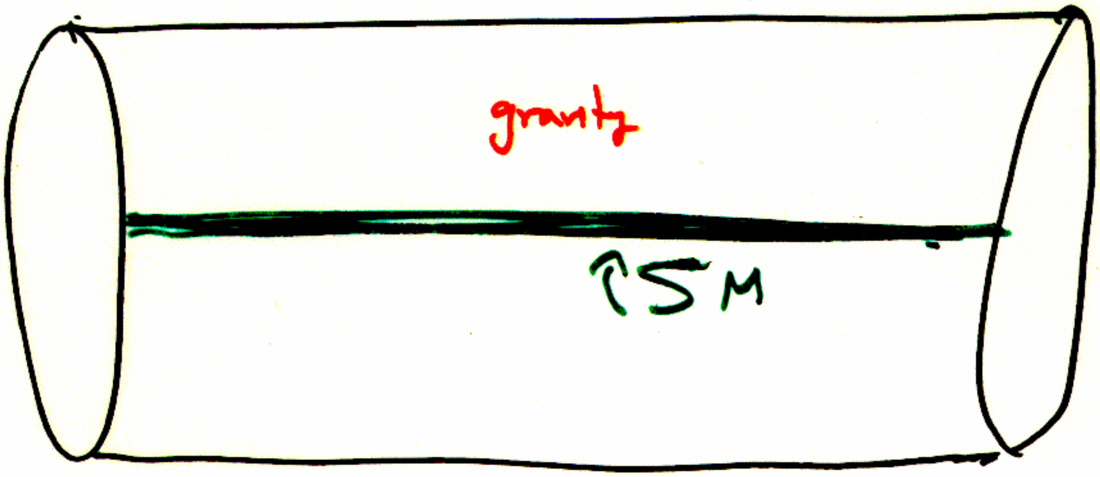
- While gravity has not been probed @ distances $\lesssim$ mm, QED, QCD, electroweak interactions certainly have been.

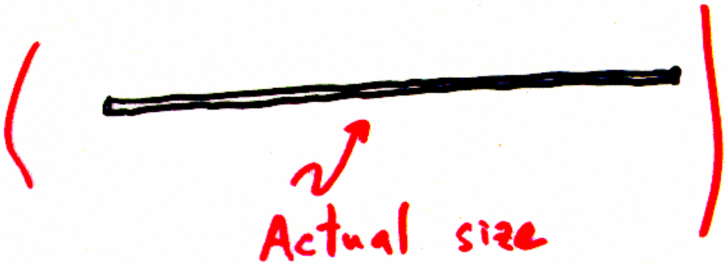
- Therefore, @ energies $\lesssim$ TeV, SM fields must be localised to a wall in the extra dimensions :



gravity in bulk, coupling suppressed by $1/M_{Pl(4+n)} \sim 1/\text{TeV}$

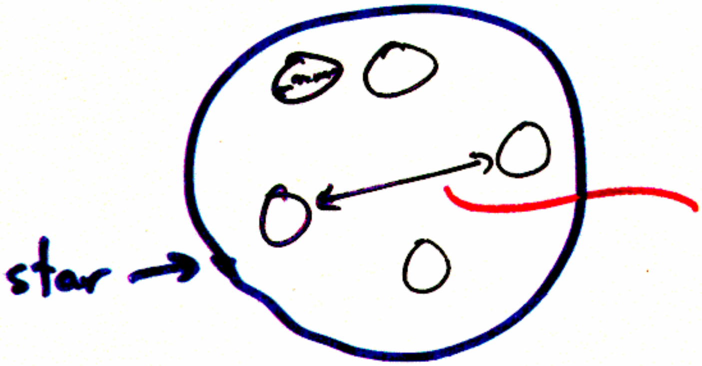
Perfectly reasonable effective theory at energies $\lesssim$ TeV, @TeV, Quant. gravity is revealed


gravity
SM


Actual size

Is this scenario alive?

- Most gravitational effects are macroscopic + do not sense modification of gravity at sub-mm distance, even if typical interparticle distances are smaller than mm.



