Temporary Acceleration of Electrons While Inside an Intense Electromagnetic Pulse

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A free electron can temporarily gain a very significant amount of energy if it is overrun by an intense electromagnetic wave. In principle this process would permit large enhancements in the center-of-mass energy of electron-electron, electron-positron and electron-photon interactions if these take place in the presence of an intense laser beam. Practical considerations severely limit the utility of this concept for contemporary lasers incident on relativistic electrons. A more accessible laboratory phenomenon is electron-positron production via an intense laser beam incident on a gas. Intense electromagnetic pulses of astrophysical origin can lead to very energetic photons via bremsstrahlung of temporarily accelerated electrons.

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The prospect of acceleration of charged particles by intense plane electromagnetic waves has excited interest since the suggestion by Menzel and Salisbury [1] that this mechanism might provide an explanation for the origin of cosmic rays. However, it has generally been recognized that if a wave overtakes a free electron, the latter gains energy from the wave only so long as the electron is still in the wave, and reverts to its initial energy once the wave has past [2–5]. There is some controversy as to the case of a 'short' pulse of radiation, for which modest net energy transfer between a wave and electron appears possible [6–9]. Acceleration via radiation pressure is negligible [10]. It has been remarked that even in the case of a 'long' pulse, some of the energy transferred from the wave to the electron can be extracted if the electron undergoes a scattering process while still inside the wave [3,5]. This paper is an elaboration of that idea. We do not discuss here the observed phenomenon that an electron ionized from an atom in a strong wave can emerge from the wave with significant energy [11].

We consider a plane electromagnetic wave (often called the background wave) with dimensionless, invariant field strength

$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{e\mathcal{E}_{\rm rms}}{m\omega_0 c} = \frac{e\mathcal{E}_{\rm rms}\lambdabar_0}{mc^2}. \qquad (1)$$

Here the wave has laboratory frequency ω_0, reduced wavelength $\lambdabar_0$, root-mean-square electric field $\mathcal{E}_{\rm rms}$, and four-vector potential A_μ; e and m are the charge and mass of the electron, and c is the speed of light.

A practical realization of such a wave is a laser been. Laser beams with parameter η close to one have been used in recent plasma-physics experiments [9] and in high-energy-physics experiments [12,13].

When such a wave overtakes a free electron, the latter undergoes transverse oscillation (quiver motion) with relativistic velocities for $\eta \gtrsim 1$ [2–5,14,15]. The $\mathbf{v} \times \mathbf{B}$ force then couples the transverse oscillation to a longitudinal drift in the direction of the wave. In the nonrelativistic limit, this effect is often said to be due to the 'ponderomotive potential' associated with the envelope of the electromagnetic pulse [3]. The resulting temporary energy transfer to the longitudinal motion of the electron can in principle be arbitrarily large.

A semiclassical description of this process exists as well. A quantum-mechanical electron inside a classical plane wave can be described by the Volkov solutions to the Dirac equation [16,17]. Such electrons are sometimes described as 'dressed' and can be characterized by a quasimomentum

$$q = p + \epsilon k_0, \qquad (2)$$

where the invariant ϵ is given by

$$\epsilon = \frac{m^2\eta^2}{2(p \cdot k_0)}, \qquad (3)$$

with $(p \cdot k_0)$ being the 4-vector product of the 4-momenta p of the electron and k_0 of a photon of the background wave. The factor ϵ need not be an integer, and can be thought of as an effective number of wave photons 'dragged' along with the electron as a result of a small difference between the large rates of absorption and emission (back into the wave) of wave photons by the electron. (Strictly speaking, the wave used in the Volkov solution is classical and hence contains no photons.) As a result, the electron inside the wave has an effective mass, $\overline{m}$, that is greater than its free mass m [3]:

$$\overline{m}^2 = q^2 = m^2(1 + \eta^2). \qquad (4)$$

From a classical view, the quasimomentum q is the result of averaging over the transverse oscillations (quiver motion) of the electron in the background wave. When discussing conservation of energy and momentum in the classical view, both transverse and longitudinal motion of the electron must be considered; but in a quantum analysis, quasimomentum is conserved and no mention is made of the classical transverse oscillations.

Throughout this paper the background wave propagates in the $+z$ direction, and the 4-momentum of a photon of this wave is written

$$k_0 = (\omega_0, 0, 0, \omega_0). \tag{5}$$

From now on we use units in which c and $\hbar$ equal one.

