## NEUTRINO OSCILLATION EXPERIMENTS USING ACCELERATORS AND REACTORS

Stanley Wojcicki Physics Department, Stanford University Stanford, CA 94305 e-mail: sgweg@slac.stanford.edu

## ABSTRACT

These lectures emphasize neutrino oscillation experiments using accelerators and reactors. We describe past, present, and proposed experiments. A brief introduction to neutrino oscillations is given at the beginning. The technology of beams and detectors for neutrino experiments is described briefly.

© 1997 by Stanley Wojcicki.

# **Table of Contents**

- 1. Introduction
- 2. Formalism of Neutrino Oscillations
  - 2.1 Phenomenology
  - 2.2 Classification of Oscillation Experiments
  - 2.3 Sensitivities
- 3. Neutrino Beams
  - 3.1 General Considerations
  - 3.2 Beams from Accelerators
    - 3.2.1 Neutrinos from Hadron Beams
    - 3.2.2 Neutrinos from Beam Dumps
    - 3.2.3 Other Accelerator-Produced Beams
  - 3.3 Neutrinos from Reactors
  - 3.4 Neutrinos from Natural Sources

### 4. Neutrino Detectors

- 4.1 Calorimeters
- 4.2 Tracking Detectors
- 4.3 Cherenkov Detectors
- 4.4 Radiochemical Detectors
- 5.  $\nu_{\mu} \rightarrow \nu_{\tau}$  Oscillation Experiments (Past and Ongoing)
  - 5.1. Disappearance Experiments
  - 5.2 Completed Appearance Experiments
  - 5.3 Statistical Analyses
  - 5.4 Current Short-Baseline Program

- 6. Oscillation Experiments Involving  $v_e$ 
  - 6.1 Reactor Disappearance Experiments
    - 6.1.1 Results from Completed Experiments
    - 6.1.2 Experiments in Progress: CHOOZ and Palo Verde
  - 6.2  $\nu_{\mu} \rightarrow \nu_{e}$  at Low Energies
    - 6.2.1 LSND Experiment
    - 6.2.2 KARMEN Experiment
  - 6.3 Searches for  $v_{\mu} \rightarrow v_{e}$  at High Energies
    - 6.3.1 BNL E776 Experiment
    - 6.3.2 Results from CCFR Experiment
    - 6.3.3 NOMAD Results on  $v_{\mu} \rightarrow v_{e}$
- 7. Future Experiments
  - 7.1 Experiments Addressing the Dark Matter Question
    - 7.1.1 COSMOS Experiment
    - 7.1.2 Outlook in Western Europe
  - 7.2 Experiments Addressing the Atmospheric Neutrino Anomaly
    - 7.2.1 K2K Experiment
    - 7.2.2 JHF Program
    - 7.2.3 MINOS Experiment
    - 7.2.4 Possibilities in Western Europe
  - 7.3 LSND Effect
    - 7.3.1 BOONE Proposal
    - 7.3.2 Possibilities at CERN
  - 7.4 Solar Neutrino Anomaly—KamLAND

### Acknowledgments

References

## **1** Introduction

The existence of the neutrino was postulated in 1930 by W. Pauli<sup>1</sup> to explain the apparent energy nonconservation in nuclear weak decays. It was another 23 years before this bold theoretical proposal was verified experimentally in a reactor experiment performed by C. Cowan and F. Reines.<sup>2</sup> The most fundamental properties of the neutrino were verified during the subsequent decade. The neutrino was shown to be left- handed in an ingenious experiment by Goldhaber, Grodzins, and Sunyar<sup>3</sup> in 1957. The distinct nature of  $v_e$  and  $v_{\mu}$  was demonstrated in 1962 in a pioneering accelerator neutrino experiment at BNL by Danby *et al.*<sup>4</sup>

The following years saw a remarkable progress in neutrino experiments, especially those utilizing accelerators as their sources. Increases in available accelerator energies and intensities, advances in neutrino beam technology, and more sophisticated and more massive neutrino detectors were all instrumental in our ability to do ever more precise neutrino experiments. The focus of those experiments, however, was until very recently mainly on using neutrinos as a probe in two different areas. Together with experiments utilizing electrons and muons, the worldwide neutrino program played a key role in measuring the nucleon structure functions. And together with a variety of other efforts (especially  $e^+e^-$  annihilations and deep inelastic electron scattering) the neutrino experiments played a key role in establishing the validity of the Standard Model (SM), through the discovery of neutral currents,<sup>5</sup> measurements of the NC/CC ratio,<sup>6</sup> and measurements of the neutrino lepton scattering cross sections.<sup>7</sup>

