THE HAWKING-UNRUH TEMPERATURE AND QUANTUM FLUCTUATIONS IN PARTICLE ACCELERATORS

K. T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544

We wish to draw attention to a novel view of the effect of the quantum fluctuations during the radiation of accelerated particles, particularly those in storage rings. This view is inspired by the remarkable insight of Hawking[1] that the effect of the strong gravitational field of a black hole on the quantum fluctuations of the surrounding space is to cause the black hole to radiate with a temperature

$$T = \frac{\hbar g}{2\pi ck},$$

where g is the acceleration due to gravity at the surface of the black hole, c is the speed of light, and k is Boltzmann's constant. Shortly thereafter Unruh[2] argued that an accelerated observer should become excited by quantum fluctuations to a temperature

$$T = \frac{\hbar a^\star}{2\pi ck},$$

where $a^\star$ is the acceleration of the observer in its instantaneous rest frame. In a series of papers Bell and coworkers[3−5] have noted that electron storage rings provide a demonstration of the utility of the Hawking-Unruh temperature, with emphasis on the question of the incomplete polarization of the electrons due to quantum fluctuations of synchrotron radiation.

Here we expand slightly on the results of Bell *et al.*, and encourage the reader to consult the literature for more detailed understanding.

Applicability of the Idea

When an accelerated charge radiates, the discrete energy and momentum of the radiated photons induce fluctuations on the motion of the charge. The insight of Unruh is that for uniform linear acceleration (in the absense of the fluctuations), the fluctuations would excite any internal degrees of freedom of the charge to the temperature stated above. His argument is very general (*i.e.*, thermodynamic) in that it does not depend on the details of the accelerating force, nor of the nature of the accelerated particle. The idea of an effective temperature is strictly applicable only for uniform linear acceleration, but should be approximately correct for other accelerations, such as that due to uniform circular motion.

A charged particle whose motion is confined by the focusing system of a particle accelerator exhibits transverse and longitudinal oscillations about its ideal path. These oscillations are excited by the quantum fluctuations of the particle's radiation, and thus provide an excellent physical example of the viewpoint of Unruh.

Further, the particles take on a thermal distribution of energies when viewed in the average rest frame of a bunch, which transforms to the observed energy spread in the laboratory. While classical synchrotron radiation would eventually polarize the spin-$\frac{1}{2}$ particles completely, the thermal fluctuations oppose this, reducing the maximum beam polarization.

It is suggestive to compare the excitation energy $U^\star = kT$, as would be observed in the particle's rest frame, to the rest energy mc^2 when the acceleration is due to laboratory electromagnetic fields E and B. Noting that $a^\star = eE^\star/m$ we find

$$\frac{U^\star}{mc^2} = \frac{\hbar e E^\star}{2\pi m^2 c^3} = \frac{\left[E_{\|} + \gamma\left(E_\perp + \beta B_\perp\right)\right]}{2\pi E_{\rm crit}},$$

where the particle's laboratory momentum is $\gamma\beta mc$, and

$$E_{\rm crit} \equiv \frac{m^2c^3}{e\hbar}.$$

For an electron,

$$E_{\rm crit} = 1.3 \times 10^{16}\,\text{volts/cm} = 4.4 \times 10^{13}\,\text{gauss}.$$

($E_{\rm crit}$ is the field strength at which spontaneous pair production becomes highly probable, *i.e.*, the field whose voltage drop across a Compton wavelength is the particle's rest energy.) We might expect that the fluctuations become noticeable when $U^\star \sim 0.1$ eV, and hence comparable to any other thermal effects in the system, such as the particle-source temperature.

For linear accelerators $E_{\|} \sim 10^6$ volts/cm at best, so $U^\star < 10^{-5}$ eV. The effect of quantum fluctuations is of course negligible because the radiation itself is of little importance in a linear accelerator.

For an electron storage ring such as LEP, $\gamma \sim 10^5$, and $B_\perp \sim 10^3$ gauss, so that $U^\star \sim 0.2$ eV. For the SSC proton storage ring, $\gamma \sim 2 \times 10^4$, while $B_\perp \sim 6 \times 10^4$ gauss, so that $U^\star \sim 2$ eV. As is well known, in essentially all electron storage rings, and in future proton rings, the effect of quantum fluctuations is quite important.

The remaining discussion is restricted to beams in storage rings (= transverse particle accelerators).

Beam-Energy Spread

An immediate application of the excitation energy $U^\star$ is to the beam-energy spread. In the average rest frame of a bunch of particles, the distribution of energies is approximately thermal, with characteristic kinetic energy $U^\star$, and momentum $p^\star = \sqrt{2mU^\star}$. The spread in laboratory energies is then given by

$$U_{\rm lab} \approx \gamma(mc^2 + U^\star \pm \beta p^\star c) \approx U_0\left(1 \pm \gamma\sqrt{\frac{\lambda_C}{\pi\rho}}\right),$$

where $U_0 = \gamma mc^2$ is the nominal beam energy, $\rho = U_0/eB_\perp$ is the radius of curvature of the central orbit, and $\lambda_C = \hbar/mc$ is the Compton wavelength. Writing this as

$$\left(\frac{\delta U}{U_0}\right)^2 \approx \frac{\gamma^2\lambda_C}{\pi\rho},$$

we obtain the standard result, as given by equation (5.48) of the review by Sands.[6]

Beam Height

The quantum fluctuations of synchrotron radiation drive the oscillations of particles about the bunch center, and set lower limits on the transverse and longitudinal beam size. If we associate a harmonic oscillator with each component of the motion about the bunch center, then each oscillator will be excited to amplitudes whose corresponding energy is $U^\star = kT^\star$.

