Limits on the Applicability of Classical Electromagnetic Fields as Inferred from the Radiation Reaction

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Can the wavelength of a classical electromagnetic field be arbitrarily small, or its electric field strength be arbitrarily large? If we require that the radiation-reaction force on a charged particle in response to an applied field be smaller than the Lorentz force we find limits on the classical electromagnetic field that herald the need for a better theory, *i.e.*, one in better accord with experiment. The classical limitations find ready interpretation in quantum electrodynamics. The examples of Compton scattering and the QED critical field strength are discussed. It is still open to conjecture whether the present theory of QED is valid at field strengths beyond the critical field revealed by a semiclassical argument.

I. INTRODUCTION

The ultimate test of the applicability of a physical theory is the accuracy with which it describes natural phenomena. Yet on occasion the difficulty of a theory in dealing with a "thought experiment" provides a clue as to limitations of that theory.

It has long since been recognized that classical electrodynamics has been surplanted by quantum electrodynamics in some respects. But one doubts that quantum electrodynamics, or even its generalization, the Standard Model of elementary particles, is valid in all domains. To aid in the search for new physics, it is helpful to review the warning signs of the past transitions from one theoretical description to another.

The debates as to the meaning of the classical radiation reaction for pointlike particles provide examples of such warning signs. One case is the "4/3 problem" of electromagnetic mass, where covariance does not imply uniqueness [1]. Such difficulties have often been interpreted as suggesting that classical electrodynamics cannot be a complete description of matter on the scale of the classical electron radius, $r_0 = e^2/mc^2$ in Gaussian units.

It seems less appreciated that the part of the classical radiation reaction that is independent of particle size provides clues as to the limits of applicability of classical electromagnetic fields. For example, a recent article in this Journal [2] ends with the sentence, "Only when all distances involved are in the classical domain is classical dynamics acceptable for electrons". While this condition is necessary, it is not sufficient. For a classical description to be accurate, an electron can only be subject to fields that are not too strong. This paper seeks to illustrate what "not too strong" means.

Considerations of strong fields have been very influential in the development of other modern theories besides quantum electrodynamics. In classical gravity, *i.e.*, general relativity, the strong-field problem is identified with black holes. One of the best known intersections between gravity and quantum electrodynamics is the Hawking radiation of black holes. In the case of the strong (nuclear) interaction, the fields associated with nuclear matter all appear to be strong, and weak fields are thought to exist only in the high-energy limit (asymptotic freedom). Such considerations led to the introduction of non-Abelian gauge theories. These constructs, when applied to the weak interaction, led to the concept of a background (Higgs) field that is strong in the sense of having a large vacuum expectation value, which in turn has the effect of generating the masses of the W and Z gauge bosons. Most recently, considerations of the strong-field (strong-coupling) limit of string theories have led to the notion of "duality", *i.e.*, the various string theories of the 1980's are actually different weak-field limits of a single strong-field theory. These string theories are noteworthy for suggesting that particles are to be considered as excitations of small, but extended quantum strings, thereby avoiding the infinite self energies that have appeared in theories of point particles since J.J. Thomson introduced the concept of electromagnetic mass in 1881 [3].

The main argument concerning classical electromagnetic fields is given in sec. 2, and is brief. This argument could have been given around 1900 by Lorentz [4–6] or by Planck [7]. who made remarks of a related nature. But the argument seems to have been first made in 1935 by Oppenheimer [8], and more explicitly by Landau and Lifshitz [9]. Additional historical commentary is given in sec. 1.1. Sections 2.1-2.5 comment on various aspects of the main argument, still from a classical perspective. A quantum view in introduced in sec. 3, and the important examples of Compton scattering and the QED critical field are discussed in secs. 3.1-2. The paper concludes in sec. 4 with remarks on the role of strong fields on the development of quantum electrodynamics, and presents two examples (secs. 4.1-2) of speculative features of strong-field QED and one of very short distance QED (sec. 4.3).

A. Historical Introduction

The relation between Newton's third law and electromagnetism has been of concern at least since the investigations of Ampère, who insisted that the force of one current element on another be along their line of centers. See Part IV, Chap. II, especially sec. 527, of Maxwell's *Treatise* for a review [10]. However, the presently used differential version of the Biot-Savart law does not satisfy Newton's third law for pairs of current elements unless they are parallel.

Perhaps discomfort with this fact contributed to the delay in acceptance of the concept of isolated electrical charges, in contrast to complete loops of current, until the late nineteenth century.

A way out of this dilemma became possible after 1884 when Poynting [11] and Heaviside [12] argued that electromagnetic fields (in suitable configurations) can be thought of as transmitting energy. The transmission of energy was then extended by Thomson [13], Poincaré [14] and Abraham [15] to include transmission (and storage) of momentum by an electromagnetic field.

That a moving charge interacting with thermal radiation should feel a radiation pressure was anticipated by Stewart in 1871 [16], who inferred that both the energy and the momentum of the charge would be affected.

In 1873, Maxwell discussed the pressure of light on conducting media at rest, and on "the medium in which waves are propagated" ([10], secs. 792-793). In the former case, the radiation of a reflected wave by a (perfectly conducting) medium in response to an incident wave results in momentum, but not energy, being transferred to the medium. The energy for the reflected wave comes from the incident wave.

The present formulation of the radiation reaction is due to Lorentz' investigations of the self force of an extended electron, beginning in 1892 [4] and continuing through 1903 [5]. The example of dipole radiation of a single charge contrasts strikingly with Maxwell's discussion of reaction forces during specular reflection. There is no net momentum radiated by an oscillating charge with zero average velocity, but energy is radiated. The external force alone can not account for the energy balance. An additional force is needed, and was identified by Lorentz as the net electromagnetic force of one part of an extended, accelerated charge distribution on another. See eq. (1) below.