I believe that we are now entering a new phase in experimental neutrino physics. The main thrust in the future will probably be twofold: better understanding of the nature of the neutrino, i.e., a study of neutrino properties; and use of the neutrino in astrophysics and cosmology as an alternative window on the universe, complementing the information obtained from studies of the electromagnetic spectrum. In these lectures I shall deal with the subject of neutrino oscillations, i.e., a part of the first program.

We believe that neutrinos are among the fundamental constituents in nature. In addition, the space around us is permeated with neutrinos which are relics of the Big Bang, to the tune of about 110 v's/cc for every neutrino flavor. But our knowledge of the neutrino's properties lags far behind our knowledge of other elementary constituents, for example, the charged leptons. A few examples may illustrate this

point. (We quote the lepton values from the latest compendium by the Particle Data Group.<sup>8</sup>)

We do not know whether neutrinos have a mass; our current information gives us only upper limits ranging from a few eV for  $v_e$  to some 20 MeV for  $v_{\tau}$ . We can contrast that with a fractional mass error of about  $3 \times 10^{-7}$  for the electron and muon and about 2 x  $10^{-4}$  for the tau.

We do not know if neutrinos are stable or decay, either into neutrinos of other flavors or into some new, as yet undiscovered, particles. In contrast, we know that electron is stable, and know the  $\mu$  lifetime with a fractional error of  $2 \times 10^{-5}$  and the  $\tau$  lifetime at the level of 0.5%.

Finally, we do not know if the neutrinos have electromagnetic structure, like for example, a magnetic moment. The electron moment is known with a precision of about one part in  $10^{11}$ ; the magnetic moment of the muon to one part in  $10^{8}$ .

The study of neutrino oscillations offers us what is potentially a most sensitive investigation or measurement of neutrino masses (neutrino mass squared differences to be precise). Observation of a non-zero neutrino mass, which would follow directly from observation of neutrino oscillations, would be a clear example of breakdown of the SM and thus an indication of physics beyond it. Many of the popular extensions of the SM do indeed predict non-zero neutrino masses and existence of neutrino oscillations.<sup>9</sup> Furthermore, neutrino oscillations are not only an attractive theoretical concept, but also a phenomenon hinted at by several experimental observations.

These observations are:

- (a) An apparent need for dark (i.e., non-shining) matter.<sup>10</sup> One example of this need is the observed deficit of sufficient matter to account for the gravitational forces needed to explain the rotation velocity of stars in spiral galaxies. Neutrinos, since they are present in abundance everywhere, could account for at least a part of this deficit if they had a finite mass.
- (b) The solar neutrino deficit, i.e., observation of fewer sun-originated neutrinos on earth than expected from the known solar luminosity.<sup>11</sup>
- (c) The atmospheric neutrino anomaly,<sup>12</sup> i.e., a measured  $v_{\mu}/v_e$  ratio for neutrinos from cosmic ray interactions in our atmosphere which is significantly smaller than predicted.
- (d) The apparent observation of  $\bar{\nu}_{e}$  in an almost pure  $\bar{\nu}_{\mu}$  beam in a Los Alamos experiment<sup>13</sup> (the LSND effect).

As discussed in parallel lectures by K. Martens,<sup>14</sup> the second and third effects could be explained by neutrino oscillations:  $v_e$  oscillations into another flavor in the case of the solar neutrino deficit and  $v_{\mu}$  oscillating into  $v_e$  or (more likely) into  $v_{\tau}$  in the case of the atmospheric neutrino anomaly. The LSND effect will be discussed later in these lectures (see Sec. 6.2.1).

These lectures start out with a very brief description of neutrino oscillation phenomenology and of the customary method of classification of neutrino oscillation experiments. The next two chapters deal with the general experimental aspects of the neutrino experiments: neutrino beams and neutrino detectors. The following two chapters discuss what is known today about neutrino oscillations from the accelerator and reactor experiments and also describe the current experimental program in the field. The final chapter concludes those lectures by discussing the current plans around the world for future accelerator and reactor experiments which could investigate more fully the four categories of hints alluded to above. The past, present, and future efforts in the non-accelerator, non-reactor area are discussed in the parallel Martens lectures.