We first consider a relativistic electron moving along the $+z$ axis with 4-momentum

$$p = (E, 0, 0, P) = \gamma m(1, 0, 0, \beta), \tag{6}$$

where E and P are the energy and the momentum of the electron prior to the arrival of the wave, $\beta \approx 1$ is the electron's velocity and $\gamma = 1/\sqrt{1-\beta^2} \gg 1$. Then

$$(p \cdot k_0) = \omega_0(E - P) = \frac{m^2\omega_0}{E + P}. \tag{7}$$

so

$$\epsilon = \frac{\eta^2(E + P)}{2\omega_0} \approx \frac{\gamma m \eta^2}{\omega_0}, \tag{8}$$

where the approximation holds for a relativistic electron. For a wave of optical frequencies (such as a laser), $\epsilon \gg 1$. The quasienergy, q_0, is then large:

$$q_0 = E(1 + \eta^2). \tag{9}$$

The electron has been accelerated from energy E outside the wave to energy $E(1 + \eta^2)$ inside the wave. Since η can in principle be large compared to 1, this acceleration can be very significant.

Can we take advantage of this acceleration in a high-energy-physics experiment?

Suppose the electron p collides head-on with a positron p', all inside the background wave. The positron 4-momentum is then

$$p' = (E', 0, 0, -P'), \tag{10}$$

where $E' \gg m$ in the relativistic case. Then

$$(p' \cdot k_0) = \omega_0(E' + P') \approx 2E'\omega_0. \tag{11}$$

The corresponding quasimomentum is

$$q' = p' + \epsilon' k_0, \tag{12}$$

where

$$\epsilon' = \frac{m^2\eta^2}{2(p' \cdot k_0)} \approx \frac{m^2\eta^2}{2E'\omega_0}. \tag{13}$$

The factor ϵ' is not large in general, and the energy of a relativistic positron (or electron) moving against an optical wave is almost unchanged.

However, the center-of-mass (cm) energy of the e^+e^- system is increased when the collision occurs inside the background wave. The cm-energy squared is

$$s = (q + q')^2 \approx 4EE'(1 + \eta^2), \tag{14}$$

which is enhanced by a factor $1 + \eta^2$ compared to the case of no background wave.

For example, the Z^0 boson could be produced in e^+e^- collisions with 33-GeV beams, if the collision took place inside a background wave of strength $\eta = 1$.

Of course, the background wave Compton scatters off the positron beam at a high rate if $\eta \gtrsim 1$, which results in substantial smearing of the energy of that beam. In practice, the cm-energy enhancement by a background wave would not be very useful in e^+e^- or ee collisions.

Note, however, that Compton scattering is insignificant when the background wave and electron move in the same direction, unless the wave is extraordinarily strong. By an application of the Larmor formula in the (classical) rest frame of the electron, we find that the fraction of the electron's (laboratory) energy radiated in one cycle of its motion in the wave is of order $\alpha\eta^2(\omega_0/E)$, where α is the fine-structure constant.

Suppose instead that the electron collides head-on with a high-energy photon of frequency ω and 4-momentum

$$p' = k = (\omega, 0, 0, -\omega). \tag{15}$$

Then eq. (14) holds on substituting ω for E'; the cm-energy squared is again enhanced by the factor $1 + \eta^2$.

The background wave can, of course, interact directly with the high-energy photon to produce e^+e^- pairs, but if $4\omega\omega_0 < m^2(1 + \eta^2)$, the pair-production rate is much suppressed [13]. Thus, there is a regime in which e + photon collisions in a strong background wave are cleaner than e^+e^- or ee collisions in the wave.

In practice, we could get the high-energy photon from Compton scattering of the background wave off an electron beam. One might not want to backscatter the wave off a positron beam because of 'backgrounds' from $e^+e^- \to Z^0$.

A physics topic of interest would be the reaction

$$k + e^- \to W^- + \nu, \tag{16}$$

which proceeds via the triple-gauge-boson coupling γWW and whose angular distribution is sensitive to the magnetic moment of the W boson [18,19]. The 'background' process

$$k + e^- \to Z^0 + e^- \tag{17}$$

could be suppressed by suitable choice of polarization of the electron and background wave.

For electron beams of 46.6 GeV as at the Stanford Linear Accelerator Center, green laser light backscatters into photons of energies up to about 30 GeV. Thus if the laser had $\eta = 1$, the cm energy would extend up to 106 GeV, well above the threshold for reactions (16-17).

However, the enhancement factors $1+\eta^2$ in the electron energy, eq. (9), and in the cm-energy squared, eq. (14), of ee or electron-photon collisions are very much dependent on the idealization that the background wave is highly collinear with the electron.