In 1897, Planck [7,17] applied the radiation reaction force of Lorentz to a model of charged oscillators and noted the existence of what are sometimes called "runaway" solutions, which he dismissed as having no physical meaning (keine physikalishe Bedeutung).

The basic concepts of the radiation reaction were brought essentially to their final form by Abraham [18,19], who emphasized the balance of both energy and momentum in the motion of extended electrons moving with arbitrary velocity.

Important contributions to the subject in the early twentieth century include those of Sommerfeld [20], Poincaré [21], Larmor [22], Lindemann [23], Von Laue [24], Born [25], Schott [26–29], Page [30], Nordström [31], Milner [32], Fermi [33], Wenzel [34], Wesel [35] and Wilson [36]. The main theme of these works was, however, models of classical charges and the related topic of electromagnetic mass.

The struggle to understand the physics of atoms led to diminished attention to classical models of charged particles in favor of quantum mechanics and quantum electrodynamics (QED). In 1935, there was apparent disagreement between QED and reported observations at energy scales of 10-100 MeV. Oppenheimer [8] then conjectured whether QED might fail at high energies and, in partial support of his view, invoked a classical argument concerning difficulties of interpretation of the radiation reaction at short distances. The present article illustrates an aspect of Oppenheimer's argument that was developed further by Landau [9].

Another response to Oppenheimer's conjecture was by Dirac [37] in 1938, when he deduced a covariant expression for the radiation reaction force (previously given by Abraham, Lorentz and von Laue in noncovariant notation) by an argument not based on a model of an extended electron. Dirac also gave considerable discussion of the paradoxes of runaway solutions and preacceleration. This work of Dirac, and most subsequent work on the classical radiation reaction, emphasized the internal consistency of classical electromagnetism as a mathematical theory, rather than as a description of nature. But, as has been remarked by Schott [26], "there is considerable danger, in a purely mathematical investigation, of losing touch with reality". Quantum mechanics had triumphed.

Research articles on the classical radiation reaction are still being produced; see, for example, Refs. [38]- [83]. Sarachik and Schappert ([61], sec. IIID) present a brief version of the argument given below in sec. 2.

Reviews of the subject include Refs. [84]- [117]. Most noteworthy in relation to the present article are the reviews by Lorentz [6], Erber [94] and Klepikov [103], which are the only ones that indicate an awareness of the problem of strong fields. The texts of Landau and Lifshitz [9] and Jackson [100] briefly mention that issue.

The radiation reaction has been a frequent topic of articles in this Journal, including Refs. [2] and [106]- [122]. The article of Page and Adams [106] is noteworthy for illustrating how the concept of electromagnetic field momentum restores the full validity of Newton's third law in an interesting example of the interaction of a pair of moving charges.

II. A GENERAL RESULT FOR THE RADIATION REACTION

Consider an electron of charge e and mass m moving in electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. The mass m is the "effective mass" in the language of Lorentz [6], now called the "renormalized" mass, for which the divergent electromagnetic self energy of a small electron is cancelled in a manner beyond the scope of this article.

Then the remaining leading effect of the radiation reaction is the "radiation resistance" which is independent of hypotheses as to the structure of the electron. Our argument emphasizes the effect of radiation resistance, since any deductions about properties of electromagnetic fields will then be as free as possible from controversy as to the nature of matter.

The (nonrelativistic) equation of motion including radiation resistance is (in Gaussian units)

$$m\dot{\mathbf{v}} = \mathbf{F}_{\rm ext} + \mathbf{F}_{\rm resist}, \tag{1}$$

where

$$\mathbf{F}_{\rm ext} = e\mathbf{E} + e\frac{\mathbf{v}}{c} \times \mathbf{B} \tag{2}$$

is the Lorentz force on the electron due to the external field,

$$\mathbf{F}_{\rm resist} = \frac{2e^2}{3c^3}\ddot{\mathbf{v}} + \mathcal{O}(\mathbf{v}/c) \tag{3}$$

is the force of radiation resistance, $\mathbf{v}$ is the velocity of the electron, c is the speed of light and the dot indicates differentiation with respect to time. Equation (3) is the form of the radiation reaction given in the original derivations of Lorentz [4] and Planck [7,17], which is sufficient for the main argument of this paper. Some discussion of the larger context of the classical radiation reaction is given in secs. 2.1-5.

If the second time derivative of the velocity is small we estimate it by taking the derivative of (1):

$$\ddot{\mathbf{v}} \approx \frac{e\dot{\mathbf{E}}}{m} + \frac{e}{m}\frac{\dot{\mathbf{v}}}{c} \times \mathbf{B} + \frac{e}{m}\frac{\mathbf{v}}{c} \times \dot{\mathbf{B}}. \tag{4}$$

We further suppose that the velocity is small (without loss of generality according to the principle of relativity; see sec. 2.4 for a relativistic discussion). so it suffices to approximate $\dot{\mathbf{v}}$ as $e\mathbf{E}/m$ in (4). Hence,

$$\ddot{\mathbf{v}} \approx \frac{e\dot{\mathbf{E}}}{m} + \frac{e^2}{m^2 c}\mathbf{E} \times \mathbf{B}. \tag{5}$$

The radiation resistance is now

$$\mathbf{F}_{\rm resist} \approx \frac{2e^2}{3c^3}\left(\frac{e\dot{\mathbf{E}}}{m} + \frac{e^2}{m^2 c}\mathbf{E} \times \mathbf{B}\right). \tag{6}$$

The first term in (6) contributes only for time-varying fields, which I take to have frequency ω and reduced wavelength $\lambda\!\!\!^{-}$; hence, $\dot{\mathbf{E}} \propto \omega\mathbf{E}$. The second term contributes only when $\mathbf{E} \times \mathbf{B} \neq 0$, which is most likely to be in a wave (with $E = B$) if the fields are large. So, for an electron in an external wave field, the magnitude of the radiation-resistance force is

$$\begin{aligned} F_{\rm resist} &\approx \frac{2}{3}eE\sqrt{\left(\frac{e^2}{mc^2}\frac{\omega}{c}\right)^2 + \left(\frac{e^3 E}{m^2 c^4}\right)^2} \\ &\approx F_{\rm ext}\sqrt{\left(\frac{r_0}{\lambda\!\!\!^{-}}\right)^2 + \left(\frac{E}{e/r_0^2}\right)^2}, \end{aligned} \tag{7}$$

where $r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ cm is the classical electron radius.