## 2 Formalism of Neutrino Oscillations

#### 2.1 Phenomenology

The underlying principle behind neutrino oscillations<sup>15</sup> is the fact that if neutrinos have mass, then a generalized neutrino state can be expressed either as a superposition of different mass eigenstates or of different flavor eigenstates. This is mainly a restatement of a well-known quantum mechanics theorem that, in general, several different basis vector representations are possible, these different representations being connected by a unitary transformation. Other well-known examples of this principle in particle physics are the  $K^0\overline{K}^0$  system (strong interaction and weak interaction eigenstates) and the quark system (weak interaction and flavor eigenstates connected by the CKM matrix).

From the study of  $e^+e^-$  annihilations at the Z<sup>0</sup> peak,<sup>16</sup> we know that there are only three neutrino flavor eigenstates if we limit the potential neutrino mass to less than  $m_z/2$ . Accordingly, the most likely situation is that we have three mass eigenstates and that the connecting unitary matrix is a 3×3 matrix. This is not rigorously required since we could have states with  $m_v > m_z/2$ , or flavor states that do not couple<sup>17</sup> to the Z<sup>o</sup>. Even though such possibilities appear *a priori* unaesthetic, there has been recently significant theoretical effort to see whether such mechanisms could be possible explanations of some of the anomalous effects seen in neutrino experiments.

Thus, for the three-flavor case, the weak eigenstates  $|\nu_{\alpha}\rangle = \nu_{e}$ ,  $\nu_{\mu}$ ,  $\nu_{\tau}$  and the mass eigenstates  $|\nu_{i}\rangle = \nu_{1}$ ,  $\nu_{2}$ ,  $\nu_{3}$  are related by

$$\begin{vmatrix} \mathbf{v}_{e} \\ \mathbf{v}_{\mu} \\ \mathbf{v}_{\tau} \end{vmatrix} = \begin{bmatrix} \mathbf{U} \\ \mathbf{U} \\ \mathbf{v}_{2} \\ \mathbf{v}_{3} \end{bmatrix},$$

i.e.,  $v_{\alpha} = Uv_i$ , where U is a unitary matrix that can be parameterized as (in analogy with the CKM matrix):

$$U = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13} & C_{12}C_{23} - S_{12}S_{23}S_{13} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13} & -C_{12}S_{23} - S_{12}C_{23}S_{13} & C_{23}C_{13} \end{bmatrix}.$$

where  $C_{ij} = \cos \theta_{ij}$  and  $S_{ij} = \sin \theta_{ij}$ , and for simplicity, we have taken the phase  $\delta = 0$ , i.e., assumed CP conservation.

The probability, then, that a state which is pure  $v_{\alpha}$  at t = 0 is transformed into another flavor  $\beta$  at a time t later (or distance L further) is

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left(\frac{\Delta m^2_{ij}L}{2E}\right)$$

with E being the energy of the neutrino and

$$\Delta m_{ij}^{2} = m^{2}(v_{i}) - m^{2}(v_{j}).$$

Thus (assuming CP invariance) we have five independent parameters: three angles,  $\theta_{12}$ ,  $\theta_{23}$ , and  $\theta_{13}$  and two  $\Delta m^2_{ij}$  (the third  $\Delta m^2_{ij}$  must be linearly related to the first two). All of the neutrino oscillation data must then be capable of being described in terms of these five parameters.

Clearly, the above expression is complicated and the relationship of experimental results to the five basic parameters is somewhat obscure. Partly due to a desire for

simplicity and partly because of the possibility (likelihood to some) that the leptonic mixing matrix U has a similar structure to the CKM matrix (i.e., is almost diagonal), it has become customary to represent the results of a single experiment in terms of oscillation between two flavors and involving only two mass eigenstates, hence only one  $\Delta m^2_{ij}$ . These two basis representations are then related by

$$\begin{bmatrix} \mathbf{v}_{\alpha} \\ \mathbf{v}_{\beta} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{v}_{1} \\ \mathbf{v}_{2} \end{bmatrix}$$

Clearly, such a representation will be a good approximation if the pattern of the U matrix is similar to the CKM matrix.