We reconsider the preceding, but now suppose that the electron makes angle $\theta \ll 1$ to the z axis, The 4-momentum of the electron is

$$p = (E, P\sin\theta, 0, P\cos\theta), \tag{18}$$

and

$$(p \cdot k_0) = E\omega_0(1 - \beta\cos\theta) \approx \frac{m\omega_0}{2\gamma}(1 + \gamma^2\theta^2). \tag{19}$$

As a consequence, the (quasi)energy of the electron inside the wave is now

$$q_0 = p_0 + \frac{m^2\eta^2\omega_0}{2(p \cdot k_0)} \approx E\left(1 + \frac{\eta^2}{1 + \gamma^2\theta^2}\right), \tag{20}$$

which reduces to eq. (9) as θ goes to zero. However, if $\theta > \eta/\gamma$, then the electron is hardly accelerated by the background wave.

Electrons of present interest in high-energy physics typically have energies in the range 1-1000 GeV, corresponding to $\gamma \approx 10^3$-10^6. This places very severe requirements on the alignment of the background wave with the electron beam. Indeed, the angular divergence of electrons beams is often larger than $1/\gamma$, so that no alignment of the background wave could impart large energy enhancements to the entire beam.

Furthermore, optical waves with $\eta \approx 1$ can only be obtained at present in focused laser beams for which the characteristic angular spread is $\Delta\theta \gtrsim 0.1$. So even if the central angle of the beam could be aligned to better than $1/\gamma$, only a very small fraction of the beam power would lie within a cone of that angle.

We also note that for the quasimomentum q to be meaningful, the electron must have resided inside the strong background field for at least one cycle. A relativistic electron moves distance $2\gamma^2(1 + \eta^2)\lambda_0$ while the background wave advances one wavelength relative to the electron [20]. However, the strong-field region of a focused laser is characterized by its Rayleigh range, which is typically a few hundred wavelengths when $\eta \approx 1$. Further, the transverse extent of the (classical) trajectory is of order $\gamma\eta\lambda_0$. Hence, in present laser systems the strong-field region is not extensive enough that the energy transfer (9) could be realized for $\gamma \gtrsim 10$.

While physical consequences of the temporary acceleration of relativistic electrons inside an intense laser beam may be difficult to demonstrate, there is also interest in the case where the electron is initially at rest, or nearly so, such as electrons ionized from gas atoms by the passage of the background laser pulse [11].

An interesting process is so-called trident production,

$$e + A \rightarrow e' + A' + e^+e^-, \tag{21}$$

of an electron-positron pair in the interaction of an ionization electron with a nucleus A of a gas atom. For a very heavy nucleus A, its final state A' has a different momentum but the same energy. Then the initial electron must provide the energy to create the e^+e^- pair as well as that for the final electron. The least energy required is when all three final-state electrons and positrons are at 'rest' (*i.e.*, they have zero net longitudinal momentum; they must always have quiver motion when they are in the wave). Thus, the minimum total quasienergy of the final-state electrons and positrons is $3\overline{m}$.

We conclude that the quasienergy q_0 of the initial electron must be at least $3\overline{m}$ for reaction (21) to occur.

If the electron is at rest prior to the arrival of the background wave its 4-momentum is

$$p = (m, 0, 0, 0). \tag{22}$$

As the electron is overtaken by a wave of strength η and 4-momentum given by (5), it takes on quasimomentum

$$q = (m(1 + \eta^2/2), 0, 0, m\eta^2/2) \equiv (\overline{m}\gamma, 0, 0, \overline{m}\gamma\beta_z). \tag{23}$$

Thus, the net longitudinal velocity of the electron inside the wave is $\beta_z = q_z/q_0 = (\eta^2/2)/(1 + \eta^2/2)$. As expected, inside a very strong wave the electron can take on relativistic longitudinal motion.

We could have trident production while the electron is still in the wave if the quasienergy $q_0 = m(1 + \eta^2/2)$ exceeds $3\overline{m}$. For an electron initially at rest, this requires $\eta \geq \sqrt{16 + 12\sqrt{2}} = 5.74$.

The trident process is still possible within a wave with $\eta < 5.74$ provided the electron has quasienergy $q_0 \geq 3\overline{m}$. This might arise, for example, because of acceleration of the electron by the plasma-wakefield effect [21].

It is conceivable that the electron creates the pair in a linearly polarized wave at a phase when its (classical) kinetic energy is high, but the final electron and the pair all appear with a lower kinetic energy corresponding to some other phase of the wave. This can't happen if the interaction takes place at a well-defined point, since the phase of the wave is a unique function space and time. It might occur if the final particles 'tunnel' to another space-time point before appearing, and the instantaneous kinetic energy is lower at that point.

However, we will find shortly that such tunneling is not consistent with energy conservation. To be as definite as possible, we consider ordinary energy along the classical trajectories, rather than quasimomentum. The latter is taken into account in the sense that the electron and positron are not created at rest, but with the transverse velocities appropriate to phase of the background wave at the spacetime point at which the pair appears. It is sufficient to consider only those trajectories with zero average momentum (*i.e.*, zero quasi-3-momentum).