Equation (7) makes physical sense only when the radiation reaction force is smaller than the external force. Here we don't explore whether the length r_0 describes a physical electron; we simply consider it to be a length that arises from the charge and mass of an electron. Rather, we concentrate on the implication of eq. (7) for the electromagnetic field. Then we infer that a classical description becomes implausible for fields whose wavelength is small compared to length r_0, or whose strength is large compared to e/r_0^2.

A. Commentary

The argument related to eq. (7) is that there are classical electromagnetic fields that lead to physically implausible behavior when radiation-reaction effects are included. This does not necessarily imply any mathematical inconsistency in the theory. Indeed, various authors have displayed solutions for electron motion coupled to an oscillator of very high natural frequency [48,95]. Such solutions are well-defined mathematically but appear "physically implausible". Of course, the mathematics might be correct in predicting the physical behavior in an unfamiliar situation. So it becomes a matter of experiment to decide whether the characterization "implausible" corresponds to physical reality or not. The experiments that produce the most influential results are typically those that reveal new phenomena in realms where prevailing theories are "implausible".

Thus far, there is no evidence for the behavior predicted by the classical equations for electrons interacting with waves of frequencies greater than c/r_0. Rather, quantum mechanics is needed for a good description of the phenomena observed in that case, Compton scattering being an early example (sec. 3.1). Laboratory studies of strong-field electrodynamics have been undertaking only recently (sec. 3.2), and deal primarily with effects not anticipated in a classical description.

The argument of sec. 2 can also be considered as a model-independent version of a restriction that Lorentz placed on his derivation of eqs. (1-3) ([6], sec. 37, eq. (73)). Namely, the derivation makes physical sense only if

$$\frac{l}{ct} \ll 1, \tag{8}$$

where l is a characteristic length of the problem, and t is a characteristic time "during which the state of motion is sensibly altered".

Lorentz would certainly have considered the classical electron radius, r_0, as an example of a relevant characteristic length. Hence, for an electron in an electromagnetic wave of (reduced) wavelength $\lambda\!\!\!^{-}$, the characteristic time of the resulting motion is $\lambda\!\!\!^{-}/c$, and Lorentz' condition (8) becomes

$$\frac{r_0}{\lambda} \ll 1, \tag{9}$$

A close variant of the above argument was also given by Planck [7].

In case of a strong field with a long (possibly infinite) wavelength, Lorentz' condition (8) can be interpreted as requiring the change in the electron's velocity to be small compared to the speed of light during the time it takes light to travel one classical electron radius. That is, we require

$$\Delta v = a\Delta t = \frac{eE}{m}\frac{r_0}{c} \ll c, \tag{10}$$

and hence,

$$E \ll \frac{mc^2}{er_0} = \frac{e}{r_0^2}. \tag{11}$$

Thus, we arrive by another (although closely related) path to the conclusion drawn previously from eq. (7). Perhaps because the limiting field strength implied by (11) is extraordinarily large by practical standards, neither Lorentz nor Planck mentioned it explicitly.

In the first sentence of his 1938 article, Dirac [37] stated that "the Lorentz model of the electron...has proved very valuable...in a certain domain of problems, in which the electromagnetic field does not vary too rapidly and the accelerations of the electrons are not too great". However, he does not elaborate on the meaning of "not too great".

Dirac's derivation of the radiation-reaction 4-force was not based on a model of an extended electron, and so the derivation was not subject to Lorentz' restriction (8). But as a co-inventor of quantum mechanics, Dirac cannot have expected his classical results to have unrestricted validity in the physical world.

In the decade after Dirac's 1938 paper, a few works [35,40,42,43,45,49] appeared that commented on the concept of a limiting field strength, typically in classical discussions of electron-positron pair creation. In sec. 3.2 we return to the issue of pair creation, but in a quantum context.

After the discovery of pulsars in 1967 there was a burst of interest in the behavior of electrons in very strong magnetic fields. Several papers appeared in which classical electrodynamics was applied [60,64,65,67,68,70,71,75], often with the intent of clarifying the boundary between the classical and quantum domains. For very large fields, classical solutions to the motion were obtained in which the electron has a damping time constant that is small compared to the period of cyclotron motion. Whether or not such highly damped solutions are "implausible", they are outside ordinary experience. Again, one must perform experiments to decide whether the classical theory is valid in this domain. If such experiments had been possible prior to the development of quantum mechanics, they would have revealed deviations from the classical theory that would have encouraged development of a new theory. Arguments such as those leading to eqs. (7), (9) and (11) would have motivated the experiments.

B. Another Strong-Field Regime

Are there any other domains in which classical electrodynamics might be called into question?

Another interpretation of Lorentz' criterion (8) is that the amplitude of the oscillatory motion of an electron in a wave of frequency ω should be small compared to the wavelength. As is well known (see prob. 2, sec. 47 of Ref. [9]), this leads to the condition that the dimensionless, Lorentz-invariant quantity,

$$\eta = \frac{eE_{\rm rms}}{m\omega c}, \tag{12}$$

should be small compared to one. Parameter η can exceed unity for waves of very low field strength if the frequency is low enough. An interesting result is that the electron can be said to have an effective mass,

$$m_{\rm eff} = m\sqrt{1+\eta^2}, \tag{13}$$

when inside a wave field [127]. An electron in a spatially varying wave experiences a force $\mathbf{F} = -\nabla m_{\rm eff} c^2$ which is often called the "ponderomotive force", but which can be regarded as a kind of radiation pressure for a case where the "reflected" wave cannot be distinguished from the incident wave, and hence as a kind of radiation reaction force in its broadest meaning.