We can now consider a state which is a pure  $|v_{\alpha}\rangle$  at t = 0. Decomposing it into mass eigenstates, we have

$$|v_{\alpha}\rangle = \cos\theta |v_{1}\rangle + \sin\theta |v_{2}\rangle.$$

At subsequent times t, we have

$$|\mathbf{v}(t)\rangle = \cos\theta e^{-iE_1 t} |\mathbf{v}_1\rangle + \sin\theta^{-iE_2 t} |\mathbf{v}_2\rangle.$$

Treating neutrinos as stable particles and assuming that  $E^2 >> m^2$ , we obtain

$$|\mathbf{v}(t)\rangle = e^{-ipt} \left[ \cos\theta e^{-\frac{1}{2}i\frac{m_1^2}{p}t} |\mathbf{v}_1\rangle + \sin\theta e^{-\frac{1}{2}i\frac{m_2^2}{p}t} |\mathbf{v}_2\rangle \right]$$

We now transform back to the flavor basis, using

$$|v_1\rangle = \cos\theta |v_{\alpha}\rangle - \sin\theta |v_{\beta}\rangle,$$
  
$$|v_2\rangle = \sin\theta |v_{\alpha}\rangle + \cos\theta |v_{\beta}\rangle,$$

and ignore the initial phase factor since eventually we shall be interested in the square of the coefficient of  $|v_{\beta}\rangle$ . We obtain

$$|\mathbf{v}(\mathbf{t})\rangle = \left[\cos^2\theta e^{-\frac{1}{2}i\frac{\mathbf{m}_1^2}{\mathbf{p}t}} + \sin^2\theta e^{-\frac{1}{2}i\frac{\mathbf{m}_2^2}{\mathbf{p}t}}\right] |\mathbf{v}_{\alpha}\rangle_{\mathbb{H}} \left[e^{-\frac{1}{2}i\frac{\mathbf{m}_2^2}{\mathbf{p}t}} - e^{-\frac{1}{2}i\frac{\mathbf{m}_1^2}{\mathbf{p}t}}\right] \sin\theta\cos\theta |\mathbf{v}_{\beta}\rangle.$$

We now take the magnitude squared of the coefficient of  $|v_{\beta}\rangle$  and use trigonometric identities to simplify the equation. This magnitude squared is then the

probability  $P(\alpha \rightarrow \beta)$ , the probability of transition of a neutrino of flavor  $\alpha$  into a neutrino of flavor  $\beta$ . If L is expressed in Km (m) and E in GeV (MeV) then the expression reduces to

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \sin^2 2\theta \sin^2 (1.27\Delta m^2 \frac{L}{E}) ,$$

where  $\Delta m^2 = m_1^2 - m_2^2$  and is expressed in eV<sup>2</sup>. This expression is obviously much simpler than the one for the three flavor case, and the results of an experiment analyzed in this formalism can be easily displayed in a two-dimensional plot since only two parameters,  $\theta$  and  $\Delta m^2$ , are involved.

#### 2.2 Classification of Oscillation Experiments

As can be seen from the last equation, results of any neutrino oscillation experiment can be displayed graphically on a two dimensional plot, the two axes traditionally being  $\sin^2 2\theta$  (abscissa) and  $\Delta m^2$  (ordinate). It is customary to use log-log representation, but sometimes  $\sin^2 2\theta$  is expressed on a linear scale. An experiment claiming a positive result delineates a contour in this space (1 $\sigma$ , 90% C.L., etc.) within which the true answer must lie if the experiment is correct. A negative result can be represented by a curve delineating the region (again at 1 $\sigma$ , 90% C.L., etc.) excluded by that particular experiment.

It is clear that if one wants to probe a region of small  $\sin^2 2\theta$ , one needs good statistics since the effect will be small. Since the neutrino flux, and hence the event rate, falls off with source-detector distance L like

$$\phi_{\rm v} \propto (1/L)^2$$

we need to be relatively close to the source to have a large event rate. In addition, we need to keep the second factor large, i.e., the argument— $1.27 \cdot \Delta m^2 \cdot \frac{L}{E}$ —has to be of the order of unity. Hence, we need

$$\Delta m^2 \approx E/L$$
,

and thus for large E/L, such an experiment will be limited to probing large values of  $\Delta m^2$ . This basically defines a <u>short baseline</u> experiment, one where the sourcedetector distance is relatively small and where the region probed extends to small values of  $\sin^2 2\theta$  but is limited to large values of  $\Delta m^2$ . On the opposite end of the spectrum are the <u>long baseline</u> experiments which try to focus on investigation of low values of  $\Delta m^2$ . Again, to keep the argument of the second factor close to unity, L/E has to be large, i.e., the detector has to be far away. But that results in a flux penalty and hence the region covered in  $\sin^2 2\theta$  is smaller. Clearly, it is the value of the ratio L/E that provides the factor determining the category of the experiment.