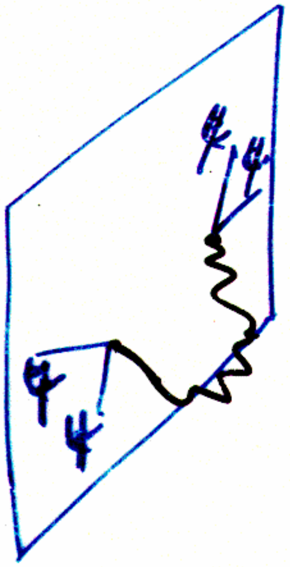
Normal Newtonian gravity between pairs, normal macroscopic energetics

- $$\frac{E_{grav}}{M} = \underbrace{\int_{r_n}^{R} d^3r \, \frac{G_{N4}}{r} \rho}_{\text{Normal piece, dominated by large distances}} + \underbrace{\int_{0}^{r_n} d^3r \, \frac{G_{N(4+n)} \rho}{r^{n+1}}}_{\text{new piece.}}$$

$$\frac{\delta E_{grav}}{E_{grav}} \sim \frac{M_{Pl}^2}{M_{Pl(4+n)}^{n+2} \, r_{typ}^{n-2} R^2} \lesssim \left(\frac{1\,mm}{R}\right)^2 \text{ irrelevant}$$

- Lab bounds.

- The theory above ~ TeV will produce higher-dimension operators suppressed by ~ TeV, constrained by compositeness searches which are at most ~ 4 TeV for electron compositeness. Since we don't know the theory above TeV we don't know the coefficients of these operators...



$$\rightarrow \mathcal{O} \sim \frac{E^2}{M^4_{Pl(4+n)}} (\bar\psi\psi)^2 .$$

Save even for

$M_{Pl(4+n)} \lesssim 1$ TeV

- Much more important: we have a massless $1/\text{TeV}$ coupled particle, the graviton.

- We know that other light particles (axions, familons), coupled with $1/\text{TeV}$, are a disaster from rare decays and stellar cooling in astrophysics – must check.

  $\sim 1/10^{2}\,\text{GeV}$

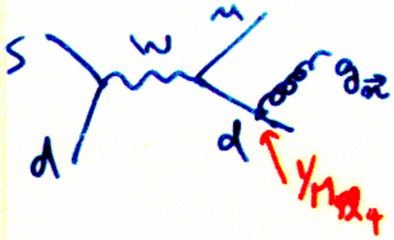
  $\rightarrow \dfrac{1}{10^{10}\,\text{GeV}}$

- Start with a very rare decay:

$K \longrightarrow \pi$ graviton.

[Note that since translation invariance is broken in extra dimensions, there is no $\vec{p}_n$ conservation, but energy must still be conserved].

- $K \to \pi$ grav. from 4-d point of view



$$Br\left(K \to \pi\, g_{\vec{n}}\right) \sim \left(\frac{m_K^2}{M_{Pl_4}^2}\right).$$

But, there are $\sim \left(m_K / \tfrac{1}{R}\right)^n = (m_K R)^n$ energetically available KK modes, so

$$Br\left(K \to \pi g\right) \sim \left(\frac{m_K^2}{M_{Pl_4}^2}\right) \times (m_K R)^n \sim \left(\frac{m_K}{M_{Pl(4+n)}}\right)^{n+2}$$

$\sim 10^{-12}$ even for $n=2$, $M_{Pl(4+n)} \sim 500\,GeV$

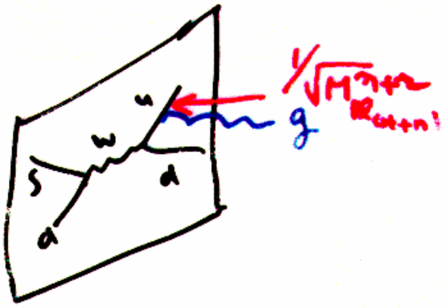
SAFE.

(familon $F \gtrsim 10^{12}\,GeV$)

- $K \to \pi$ grav. from $(4+n)$ d point of view:

$$g_{AB} = \eta_{AB} + \frac{h_{AB}}{\sqrt{M_{Pl(4+n)}^{n+2}}}$$

canonically normalized in $(4+n)$d.