Debates continue regarding energy-transfer mechanisms between electrons and strong classical waves (as represented by a laser beam with $\eta \gtrsim 1$). To what extent can net energy be exchanged between a free electron and a laser pulse in vacuum? Does a classical discussion suffice? Our understanding suggests that quantum aspects should be unimportant even for $\eta \gg 1$ so long as condition (11) is satisfied, but full understanding has been elusive. Detailed discussion of this matter is deferred to a future article.

C. Utility of the Classical Radiation Reaction

Besides provoking extensive discussion on the validity of classical electrodynamics, the radiation reaction has enjoyed some well-known success in classical phenomenology. In particular, the topics of linewidth of radiation by a classical oscillator and resonance width in scattering of waves off such an oscillator show how partial understanding of atomic systems can be obtained in a classical context. Also, the radiation reaction is very important in antenna engineering where the power source must provide for the energy (and momentum, if any) radiated as well as that consumed in Joule losses. It is worth noting that these successes hold where the electron is part of an extended system.

In contrast, the radiation reaction has been almost completely negligible in descriptions of the radiation of free electrons for practical parameters in the classical domain (*i.e.*, outside the domain of quantum mechanics). That this might be the case is the main argument of sec. 2. Section 3 discusses effects of the radiation reaction in the quantum domain.

D. Relativistic Radiation Reaction

For purposes of additional commentary, it is useful to record relativistic expressions for the radiation reaction.

The relativistic version of (1) in 4-vector notation is

$$mc^2 \frac{du^\mu}{ds} = F^\mu_{\rm ext} + F^\mu_{\rm resist}, \tag{14}$$

with external 4-force $F^\mu_{\rm ext} = \gamma(\mathbf{F}_{\rm ext} \cdot \mathbf{v}/c, \mathbf{F}_{\rm ext})$, and radiation-reaction 4-force given by

$$F^\mu_{\rm resist} = \frac{2e^2}{3} \frac{d^2 u^\mu}{ds^2} - \frac{Ru^\mu}{c}, \tag{15}$$

where

$$R = -\frac{2e^2 c}{3} \frac{du_\nu}{ds} \frac{du^\nu}{ds} = \frac{2e^2\gamma^6}{3c^3} \left[\dot{\mathbf{v}}^2 - \frac{(\mathbf{v} \times \dot{\mathbf{v}})^2}{c^2} \right] \geq 0 \tag{16}$$

is the invariant rate of radiation of energy of an accelerated charge, $u^\mu = \gamma(1, \mathbf{v}/c)$ is the 4-velocity, $\gamma = 1/\sqrt{1 - v^2/c^2}$, $ds = c d\tau$ is the invariant interval and the metric is $(1, -1, -1, -1)$.

The time component of eq. (14) can be written

$$\frac{d\gamma mc^2}{dt} = \mathbf{F}_{\rm ext} \cdot \mathbf{v} + \frac{d}{dt} \left(\frac{2e^2 \gamma^4 \mathbf{v} \cdot \dot{\mathbf{v}}}{3c^2} \right) - R, \tag{17}$$

and the space components as

$$\frac{d\gamma m\mathbf{v}}{dt} = \mathbf{F}_{\rm ext} + \frac{2e^2\gamma^2}{3c^3} \left[\ddot{\mathbf{v}} + \frac{3\gamma^2}{c^2} (\mathbf{v} \cdot \dot{\mathbf{v}})\dot{\mathbf{v}} + \frac{\gamma^2}{c^2} (\mathbf{v} \cdot \ddot{\mathbf{v}})\mathbf{v} + \frac{3\gamma^4}{c^4} (\mathbf{v} \cdot \dot{\mathbf{v}})^2 \mathbf{v} \right]. \tag{18}$$

Keeping terms only to first order in velocity, eqs. (17-18) become

$$\frac{dmv^2/2}{dt} = \mathbf{F}_{\rm ext} \cdot \mathbf{v} + \frac{2e^2 \mathbf{v} \cdot \ddot{\mathbf{v}}}{3c^3}, \tag{19}$$

and

$$\frac{dm\mathbf{v}}{dt} = \mathbf{F}_{\rm ext} + \frac{2e^2 \ddot{\mathbf{v}}}{3c^3} + \frac{2e^2 (\mathbf{v} \cdot \dot{\mathbf{v}})\dot{\mathbf{v}}}{c^3}. \tag{20}$$

Equations (17-18) were first given by Abraham [19]. Von Laue [24] was the first to show that these equations can be obtained by a Lorentz transformation of the nonrelativistic results (19-20). The covariant notation of eqs. (14-16) was first applied to the radiation reaction by Dirac [37]. An interesting discussion of the development of eqs. (17-18) has been given recently by Yaghjian [104].

E. Terminology

During a century of discussion of the radiation reaction a variety of terminology has been employed. In this article I use the phrase "radiation reaction" to cover all aspects of the physics of "Rückwirkung der Strahlung" as introduced by Lorentz and Abraham. This usage contrasts with a proposed narrow interpretation discussed at the end of this section.

"Æthereal friction" was the first description by Stewart [16] in 1871, which he used in only a qualitative manner.

In 1873, Maxwell wrote on the "pressure exerted by light" in secs. 792-793 of his *Treatise* [10].

Lorentz used the French word "résistance" in describing eq. (3) when he presented it in 1892, and used the English equivalent "resistance" in his 1906 Columbia lectures [6].

