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Building Models for J/ψ Elastic and Inelastic Cross Section

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Abstract

In the following article we are going to build one model for J/ψ inelastic cross section based on quantum chroma mechanics. And we will use this model to simulate z distribution of inelastic J/ψ photoproduction.

1 Kinematics

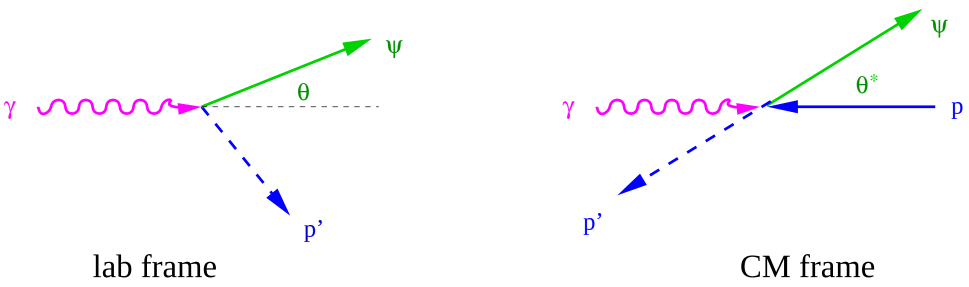

Figure 1: Kinematics of elastic J/ψ photoproduction in lab and center-of-momentum frames.

Fig. 1 [1] shows the knienatics in laboratory and center of momentum frames for J/ψ photoproduction. Two essential variables s and W are defined in lab frame by:

$$W^2 = s = (k_\mu + p_\mu)^2 = m_p^2 + 2m_p k \tag{1}$$

Table 1 [1] gives some other useful variables in the lab and CM frames.

2 Overview

From the work of Nathan Baillie and Robert Fersch [2], we get a satisfying model, Eq. 2, for the total elastic cross-section σ of J/ψ photo production.

lab	CM
$k_\mu = (k, 0, 0, k)$	$k^*_\mu = (k^*, 0, 0, k^*)$
$p_\mu = (m_\psi, 0, 0, 0)$	$p^*_\mu = (E^*_p, 0, 0, -k^*)$
$k'_\mu = (E_\psi, p\sin\theta, 0, p\cos\theta)$	$k'^*_\mu = (E^*_\psi, k'^*\sin\theta^*, 0, , k'^*\cos\theta^*)$
$p'_\mu = (k + m_p - E_\psi, -p\sin\theta, 0, k - p\cos\theta)$	$p'^*_\mu = (E'^*_p, -k'^*\sin\theta^*, 0, -k'^*\cos\theta^*)$
	$E^*_p = \sqrt{m_p^2 + k^{*2}}$
$E_\psi = \sqrt{m_\psi^2 + p^2}$	$E^*_\psi = \sqrt{m_\psi^2 + k'^{*2}}$
	$E'^*_p = \sqrt{W^2 + k'^{*2}}$

Table 1: Kinematic variables for J/ψ photoproduction. The stars indicate the CM frame, the primes denote variables for the final reaction states, and the subscript μ indicates a 4-momentum of the form (E, p_x, p_y, p_z). The variable k is the incident photon momentum, p is the outgoing J/ψ momentum, θ is the angle of the J/ψ with respect to the photon direction, m_p is the nucleon mass, m_ψ is the J/ψ mass, and W is the invariant mass.

$$\sigma = \alpha_1 \cdot (1 - \frac{W_{th}^2}{W^2})^{\alpha_2} \cdot W^{\alpha_3} \tag{2}$$

with threshold energy $W_{th} = 4.0$ GeV, $\alpha_1 = 2.37$, $\alpha_2 = 1.53$ and $\alpha_3 = 0.740$ when W is expressed in GeV and σ in nb.
By a little algebra, we can derive the relationship between total cross section σ and differential cross section $d\sigma/dt$ as:

$$\sigma = \frac{d\sigma/dt}{be^{b(t-t_{min})}} \tag{3}$$

Eq. 3 can be proved by the integration over t from infinity to t_{min}. In Eq. 3, b can be obtained from data fitting. We simply list the fitting result:

$$b = b_0 + b_1 \ln W \tag{4}$$

with $b_0 = 0.35\ GeV^{-2}$ and $b_1 = 0.87\ GeV^{-2}$ and $\chi^2/ndf = 20.6/10 = 2.1$. If t and W are known in elastic scattering, t_{min} in Eq. 3 can be derived from the kinematics based on energy conservation and momentum conservation from the equations below:

$$\begin{aligned} s &= m_p^2 + 2m_p k && (5) \\ k^* &= \frac{m_p}{\sqrt{s}} k && (6) \\ k'^{*2}_{el} &= k^{*2} + \frac{m_\psi^2(m_\psi - 2s - sm_p^2)}{4s} && (7) \\ t_{min} &= m_\psi^2 - 2k^*(\sqrt{m_\psi^2 + k'^{*2}_{el}} - k'^{*2}_{el}) && (8) \end{aligned}$$

k'^{*}_{el} in Eq. 8 is the final center-of-mass momentum for elastic J/ψ production. With Eq. 2 to Eq. 8, we can calculate differential cross section $d\sigma/dt$ in terms of t and W. If the range of t is further known, we can integrate $d\sigma/dt$ over t to get the cross section $\Delta\sigma$ for this certain range of t. This result will be very useful in a later section of this note.
After the work on elastic cross section, they began to make a model for the inelastic cross section. The starting point is trying to find a function to fit the double differential cross section $d^2\sigma/dp_t^2dz$. In order to do that they made an assumption that the differential cross section can be written as:

$$\frac{d^2\sigma}{dp_t^2dz} = f(z)g(p_t^2)h(W) \tag{9}$$

and then from this assumption, Eq. 9, they obtained a fit to the form:

$$\frac{d^2\sigma}{dp_t^2dz} = \alpha_6 \frac{(1-\frac{W_{th}^2}{W^2})^{\alpha_2}W^{\alpha_3}}{(p_t^2+\frac{M_\psi^2}{\gamma})^{\alpha_0+\alpha_1 z}}(1+\alpha_4 e^{\alpha_5 z}) \tag{10}$$

with $\alpha_0 = 2.165$, $\alpha_1 = 0.8434$, $\alpha_2 = 1.528$, $\alpha_3 = 0.7419$, $\alpha_4 = 0.193$, $\alpha_5 = 3.632$, $\alpha_6 = 4.44$, $\gamma = 5$, and $W_{th} = 4.03$ GeV. However, Eq. 10 is a purely empirical form and the factorization in Eq. 9 is not a necessary consequence of QCD. In this technical note we wish to:

1. Build a model for J/ψ inelastic double differential cross section $d^2\sigma/dp_t^2dz$ based on perturbative QCD;
2. Obtain unknown parameters in our model from data fitting and;
3. Apply this model for the simulation of E160 J/ψ production at SLAC.

3 Formulas

From the color singlet model (with $\hbar$=c=1) in pQCD [3], the double differential cross section $d^2\sigma/dp_t^2dz$ for J/ψ production by real photons (Q^2=0) from a nucleon is given by:

$$\frac{d^2\sigma}{dp_t^2dz} = \alpha_1 \frac{G(x)}{s[m_\psi^2(1-z)+p_t^2]}[\frac{1}{M_T^4}+\frac{(1-z)^4}{K_T^4}+\frac{z^4p_t^4}{M_T^4K_T^4}] \tag{11}$$

with z, p_t, M_T and K_T defined by:

$$\begin{aligned} z &= \frac{E_\psi}{k} && (12)\\ p_t &= p\sin\theta && (13)\\ M_T^2 &= m_\psi^2+p_t^2 && (14)\\ K_T^2 &= m_\psi^2(1-z)^2+p_t^2 && (15)\\ & && (16) \end{aligned}$$

m_ψ is the mass of the J/ψ. G(x) is the gluon distribution of the target nucleon. x is defined by:

$$x = \frac{\hat{s}}{s} = \frac{1}{s}[\frac{m_\psi^2}{z} + \frac{p_t^2}{(1-z)z}] \tag{17}$$

We should pay attention to Eq. 17, for the condition

$$0 \leq x \leq 1 \tag{18}$$

must be satisfied. So, the range of p_t^2 and z allowed in this model can be calculated. The range of p_t^2 is:

$$0 \leq p_t^2 \leq \frac{(s - m_\psi^2)^2}{4s} \tag{19}$$

The range of z is not as simple as p_t^2. We should first compare m_ψ/k (k is the energy of incident) with μ_- which is defined by:

$$\mu_- = \frac{m_\psi + s - \sqrt{(m_\psi^2 - s)^2 - 4sp_t^2}}{2s} \tag{20}$$

We also have corresponding μ_+ defined by:

$$\mu_+ = \frac{m_\psi + s + \sqrt{(m_\psi^2 - s)^2 - 4sp_t^2}}{2s} \tag{21}$$

If $m_\psi/k \geq \mu_-$, then

$$\frac{m_\psi}{k} \leq z \leq \mu_+ \tag{22}$$

If $m_\psi/k < \mu_-$, then

$$\mu_- \leq z \leq \mu_+ \tag{23}$$

In this note, we take the gluon trial function as:

$$xG(x) = N(1-x)^n \tag{24}$$

However, if combine Eq. 11 Eq. 17 and Eq. 24 together, we find a problem that $d^2\sigma/dp_t^2dz$ does not depend on s anymore. From the suggestion of Keith, we revise Eq. 11 a little bit, which means we add a term $1/s^{\alpha_3}$. Thus we rewrite the formula for double differential cross section $d^2\sigma/dp_t^2dz$ explicitly as:

$$\frac{d^2\sigma}{dp_t^2dz} = \alpha_1 \frac{(1-x)^{\alpha_2}}{(sx)s^{\alpha_3}[m_\psi^2(1-z) + p_t^2]}[\frac{1}{M_T^4} + \frac{(1-z)^4}{K_T^4} + \frac{z^4p_t^4}{M_T^4K_T^4}] \tag{25}$$

So our mission next is to find out α_1,α_2 and α_3 from data fitting.

4 Data Fitting Result

We have about one hundred double differential $d^2\sigma/dp_t^2dz$ data points from EMC and H1, and around 10 total cross section σ data points from HERA, Fermi, Cern and EMC. Here are the principles of our data fitting.

1. The value of χ^2/ndf in double differential cross section $d^2\sigma/dp_t^2dz$ fitting should be as small as possible.

2. After we get the parameters, i.e. α_1 to α_3 in Eq. 25, from Step 1, we can integrate Eq. 25 to calculate total cross section σ. In comparison to the experiment data of total cross section σ, the value of χ^2/ndf should also be as small as possible.

3. Discuss the physics meaning.

Shortly speaking, we get a final expression of $d^2\sigma/dp_t^2dz$ between the balance of Step 1 and Step 2 as:

$$\frac{d^2\sigma}{dp_t^2dz} = 6000\frac{(1-x)^{1.9}}{xs^{0.69}[m_\psi^2(1-Z)+p_t^2]}[\frac{1}{M_T^4} + \frac{(1-Z)^4}{K_T^4} + \frac{Z^4p_t^4}{M_T^4K_T^4}] \tag{26}$$

The unit of mass and p_t^2 in Eq. 26 is GeV and $d^2\sigma/dp_t^2dz$ results in nb/GeV. We will talk about Step 3 shortly.

5 Comparison of Two Models

First, we compare the χ^2/ndf in two models for double differential cross section $d^2\sigma/dp_t^2dz$. In model 1, namely Eq. 10, the value of χ^2/ndf is 265/111. And in model 2, namely Eq. 25, the value of χ^2/ndf is 304/111, greater than model 1. Therefore, model 1 is better in this fitting.
Second, we compare the χ^2/ndf for total cross section σ as a function of k. Please see Fig. 2. From this figure, it is obvious that model 2 has a better χ^2/ndf than model 1.
Not being able to tell which model is better from χ^2/ndf, we seem to conclude that model 2 defeats model 1 because model 2 is derived from QCD. However, it is time to discuss Step 3 in section Data Fitting Result. From Eq. 26, we have the distribution of gluon as:

$$xG(x) = N(1-x)^{1.9} \tag{27}$$

As a trial function of gluon distribution, from the knowledge we have, the power of $(1-x)$ should be at least 5. It is larger than our result 1.9 in Eq. 26. So,

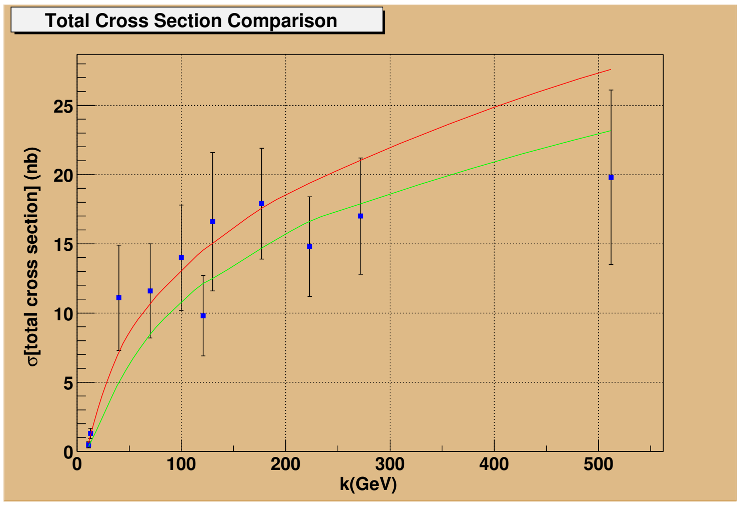

Figure 2: Total inelastic cross section in nb versus beam energy k in GeV. Red line is total cross section calculated from Model 1. Green line is total cross section calculated from Model 2. Blue dots with error bar are experiment data.

model 1 and model 2 are neither successful in physics. What we can do next is to treat α_1 in Eq. 11 as a function of s and meanwhile keep the power of $(1-x)$ greater than 5.