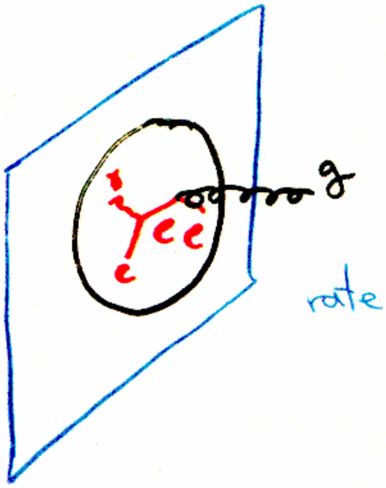


$$\longrightarrow \frac{1}{M_{Pl(4+n)}^{n+2}} \text{ in the rate } \checkmark$$

• Astrophysical constraints.

Analagous to axions, worry is that stars can lose energy too rapidly, by emitting gravitons into the extra dimensions:

rate $\sim 1/F^2$



rate $\sim \frac{T^n}{M_{Pl(4+n)}^{n+2}}$

[Note: if $n=7$, the first KK excitation is @ $1/R \sim 100$ MeV, too heavy to be produced even in SN 1987 A where $T_{SN} \sim 20-60$ MeV [$T_{redgiant} \sim 10$ keV, $T_{sun} \sim$ keV]]

So we have a very rough dictionary:

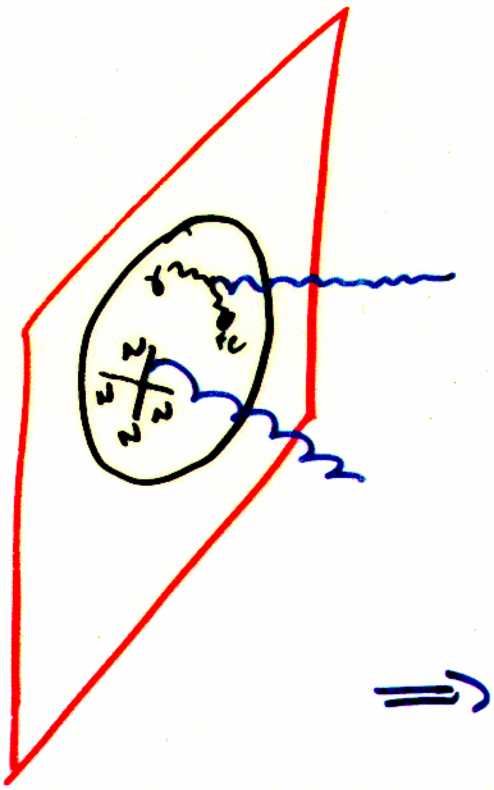
$$\frac{1}{F^2} \longleftrightarrow \frac{T^n}{M_{Pl(4+n)}^{n+2}} \Longrightarrow$$

For $n>2$, everything OK for $M_{Pl(4+n)} \lesssim 1$ TeV

For $n=2$, need $M_{Pl6} \sim 10$ TeV

($M_s \sim$ TeV)

Strongest bounds come from SN1987A :



$$\mathcal{L}_{SN1987A} \sim 10^{53}\,\text{egr}\,s^{-1} \sim 10^{26}\,\text{TeV}^2$$

$$\mathcal{L}_{\substack{\text{grav.}\\ (\text{Primakoff})}} \sim 10^{57}\,\frac{T^{n+4}}{M_{(4+n)}^{n+2}} < \mathcal{L}_{SN}$$

$$\Rightarrow \quad M_6 \gtrsim 10\,\text{TeV} \quad (n=2)$$

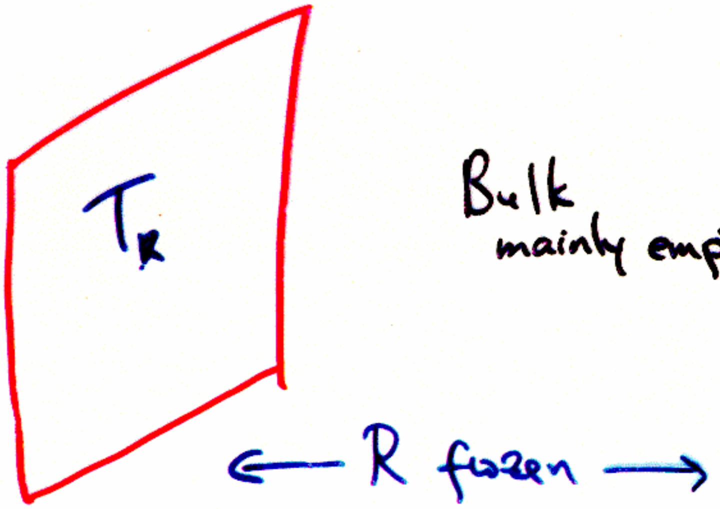
$$< 1\,\text{TeV} \quad n>2$$

ok .