Planck [7,17] also discussed eq. (3), which he described as "Dämpfung" (damping) and "Dämpfung durch Strahlung" (literally, "damping by radiation" but translated more smoothly as "radiation damping"). The term "Strahlungsdämpfung" (radiation damping) does not, however, appear in the German literature until 1933 [34].

Around 1900, Larmor [22] used the terms "frictional resistance" and "ray pressure" to describe a result meant to quantify Stewart's insight, but which analysis has not stood the test of time.

The massive analyses of Abraham were accompanied by the introduction of several new terms. The title of Abraham's 1904 article [19] included the term "Strahlungsdruck" (radiation pressure). This use of the phrase "radiation pressure" can, however, be confused with the simpler concept of the pressure that results when a wave is reflected from a conducting surface [10]. Perhaps for this reason, Abraham also introduced the phrase "Reaktionskraft der Strahlung", which I translate as "radiation reaction force". This appears to be the origin of the phrase "radiation reaction", although in German that phrase remained a qualifier to "Kraft" (force) for many years. The variant "Strahlungsreaktion" (radiation reaction) appeared for the first time in 1933 [35].

Lorentz' 1903 Encyklopädie article [5] introduced the topic of the radiation reaction with the phrase "Rückwirkung des Äthers" (back interaction of the æther). In his 1905 monograph [84], Abraham used the variant "Rückwirkung der Strahlung" (back interaction of radiation, which could also be translated agreeably as "radiation reaction").

In England in 1908, the Adams Prize examiners chose the topic of the radiation reaction, suggesting the cumbersome title "The Radiation from Electric Systems or Ions in Accelerated Motion and the Mechanical Reactions on their Motion which arise from it". The winning essay by Schott [26] adopted much of this title, but in the text Schott refers to "radiation pressure" and indicates that he follows Abraham in this. In his 1915 article, Schott

[27] also used the phrase "reaction due to radiation" and indicated that it was equivalent to his use of the phrase "radiation pressure".

Schott also introduced other terms that seem less than ideal descriptions of the phenomena associated with the radiation reaction. His argument of 1912 [26] is less crisp than one he gave in 1915 [27], so I follow the latter here. Schott considered the rate at which a radiating charge loses energy, and deduced eq. (17) in essentially that form. Schott noted that term R is just the rate of radiation of energy by an accelerated charge, which he described as an "irreversible" process. He then interpreted the term

$$Q = \frac{2\gamma^4 e^2 \mathbf{v}\cdot\dot{\mathbf{v}}}{3c^3}, \tag{21}$$

as an energy stored "in the electron in virtue of its acceleration" and gave it the name "acceleration energy". Schott considered the term $\dot{Q}$ in eq. (17) to be a "reversible" loss of energy.

Insights related to the concept of the "acceleration energy" have been useful in resolving the paradox of whether a charge radiates if its acceleration is uniform, *i.e.*, if $\ddot{\mathbf{v}} = 0$. In this case the radiation reaction force (3) vanishes and many people have argued that this means there is no radiation [25,31,86,124]. But as first argued by Schott [27], in the case of uniform acceleration "the energy radiated by the electron is derived entirely from its acceleration energy; there is as it were internal compensation amongst the different parts of its radiation pressure, which causes its resultant effect to vanish". This view is somewhat easier to follow if "acceleration energy" means energy stored in the near and induction zones of the electromagnetic field [52,93].

Schott's use of the word "irreversible" to describe the process of radiation seems inapt. He may have meant that in a classical universe containing only one electron and an external force field, the radiated energy can never return to the electron. But as noted by Planck [125], "the fundamental equations of mechanics as well as those of electrodynamics allow the direct reversal of every process as regards time". For example, "if we now consider any radiation processes whatsoever, taking place in a perfect vacuum enclosed by reflecting walls, it is found that, since they are completely determined by the principles of classical electrodynamics, there can be in their case no question of irreversibility of any kind". However, "an irreversible element is introduced by the addition of emitting and absorbing substance". Thus, consistent use of the word "irreversible" goes beyond classical electron theory. These views of Planck were seconded by Einstein [126] and elaborated upon in the absorber theory of radiation of Wheeler and Feynman [89].

As another counterexample to the view that radiation is irreversible, a theme of contemporary accelerator physics is that every radiation process can be inverted to produce energy gain, not loss. Hence, there are now devices that accelerate electrons based on inverse Čerenkov radiation, inverse free-electron radiation, inverse Smith-Purcell radiation, inverse transition radiation, *etc.* Uniform acceleration is the inverse of uniform deceleration, and the inverse transformation is especially simple here: since $\mathbf{F}_{\rm resist}$ vanishes, it suffices to reverse the sign of the external force. These inverse radiation processes will be the subject of a future contribution to this Journal.

Schott's use of "irreversible" as applied to the term $-Ru^\mu/c$ of the radiation reaction has not been followed in the German literature. See Ref. [93] for an interesting contrast.

The English phrase "radiation reaction" appears to have been first used by Page in 1918 [30].

In his 1938 paper, Dirac [37] used the phrase "the effect of radiation damping on the motion of the electron". As a consequence, most subsequent papers use "radiation damping" interchangeably with "radiation reaction" as a general description of the subject. Thus, in German there appeared the use of "Strahlungsdämfung" [34] (already in 1933), in French, "force de freinage" [43] (braking force, compare "rayonnement freinage" = Bremsstrahlung), and in Russian the equivalent of "radiation damping" must have been used as well [9]. Dirac seconded Schott's use of the terms "irreversible" and "acceleration energy", and these become fairly common in the English literature thereafter. Indeed, "acceleration energy" becomes "Schott acceleration energy", or just "Schott energy".