Thus, long baseline experiments are able to probe low values of  $\Delta m^2$  but their reach in sin<sup>2</sup>2 $\theta$  is more limited. Solar neutrino studies are clearly long baseline experiments; the initial reactor and accelerator oscillation searches would be classified as short baseline experiments. We illustrate the regions covered by each kind of experiment graphically in Fig. 1.



FIG. 1. A rough illustration of the regions in oscillation parameter space that might be covered by a long baseline experiment (solid line) and a short baseline experiment (dashed line).

An alternative classification of experiments is between <u>appearance</u> and <u>disappearance</u> experiments. Considering a search for the possible oscillation  $\nu_{\alpha} \rightarrow \nu_{\beta}$ , the latter kind of experiment would measure the  $\nu_{\alpha}$  interaction rate at one or more locations and compare it with the expected signal, based on the knowledge of the neutrino flux at (or near) the source. Use of two detectors, one near and one far from

the source, can reduce systematic errors in this kind of an experiment. Such experiments cannot see very small signals because their observation would involve subtraction of two large numbers from each other; they also cannot tell the mode of oscillation, i.e., whether we see  $v_{\alpha} \rightarrow v_{\beta}$  or  $v_{\alpha} \rightarrow v_{\gamma}$ , since only the  $v_{\alpha}$  interaction rate is measured. Study of solar neutrinos is clearly via disappearance experiments.

Appearance experiments try to detect the potentially created new flavor, i.e.,  $v_{\beta}$  in our case. Their sensitivity for small signals is much better and is generally limited by the knowledge of the amount of  $v_{\beta}$  in the initial beam and the ability of the detector to distinguish clearly  $v_{\beta}$  from  $v_{\alpha}$ . Searches for  $v_{\tau}$ , identified by  $\tau$  production and decay in emulsion with essentially no background in a predominantly  $v_{\mu}$  beam are examples of appearance experiments.

#### 2.3 Sensitivities

In this section we discuss how the reach of a given experiment depends on the experimental parameters, i.e., L, E, and N, the number of events. We distinguish between two qualitatively different situations: a background-free experiment (e.g., search for  $v_{\tau}$  in emulsion), and an experiment relying on a statistical subtraction, e.g., a disappearance experiment or a measurement of the NC/CC ratio. The reach can be parametrized by the lowest value of  $\Delta m^2$  accessible and by the lowest value of  $\sin^2 2\theta$  that can be explored.

The number of signal events,  $N_{\beta}$ , is given by

$$N_{\beta} = N_{\alpha} \sin^2 2\theta \, \sin^2 (1.27 \cdot \Delta m^2 \cdot \frac{L}{E}),$$

where  $N_{\alpha}$  is the expected number of events of the original flavor in the absence of oscillations at a given location L and varies as

$$N_{\alpha} \propto N_{\alpha}^{o} \left(\frac{1}{L}\right)^{2},$$

with  $N_{\alpha}^{o}$  being the number of  $v_{\alpha}$  interactions at the source (L = 0). We may write

$$N^{o}_{\alpha} = If(E)$$

where I is the total proton intensity on target, and f(E) is a function describing energy dependence of the neutrino flux which is determined by the initial hadronic production spectrum, details of the focusing system, length of the decay volume, and energy

dependence of the neutrino cross section (which is proportional to E in the GeV region).