6 Feed Down From ψ(2s)

Though not successful in physics, these models are still applied in E160 J/ψ product simulation to show us a rough idea for further work.
The basic graph we want to get is the number of J/ψ events for z within a certain range of t at specific k. We know that besides J/ψ there is ψ(2s) particle production in elastic scattering. More detailed knowledge is that the ratio of J/ψ production to ψ(2s) production is 7:1. And particle ψ(2s) will decay rapidly to other particles which also include ψ particle. All the possible decay processes of ψ(2s) to J/ψ are listed as:

$$\psi(2s) \longrightarrow \left\{ \begin{array}{c} h_{1c}^0 \\ \chi_{0c}^0 \\ \chi_{1c}^0 \end{array} \right\} \longrightarrow J/\psi$$

Inevitably, these decays will form a background for the J/ψ particle generated directly from inelastic scattering. And we assume that J/ψ from ψ(2s) decay generated in elastic scattering is the only source of background.
To simulate this background, we use the JETSET package to complete this

process to get the final energy distribution of J/ψ from $\psi(2s)$ decay. The subroutine ~fridgy/exp160/modeldecayf.f on the SLAC unix cluster performs the calculation. We want to emphasize here that this simulation is independent of inelastic model. Because it is derived directly from elastic scattering (please refer back to section 2 to see how to calculate cross section $\Delta\sigma$ for a certain range of t) and from the properties of J/ψ and $\psi(2s)$.

Fig. 3 is the simulation result. The black and red z distribution of event

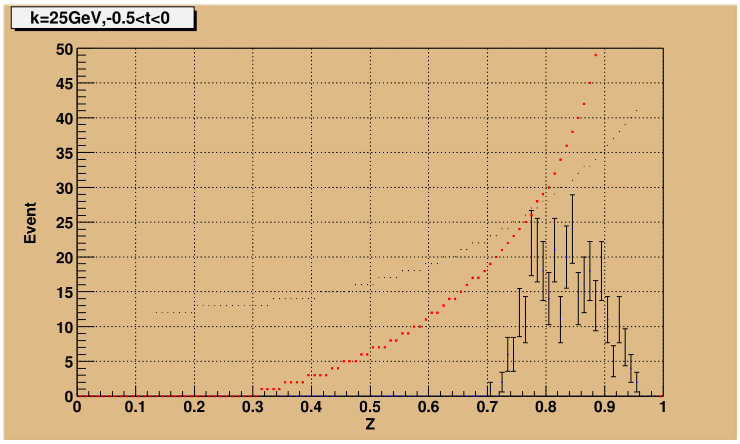

Figure 3: Blue dots with statistical error bar are J/ψ particles from $\psi(2s)$ deay. Blac dots are J/ψ calculated from model 1. It shows a threhold at $z = \frac{m_\psi}{k}$. Red dots are calculated from model 2. It does not have the threhold as model 1, but rises rapidly when z is greater than 0.9.

number is calculated by the integration of the double differential cross section $d^2\sigma/dp_t^2dz$ over the given range of t and z from two models separately. Because the step of z is very small, i.e. 0.05 in our calculation, we can treat the cross section $\Delta\sigma$ for given range of t and z just as the product of dσ/dz and z step. There is also an executable program called foo under the same directory. The function of foo is: once you give the value of incident photon energy k in GeV and the range of t, foo will generate a root file like Fig. 3 only based on model 1.

7 Things Need to Complete

What we wish to do is to build a model to extract information on $\psi(2s)$ decays, inelastic, and elastic J/ψ production from our real data. So far there are two things to do.
One is : Keep revising the QCD model. It has been stated in section Comparison of Two Models.
Two is : Fit J/ψ particle distribution from $\psi(2s)$ decay as a function of k, t,

for example, a Gaussian distribution. This function will help us extracting the J/ψ particle only generated in inelastic scattering from real data.

References

[1] Keith Griffioen: Kinematics for J/ψ Photoproduction, RPC-TN-22

[2] Nathan Baillie, Robert Fersch, Keith Griffioen: Inelastic and Elastic Cross Section Curve Fits for J/ψ Photoproduction

[3] J. Ashman *et al.*: Muoproduction of J/ψ and the gluon distribution of the nucleon, Z. Phys. C56(1992)21