The terminology of Schott and Dirac was taken a step further by Rohrlich in 1961 [53] and 1965 [98], who proposed that only the second term in the covariant expression (15) is entitled to be called "the radiation reaction". The first term of (15) is to be called the "Schott term". A motivation for this terminology appears to be that in the case of uniform acceleration, expression (15) vanishes by virtue of cancellation of its two nonzero terms. Then the broadly defined "radiation reaction" (*i.e.*, eq. (15)) vanishes, but the radiation does not (although it takes considerable effort to demonstrate this [52]). The terminology of Rohrlich has the merit that the paradox "how can there be radiation if there is no radiation reaction" is avoided in this case since only the (nonvanishing) term $-Ru^\mu/c$ is called the "radiation reaction".

However, this terminology is at odds with the origins of the subject, which emphasize the low-velocity limit, eqs. (19-20). Here, the radiated momentum enters only in terms of order v^2/c^2, so the direct back reaction of the radiated momentum (*i.e.*, $-Ru^\mu/c$) plays no role in the nonrelativistic limit. Thus, according to Rorhlich's terminology there is no "radiation reaction" in the nonrelativistic limit.

But the original, and continuing, purpose of the concepts of the radiation reaction is to describe how a charge reacts to the radiation of energy when it does not radiate net momentum. To define the "radiation reaction" to be zero in this circumstance is counterproductive.

It appears that the terminology of Rohrlich has been adopted only by three subsequent workers [59,61,102].

III. A QUANTUM INTERPRETATION

To go further, we pass beyond the realm of classical electromagnetism. The remainder of this paper is not a direct consequence of that theory, but considers how only a modest admixture of quantum concepts greatly clarifies the hints deduced by classical argument.

A simple device is to multiply and divide eq. (7) by Planck's constant $\hbar$, which was introduced by him [123] shortly after his work on eq. (1) [7]. Then we can write

$$\begin{aligned} F_{\rm resist} &\approx F_{\rm ext}\sqrt{\left(\frac{e^2}{\hbar c}\frac{\hbar}{mc}\frac{\omega}{c}\right)^2 + \left(\frac{e^2}{\hbar c}\frac{e\hbar}{m^2c^3}E\right)^2} \\ &\approx \alpha F_{\rm ext}\sqrt{\left(\frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}}\right)^2 + \left(\frac{E}{E_{\rm crit}}\right)^2}, \end{aligned} \tag{22}$$

where $\alpha = e^2/\hbar c$ is the QED fine structure constant, $\lambda\!\!\!^{-}_C = \hbar/mc$ is the reduced Compton wavelength of an electron and

$$E_{\rm crit} = \frac{m^2c^3}{e\hbar} = 1.6\times 10^{16}\ \mathrm{V/cm} = 3.3\times 10^{13}\ \mathrm{Gauss} \tag{23}$$

is the QED critical field strength, discussed in sec. 3.2 below.

Thus, our naïve quantum theory (classical electromagnetism plus $\hbar$) leads us to expect important departures from classical electromagnetism for waves of wavelength much shorter than the Compton wavelength of the electron, and for fields of strength larger than the QED critical field strength.

A. The Radiation Reaction and Compton Scattering

Compton scattering [128] was one of the earlier predictions of quantum theory and its observation had an important historical role in widespread acceptance of photons as quanta of light. Compton scattering is distinguished from Thomson scattering of classical electromagnetism in that wavelengths of the photons involved in Compton scattering are not small compared to the Compton wavelength of the electron, when measured in the frame in which the electron is initially at rest. Hence Compton scattering appears to be exactly the kind of example discussed above in which the radiation reaction should be important.

A description of a quantum scattering experiment relates the energy and momentum (plus relevant internal quantum numbers) of the initial state to those of the final state without discussion of forces. Yet, we can identify various correspondences between the quantum and classical descriptions.

In the case of Compton scattering, the initial photon corresponds to the external force field on the electron, while the final photon corresponds to the radiated wave. The quantum changes in momentum (and energy) of the electron in the scattering process can be said to correspond to classical time integrals of force (and of $\mathbf{F}\cdot\mathbf{v}$). Conservation of momentum (and energy) is described in the scattering process by including the back reaction of the final photon on the electron as well as the direct reaction of the initial photon. Thus, the quantum description, which incorporates conservation of momentum (and energy), can be said to include automatically the (time-integrated) effects of the radiation reaction.

Compton scattering is an electromagnetic scattering process in which large changes in momentum (and energy) of the electron are observed (in the frame in which the electron is initially at rest). It can therefore be said to correspond to a situation in which the radiation reaction is large, in agreement with the semiclassical inferences of secs. 2 and 3.

The correspondence between quantum conservation of energy and the classical radiation reaction appears to involve only the second term, $-Ru^\mu/c$, in expression (15) for the radiation-reaction force. Since the electron has constant (though different) initial and final velocities in a scattering experiment, the "acceleration energy" Q of eq. (21) is zero both before and after the scattering, and the equivalent of $\dot{Q}$ cannot be expected to appear in the quantum description (at "tree level", in the technical jargon) of Compton scattering.

Effects corresponding to the near-field "acceleration energy" can be said to occur in quantum electrodynamics in the case of so-called vertex corrections and propagator (mass) corrections, in which a virtual photon is emitted and absorbed by the same electron. These "loop corrections" to the behavior of a quantum point charge implement the equivalent of the self interaction of an extended charge, but diverge when the emission and absorption occur at the same spacetime point. They are the source of the famous infinities of QED that are dealt with by "renormalization". See also sec. 4.1 below.

In the early 1940's, Heitler [129,87] and coworkers [130,131] formulated a version of QED in which radiation damping played a prominent role. Following the suggestion of Oppenheimer [8], they hoped that this theory would provide a general method of dealing with the divergences of QED. By selecting a subset of "loop corrections", they deduced an expression for Compton scattering that corresponds to classical Thomson scattering plus classical radiation damping. While this result is suggestive, it does not appear to be endorsed in detail by subsequent treatments of "renormalization" in QED.