For circular polarization of the background wave, the electron trajectory is a circle in the plane perpendicular to the z axis, with radius a/ω_0, velocity $\beta = a$ and Lorentz factor

$$\gamma_{\rm circ} = \frac{1}{\sqrt{1 - a^2}} = \sqrt{1 + \eta^2}, \tag{24}$$

where parameter a is given by

$$a^2 = \frac{\eta^2}{1+\eta^2}, \qquad 0 \le a^2 \le 1. \tag{25}$$

For a background wave that is linearly polarized in the x-direction, the trajectory can be parametrized as [4,14]

$$x = \sqrt{2}\frac{a}{\omega_0}\sin\delta, \qquad z = \frac{a^2}{4\omega_0}\sin 2\delta, \tag{26}$$

where $\delta = \omega_0\tau\sqrt{1+\eta^2} = \omega_0\tau/\sqrt{1-a^2}$, and τ is the proper time. Expression (26) describes the well-known figure-8 trajectory. Now $dx/d\tau = (dx/dt)(dt/d\tau) = \gamma\beta_x$, so $\gamma^2 = 1 + \gamma^2\beta^2 = 1 + (dx/d\tau)^2 + (dz/d\tau)^2$. We find that

$$\gamma_{\rm lin} = \frac{1 + \frac{1}{2}[a^2 - (\omega_0 x)^2]}{\sqrt{1-a^2}}. \tag{27}$$

From expression (26) for the x-trajectory we see that $0 \le (\omega_0 x)^2 \le 2a^2$, so

$$\gamma_{\rm min} = \frac{1+\eta^2/2}{\sqrt{1+\eta^2}}, \qquad \text{and} \qquad \gamma_{\rm max} = \frac{1+3\eta^2/2}{\sqrt{1+\eta^2}}. \tag{28}$$

These values surround the result that $\gamma_{\rm circ} = \sqrt{1+\eta^2}$ always for circular polarization. For small η, $\gamma_{\rm min} \approx 1 + \eta^4/8$, $\gamma_{\rm max} \approx 1 + \eta^2$, and $\gamma_{\rm circ} \approx 1 + \eta^2/2$; for large η, $\gamma_{\rm min} \approx \eta/2$, $\gamma_{\rm max} \approx 3\eta/2$, and $\gamma_{\rm circ} \approx \eta$.

Suppose an electron interacts with a nucleus at the place where its Lorentz factor is $\gamma_{\rm max}$ and reappears along with an electron-positron pair at a location where $\gamma_{\rm min}$ holds at that moment. The nucleus absorbs the excess momentum of the initial electron. Conservation of (ordinary) energy requires that $\gamma_{\rm max} = 3\gamma_{\rm min}$. But this is not satisfied for any value of η according to (28). That is, the hypothetical tunneling process is not possible under any circumstances.

In sum, even when in a background wave an electron can produce positrons off nuclei only if the electron has sufficient longitudinal momentum that the corresponding (quasi)energy is three times the (effective) electron mass.

We close by returning to the astrophysical context that began the historical debate on acceleration by intense electromagnetic waves. Gunn and Ostriker [22] have given an extensive discussion the possibility of electron acceleration in the rotating dipole field of a millisecond pulsar, where the field strength η can be of order 10^{10}. Their argument does not primarily address free electrons overtaken by a wave, but rather electrons 'injected' or 'dropped at rest' into the wave. Neutron decay is a candidate process for 'injection'. In very strong fields ($\eta \gg 1$) this decay takes place together with the absorption by the electron (and proton) of a very large number of wave photons, so that the electron is created with (quasi)energy $\approx m\eta^2/2$ (compare eq. (23)) [23]. Because the fields of the pulsar fall off as $1/r$ where (coincidentally) $r_{\rm pulsar} \approx \lambda_0$, the wavelength of the rotating dipole radiation, the field is in some sense 'short', and the electron may emerge with some fraction of the large energy it had at the moment of its creation.

An example closer to the theme of the present paper would be an electron that is overtaken by the intense electromagnetic pulse of a supernova (or other transient astrophysical occurrence, perhaps including gamma-ray bursters) and thereby temporarily accelerated to energy $m\eta^2/2$. Such pulses could have significant fields at optical frequencies, where the transverse scale, $\eta\lambda_0$, of the motion of accelerated electrons is less than the Chandrasekhar radius for $\eta < 10^{10}$. In general, the electron has low energy before and after the passage of the pulse. However, high-energy photons can arise via bremsstrahlung of the electron when it interacts with a plasma nucleus while still in the pulse. In this view, the primary astrophysical evidence of temporarily accelerated electrons would be high-energy photons which, of course, could transfer some of their energy to protons and other charged particles in subsequent interactions.

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