Cosmology.

Of course all of early-universe cosmology must be rethought in this framework. As long as extra dimensions have stabilised @ temperatures ~ 1 MeV (by nucleosynthesis) the universe may be normal enough for everything to work.

Parametrise ignorance in following question: what is the highest temperature T_* at which the universe can be "normal"?



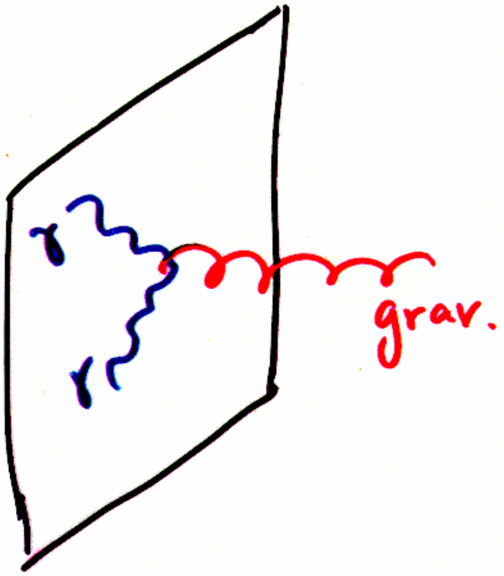
Cosmological Time Scales

Normal: Cooling by adiabatic expansion

$$\dot{T}/T \sim \dot{R}/R = H \sim T^2/M_{pl} \equiv 1/t_H \, .$$

dominates at late times

New: Cooling by evaporation



$$\dot{T}/T \sim \frac{T^{n+3}}{M_{(4+n)}^{n+2}} \equiv 1/t_{ev.}$$

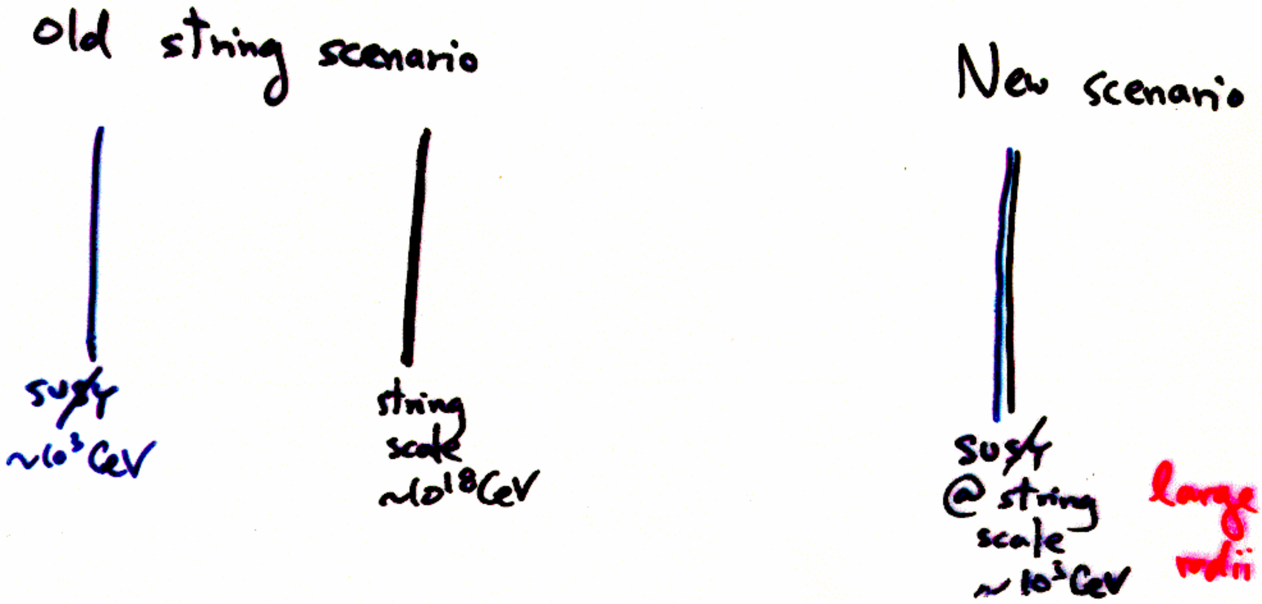
dominates at early times

For normalcy, we want $t_{ev} \gg t_H$

$$\Rightarrow \quad T_* \lesssim 10^{\frac{6n-9}{n+1}} \text{ MeV} \qquad (M_{4+n} \sim \text{TeV})$$

$$(\underset{n=2}{10 \text{ MeV}} - \underset{n=6}{10 \text{ GeV}})$$

- Are we "pushing the limits of parameter space"?
  - Experimentally: Yes! Theory is testable, with all sorts of new physics at dangerously low energies.
- Theoretically: No. All of our scales are motivated by solving the hierarchy problem.