B. The Critical Field

The second term under the radical in eq. (22) may be less familiar. The concept of a critical field in quantum mechanics began with Klein's paradox [132]: an electron that encounters an (electric) potential step appears to be reflected with greater than unit probability in Dirac's theory.

Sauter [133] noted that this effect arises only when the potential gradient is larger than the critical field, $m^2c^3/e\hbar$. The resolution of the paradox is due to Heisenberg and Euler [134], who noted that electrons and positrons can be spontaneously produced in critical fields – a very extreme form of the radiation reaction. The critical field has been discussed at a sophisticated level by Schwinger [135] and by Brezin and Itzykson [136], among many others.

An electron that encounters an electromagnetic wave of critical strength produces not only Compton scattering of the wave photons but also electron-positron pairs. These effects have recently been observed in experiments in which the author participated [137,138].

There is speculation that critical magnetic fields exist at the surface of neutron stars [139–141], and may be responsible for some aspects of pulsar radiation.

Pomeranchuk [40] noted that the Earth's magnetic field appears to have critical strength from the point of view of an electron of energy 10^{19} eV, which energy is at the upper limit of observation of cosmic rays.

The critical field arises in discussion of the radiation, commonly called synchrotron radiation, of electrons moving in circular orbits under the influence of a magnetic field B. If an electron of laboratory energy $\mathcal{E} \gg mc^2$ moves in an orbit with angular velocity ω_0, then the characteristic frequency of the synchrotron radiation is

$$\omega \approx \gamma^3\omega_0, \tag{24}$$

where $\gamma = \mathcal{E}/mc^2$ is the Lorentz boost to the rest frame of the electron. For motion in a magnetic field B, the cyclotron frequency ω_0 can be written

$$\hbar\omega_0 = \frac{mc^2 B}{\gamma B_{\rm crit}}, \tag{25}$$

where $B_{\rm crit} = m^2c^3/e\hbar$. Thus, the characteristic energy of synchrotron-radiation photons (often called the critical energy) is

$$\hbar\omega \approx \mathcal{E}\frac{\gamma B}{B_{\rm crit}}. \tag{26}$$

Hence an electron radiates away roughly 100% of its energy in a single synchrotron-radiation photon if the magnetic field in the electron's rest frame, $B^\star = \gamma B$, has critical field strength. In this regime a classical theory of synchrotron radiation is inadequate [142,143].

Critical electric fields can be created for short times in the collision of nonrelativistic heavy ions, resulting in positron production [144–146].

As a final example of the inapplicability of classical electromagnetism in strong fields, the performance of future high-energy electron-positron colliders will be limited by the disruptive (quantum) effect of the critical fields experienced by one bunch of charge as it passes through the oncoming bunch [147].

IV. DISCUSSION

In this paper I have followed the example of Landau in using the argument of sec. 2 to suggest limitations to the concepts of classical electrodynamics. However, this line of argument appears to have played no role in the early development of quantum mechanics. Rather, the argument was used in the 1930's to suggest that quantum electrodynamics might have conceptual limitation when carried beyond the leading order of approximation [8,134,148–152]. The history of this era has been well reviewed in the recent book by Schweber [153].

While the program of renormalization, of which Lorentz was an early advocate in the classical context [6], appears to have been successful in eliminating the formal divergences that were so troublesome in the 1930's, quantum electrodynamics is still essentially untested for fields in excess of the critical field strength (23) [138]. It still may be the case that this realm contains new physical phenomena that will validate the cautionary argument of Oppenheimer [8].

We close with three examples to stimulate additional discussion. Two are from strong-field electrodynamics; while not necessarily suggesting defects in the theory, they indicate that not all aspects of QED are integrated in the most familiar presentations. The third example considers the case of extraordinarily short wavelengths.

A. The Mass Shift of an Accelerated Charge

We can rewrite the nonrelativistic expressions (1-3) for the radiation reaction as

$$\frac{d}{dt}\left(m\mathbf{v} - \frac{2e^2\dot{\mathbf{v}}}{3c^3}\right) = \mathbf{F}_{\rm ext}, \tag{27}$$

and the relativistic expressions (14-16) as

$$\frac{d}{ds}\left(mc^2u^\mu - \frac{2e^2}{3}\frac{du^\mu}{ds}\right) = F^\mu_{\rm ext} - \frac{Ru^\mu}{c}. \tag{28}$$

These forms suggest the interpretation that the momentum, $m\mathbf{v}$, of a moving charge is decreased by amount $2e^2\dot{\mathbf{v}}/3c^2$ if that charge is accelerating as well [52,53].

If we take mc as the scale of the ordinary momentum, then the effect of acceleration, eE/m, due to an electric field E becomes large in eq. (27) only when $E \gtrsim m^2c^4/e^3 = e/r_0^2$, *i.e.*, when the electric field is large compared to the classical critical field found in sec. 2.

This interpretation has been seconded by Ritus [154] based on a semiclassical analysis (classical electromagnetic field, quantum electron) of the behavior of electrons in a strong, uniform electric field. He finds that the mass of an electron (= eigenvalue of the mass operator) obeys

$$m = m_0\left(1 - \frac{\alpha E}{2E_{\rm crit}} + \mathcal{O}(E^2/E^2_{\rm crit})\right), \tag{29}$$

and remarks on the relation between this result and the classical interpretations (27-28). The mass shift of an accelerated charge becomes large when $E \gtrsim E_{\rm crit}/\alpha = e/r_0^2$, as found above.

The physical meaning of Ritus' result remains somewhat unclear. For example, a mass shift of the form (29) does not appear in Ritus' treatment of Compton scattering in intense wave fields [155] (which treatment agrees with other works), although the effective mass (13) does appear.