For example, consider the vertical betatron oscillations which determine the beam height. The frequency of these oscillations is $\omega = \nu_z\omega_0 = \nu_z c/R$, where ν_z is the vertical betatron number, and $R = L/2\pi$ is the mean radius of the storage ring. In the average rest frame of a bunch the oscillation frequency appears to be $\omega^\star = \gamma\omega$, and the spring constant in this frame is given by $k^\star = m\omega^{\star 2} = \gamma^2 m\omega^2$. The typical amplitude of oscillation in this frame is then

$$\frac{1}{2}k^\star z^{\star 2} \approx U^\star = \frac{\hbar a^\star}{2\pi c} = \frac{\hbar\gamma^2 a}{2\pi c} = \frac{\hbar\gamma^2 c}{2\pi\rho},$$

noting that in uniform circular motion the acceleration is transverse. For the vertical oscillation the lab frame amplitude z is the same as $z^\star$. Combining the above we find

$$z^2 = \frac{\lambda_C R^2}{\pi\nu_z^2\rho},$$

which reproduces the standard result, such as equation (5.107) of Sands.[6]

An analogous argument is given in ref. 5 to derive the beam height in a weakly focused storage ring.

Bunch Length and Beam Width

A similar analysis can be given for oscillations in the plane of the orbit. However, radial and longitudinal excursions are also directly coupled to energy excursions, which proves to be the stronger effect. As the present method finds the standard result for the beam-energy spread, the usual results for bunch length and beam width follow at once. [In ref. 6, use equations (5.64) and (5.93) to yield expressions (5.65) and (5.95).]

Beam Polarization

Sokolov and Ternov[7] predicted that quantum fluctuations in synchrotron radiation limit the transverse polarization of the beam to 92%. In the absense of quantum fluctuations the polarization should reach 100% after long times. Bell and Leinaas[3] realized that the thermal character of the fluctuations provides an alternate view of the depolarizing mechanism. In ref. 5 they provide a detailed justification that the thermodynamic arguments are fully equivalent to the original QED calculation of Sokolov and Ternov. In the process they find that for circular motion in a weakly focused ring (betatron), the effective temperature due to quantum fluctuations is

$$kT = \frac{13}{96}\sqrt{3}\frac{\hbar a^\star}{c},$$

which is about 1.5 times Unruh's result for linear acceleration.

Radiation Spectrum

Because of the quantum fluctuations the motion of the particles departs from the central orbit, and a classical calculation of the synchrotron-radiation spectrum is incorrect in principle. The deviations become significant only when the characteristic energy of the radiation approaches the beam energy, *i.e.*, when $\gamma B_\perp/E_{\rm crit} \sim 1$, and the prominent effect is the cutoff at the high-energy end of the spectrum.

In the regime where the quantum corrections to the radiation spectrum are small the author has given an estimate of their size.[8] For this we imagine the accelerated charge is surrounded (in its rest frame) by a bath of photons with a Planck spectrum of temperature $kT = \hbar a^\star/2\pi c$. The correction to the classical spectrum is considered to arise from the Thomson scattering of these virtual photons off the charged particle. In the lab frame the spectral correction is proportional to the Lorentz transform of the Planck spectrum, whose peak photon energy is then $2\gamma kT = \hbar\gamma^3 c/\pi\rho$, essentially the same as that of the classical spectrum. On integrating over energy, the total rate of the correction term is the classical (Larmor) rate times

$$\frac{\alpha}{60\pi}\left(\frac{\gamma B_\perp}{E_{\rm crit}}\right)^2,$$

which is indeed very small at present storage rings.

Acknowledgements

I would like to thank Ian Affleck and Heinrich Mitter for several discussions on this topic. This work was supported in part by the U.S. Department of Energy under contract DOE-AC02-76ER-03072.

---

[1]S.W. Hawking, 'Black-Hole Explosions,' Nature **248**, 30 (1974).

[2]W.G. Unruh, 'Notes on Black-Hole Evaporation,' Phys. Rev. D **14**, 870 (1976).

[3]J.S. Bell and J.M. Leinaas, 'Electrons as Accelerated Thermometers,' Nuc. Phys. **B212**, 131 (1983).

[4]J.S. Bell, R.J. Hughes and J.M. Leinaas, 'The Unruh Effect in Extended Thermometers,' Z. Phys. C **28**, 75 (1985).

[5]J.S. Bell and J.M. Leinaas, 'The Unruh Effect and Quantum Fluctuations of Electrons in Storage Rings,' CERN-TH.4468/86 (June, 1986).

[6]M. Sands, 'The Physics of Electron Storage Rings,' SLAC-121, (1970); also in Proc. 1969 Int. School of Physics, 'Enrico Fermi,' ed. by B. Touschek (Academic Press, 1971), p. 257.

[7]A.A. Sokolov and I.M. Ternov, 'On Polarization and Spin Effects in the Theory of Synchrotron Radiation,' Sov. Phys. Dokl. **8**, 1203 (1964).

[8]K.T. McDonald, 'Fundamental Physics During Violent Acceleration,' in *Laser Acceleration of Particles*, AIP Conf. Proc. No. 130, ed. by C. Joshi and T. Katsouleas (Malibu, 1985), p. 23.