Experimental Signals

- Sub-mm measurements of gravitational strength forces in next 1-2 years, perhaps down to ~1 micron.

$\Rightarrow$ Could observe transition from $1/r^2 \to 1/r^4$ Newtonian gravity for $n=2$ new dimension.

- For any n, in a string scenario, there could be light particles of mass $\sim \frac{m_{EW}^2}{M_{Pl_4}}$ $\sim (1\,\text{mm})^{-1}$ mediating observable grav. strength Yukawa forces at sub-mm distances.

- For any n, repulsive forces 10^6-10^8 × gravity, from gauge fields in the bulk coupled to lin. comb. of B, L #.

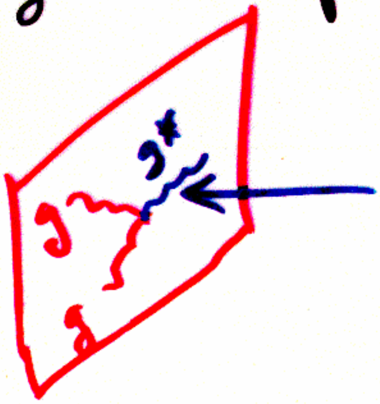
Quantum Gravity @ LHC, NLC

- Since $M_{Pl(4+n)} \sim$ TeV, LHC + NLC are Planck-scale machines, and could probe quantum-gravity.

- Example: Strings @ ~ TeV.

Novel phenomena include: production of higher string vibrational modes (Regge excitation) for every SM particle.

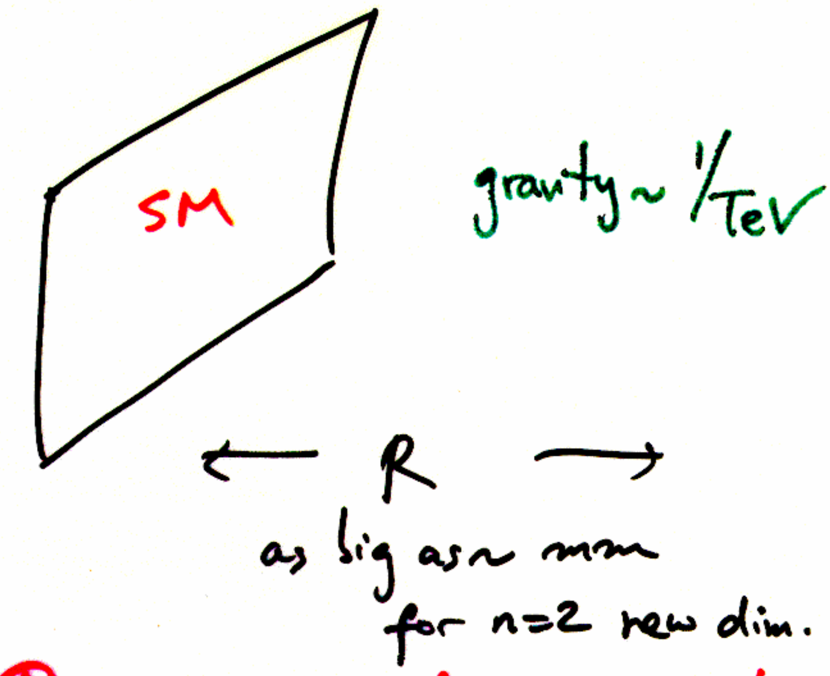
e.g.



Spin 2 "heavy gluon".

Conclusions

- There is a new framework for solving the hierarchy problem which does not rely on SUSY or technicolor.



- Raises exciting prospect that LHC, NLC are quant. gravity machines, and that new submm forces may soon be discovered.
- Many open questions (origin of R? Unification? Early universe?