B. Hawking-Unruh Radiation

According to Hawking [156], an observer outside a black hole experiences a bath of thermal radiation of temperature

$$T = \frac{\hbar g}{2\pi c k}\ , \tag{30}$$

where g is the local acceleration due to gravity and k is Boltzmann's constant. In some manner, the background gravitational field interacts with the quantum fluctuations of the electromagnetic field with the result that energy can be transferred to the observer as if he(she) were in an oven filled with black-body radiation. Of course, the effect is strong only if the background field is strong.

An extreme example is that if the temperature is equivalent to 1 MeV or more, virtual electron-positron pairs emerge from the vacuum into real particles.

As remarked by Unruh [157], this phenomenon can be demonstrated in the laboratory according to the principle of equivalence: an accelerated observer in a gravity-free environment experiences the same physics (locally) as an observer at rest in a gravitational field. Therefore, an accelerated observer (in zero gravity) should find him(her)self in a thermal bath of radiation characterized by temperature

$$T = \frac{\hbar a^\star}{2\pi c k}\ , \tag{31}$$

where $a^\star$ is the acceleration as measured in the observer's instantaneous rest frame.

This topic has been reviewed in the Journal in Ref. [158].

The Hawking-Unruh temperature finds application in accelerator physics as the reason that electrons in a storage ring do not reach 100% polarization despite emitting polarized synchrotron radiation [159]. Indeed, the various limiting features of performance of a storage ring that arise due to quantum fluctuations of the synchrotron radiation can be understood quickly in terms of eq. (31) [160].

Here we consider a more speculative example. Suppose the observer is an electron accelerated by an electromagnetic field E. Then, scattering of the electron off photons in the apparent thermal bath would be interpreted by a laboratory observer as an extra contribution to the radiation rate of the accelerated charge [161]. The power of the extra radiation, which I call Unruh radiation, is given by

$$\begin{aligned}\frac{dU_{\rm Unruh}}{dt} &= (\text{energy flux of thermal radiation}) \\ &\quad \times(\text{scattering cross section}).\end{aligned} \tag{32}$$

For the scattering cross section, we use the well-known result for Thomson scattering, $\sigma_{\rm Thomson} = 8\pi r_0^2/3$. The energy density of thermal radiation is given by the usual expression of Planck:

$$\frac{dU}{d\nu} = \frac{8\pi}{c^3}\frac{h\nu^3}{e^{h\nu/kT}-1}, \tag{33}$$

where ν is the frequency. The flux of the isotropic radiation on the electron is just c times the energy density. Note that these relations hold in the instantaneous rest frame of the electron. Then

$$\frac{dU_{\rm Unruh}}{dt d\nu} = \frac{8\pi}{c^2}\frac{h\nu^3}{e^{h\nu/kT}-1}\frac{8\pi}{3}r_0^2. \tag{34}$$

On integrating over ν we find

$$\frac{dU_{\rm Unruh}}{dt} = \frac{8\pi^3\hbar r_0^2}{45c^2}\left(\frac{kT}{\hbar}\right)^4 = \frac{\hbar r_0^2 a^{\star 4}}{90\pi c^6}, \tag{35}$$

using the Hawking-Unruh relation (31). The presence of $\hbar$ in eq. (35) reminds us that Unruh radiation is a quantum effect.

This equals the classical Larmor radiation rate, $dU/dt = 2e^2 a^{\star 2}/3c^3$, when

$$E^\star = \sqrt{\frac{60\pi}{\alpha}}E_{\rm crit} \approx \frac{E_{\rm crit}}{\alpha}, \tag{36}$$

where $E_{\rm crit}$ is the QED critical field strength introduced in eq. (23). In this case, the acceleration $a^\star = eE^\star/m$ is about 10^{31} Earth g's.

The physical significance of Unruh radiation remains unclear. Sciama [162] has emphasized how the apparent temperature of an accelerated observed should be interpreted in view of quantum fluctuations. Unruh radiation is a quantum correction to the classical radiation rate that grows large only in situations where quantum fluctuations in the radiation rate become very significant. This phenomenon should be contained in the standard theory of QED, but a direct demonstration of this is not yet available. Likewise, the relation between Unruh radiation and the mass shift of an accelerated charge, both of which become prominent at fields of strength $E_{\rm crit}/\alpha$, is not yet evident.

C. Can a Photon Be a Black Hole?

While quantum electrodynamics appears valid in all laboratory studies so far, which have explored photons energies up to the TeV energy scale, will this success continue at arbitrarily high energies (*i.e.*, arbitrarily short wavelengths)?

Consider a photon whose (reduced) wavelength $\lambdabar$ is the so-called Planck length [163],

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33} \text{ cm}, \tag{37}$$

where G is Newton's gravitational constant. The gravitational effect of such a photon is quite large. A measure of this is the "equivalent mass",

$$m_{\rm equiv} = \frac{\hbar\omega}{c^2} = \frac{\hbar}{c\lambdabar} = \frac{\hbar}{cL_P}. \tag{38}$$

The Schwarzschild radius corresponding to this equivalent mass is

$$R = \frac{2Gm_{\rm equiv}}{c^2} = \frac{2\hbar G\omega}{c^3 L_P} = 2L_P = 2\lambdabar. \tag{39}$$

A naïve interpretation of this result is that a photon is a black hole if its wavelength is less than the Planck length. Among the scattering processes involving such a photon and a charged particle would be the case in which the charged particle is devoured by the photon, which would increase the energy of the latter, making its wavelength shorter still.

At very short wavelengths, electromagnetism and gravitation become intertwined in a manner that requires new understanding. The current best candidate for the eventual theory that unifies the fundamental interactions at short wavelengths is string theory. Variants of the preceding argument are often used to motivate the need for a new theory.

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