To investigate sensitivity at low  $\Delta m^2$  ( $\Delta m^2 \ll 1 \text{ eV}^2$ ), i.e., the lowest value of  $\Delta m^2$  that can be detected, we can write

$$N_{\beta} = N_{\alpha} \sin^2 2\theta \cdot (1.27 \cdot \Delta m^2 \cdot \frac{L}{E})^2 \propto N_{\alpha}^{o} \cdot (\frac{1.27 \cdot \Delta m^2}{E})^2$$

where  $N_{\beta}$  is the number of  $\beta$  flavor events detected necessary to establish presence of a signal. For the truly background-free case,  $N_{\beta} = 1$  (or 2 or 3 for very small background case). Thus the sensitivity for background-free case is  $\Delta m^2 \propto \sqrt{1/N_{\alpha}^{o}}$ , i.e, <u>independent</u> of L.

For the statistical case, the figure of merit for determination of sensitivity is the quantity  $\delta$  defined by

$$\delta = N_{\beta} / \sqrt{N_{\alpha}},$$

i.e., the number of standard deviations away from zero, namely from no effect. For low  $\Delta m^2$  we have

$$\delta \propto N_{\alpha}^{o} \cdot \left(\frac{1.27 \cdot \Delta m^{2}}{E}\right)^{2} / \sqrt{N_{\alpha}} = N_{\alpha}^{o} \left(\frac{1.27 \cdot \Delta m^{2}}{E}\right)^{2} / \sqrt{N_{\alpha}^{o}} \frac{1}{L} = \sqrt{N_{\alpha}^{o}} \left(\frac{\Delta m^{2}}{E}\right)^{2} L.$$

Thus, the sensitivity in  $\Delta m^2$  in this case goes as  $(N_{\alpha}^{o})^{-1/4} (\frac{E}{\sqrt{L}})$ . (Note that in this definition of sensitivity a lower number means further reach, and that  $N_{\alpha}^{o}$  has very likely a strong dependence on E as discussed above). The above arguments illustrate the importance of choosing as small a value of E/L as feasible for good low  $\Delta m^2$  sensitivity; because of fourth root dependence on  $N_{\alpha}^{o}$ , it is laborious and expensive to gain more sensitivity by increasing the running time (or the neutrino flux or the tonnage of the detector).

We turn now to the question of sensitivity in  $\sin^2 2\theta$ . Maximum sensitivity is generally taken as one that will occur at values of  $\Delta m^2$  high enough so that we shall have

$$\overline{\sin^2\left(1.27\cdot\Delta m^2\cdot L/E\right)} = \frac{1}{2}$$

where the average is over the energy spectrum. Hence we have

$$N_{\beta} = \frac{1}{2} N_{\alpha} \sin^2 2\theta \propto N_{\alpha}^{o} \left(\frac{1}{L}\right)^2 \sin^2 2\theta.$$

For the background-free case the sensitivity in  $\sin^2 2\theta$  will vary inversely with  $N^o_{\alpha}$  (i.e.,  $\propto 1/N^o_{\alpha}$ ) and as L<sup>2</sup>. For statistical analyses

$$\delta \propto N_{\alpha}^{o} \left(\frac{1}{L}\right)^{2} \sin^{2}2\theta / \sqrt{N_{\alpha}^{o}} \frac{1}{L} = \sqrt{N_{\alpha}^{o}} \frac{1}{L} \sin^{2}2\theta.$$

The sensitivity will be proportional to  $1/\sqrt{N_{\alpha}^{o}}$  and L. Thus, clearly a mistake in the proper choice of E/L is less costly in the reach for sin<sup>2</sup>2 $\theta$  than for  $\Delta m^{2}$ .

Knowing now the dependence of the intercepts of our sensitivity contour, it remains to ask about the shape of the contour in the intermediate region. Taking the log of our probability equation for low  $\Delta m^2$  we have

$$\log N_{\beta} = \log N_{\alpha} + \log \left( \sin^2 2\theta \right) + 2\log \left( 1.27 \cdot \frac{L}{E} \right) + 2\log \left( \Delta m^2 \right)$$

Thus the slope of the sensitivity curve on a log-log plot will be 1/2. The general shape of a typical sensitivity plot is shown in Fig. 2. The turnaround point corresponds roughly to

$$1.27 \cdot \Delta m^2 \cdot \frac{L}{E} = \frac{\pi}{2},$$

and the sharpness of the wiggles near that region increases for a relatively narrow beam energy spectrum and is washed out for a broad spectrum.



FIG. 2. A typical shape of a sensitivity plot for an oscillation experiment. We note the dependence of the limiting points on initial flux and the values of L and E. One must remember, of course, the additional, implicit dependence of  $N_0$  on E as discussed in the text.