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Nonlinear Boundary-Value Problem for the Heat Mass Transfer Model of W. Fushchych

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Abstract

We find a numerical-analytic solution of a nonlinear boundary-value problem for the biparabolic differential equation, which describes the mass and heat transfer in the model of W. Fushchych.

We consider the following generalized law of heat conduction:

$$q + \tau_q \frac{\partial q}{\partial t} = -\lambda \frac{\partial}{\partial x} \left(T + \tau_T \frac{\partial T}{\partial t} \right), \tag{1}$$

where λ is the heat conduction coefficient, T is the temperature, q is the heat flux, τ_q, τ_T are the relaxation coefficients.

Using (1), we obtain the following partial differential equation of heat conduction in relaxing media [1]:

$$\left(L_1 + R \frac{\partial}{\partial t} L_2 \right) T(x, t) = 0, \tag{2}$$

where

$$L_1 \equiv \frac{\partial}{\partial t} - a \frac{\partial^2}{\partial x^2}, \qquad L_2 \equiv \frac{\partial}{\partial t} - \frac{\tau_T}{\tau_q} \frac{\partial^2}{\partial x^2},$$

a is the temperature conduction coefficient.

Let us take note of the fact that equation (2) satisfies no requirements of symmetry [2].

A more accurate mathematical model of heat and mass transfer consider in [2]. This model based on the following law of heat conduction:

$$q + \tau_r \frac{\partial q}{\partial t} = -\lambda \frac{\partial}{\partial x} (T + 2\tau_r L_1 T), \tag{3}$$

where τ_r is the relaxation time.

In accordance with (3), we obtain the biparabolic differential equation for heat conduction [2–4]

$$(L_1 + \tau_r L_1^2) T(x, t) = 0. \tag{4}$$

It is well known that equation (4) is invariant with respect to the Galilei group $G(1, 3)$ [2–4].

We shall consider the process of burning in active media in accordance with the bi-parabolic equation (4). Solution of this problem reduces to solution of a boundary–value problem of the following form:

$$(L+\tau_r L^2)T(x,t)=\left(1+\tau_r\frac{\partial}{\partial t}\right)Q(T), \tag{5}$$

$$T(0,t)=T''_{xx}(0,t)=T(l,t)=T''_{xx}(l,t)=0, \tag{6}$$

$$T(x,0)=\theta(x),\qquad T'_t(x,0)=\psi(x), \tag{7}$$

where $\theta(x),\psi(x)$ are known functions, $Q(T)=T^m(m\geq 1)$ is the potential of heat sources, l is the scale length.

Introducing the integral transform

$$\overline{T}_n(t)=\int\limits_0^l T(x,t)\sin(\lambda_n x)\,dx \qquad \left(\lambda_n=\frac{n\pi}{l}x\right), \tag{8}$$

we obtain the following Cauchy prroblem:

$$\begin{aligned}&\tau_r\frac{d^2\overline{T}_n(t)}{dt^2}+\nu_n^{(1)}\frac{d\overline{T}_n(t)}{dt}+\nu_n^{(2)}\overline{T}_n(t)=\overline{\Phi}(t),\\ &\overline{T}_n(0)=\alpha_n,\qquad \overline{T}_n{}'(0)=\beta_n,\end{aligned} \tag{9}$$

where

$$\nu_n^{(1)}=1+2\tau_r\lambda_n^2,\qquad \nu_n^{(2)}=\lambda_n^{(2)}(1+\tau_r\lambda_n^2), \tag{10}$$

$$\left\{\begin{array}{c}\alpha_n\\ \beta_n\end{array}\right\}=\int\limits_0^l\left\{\begin{array}{c}\varphi(x)\\ \psi(x)\end{array}\right\}\sin(\lambda_n x)\,dx, \tag{11}$$

$$\overline{\Phi}_n(t)=\int\limits_0^l\left(1+\tau_r\frac{\partial}{\partial\tau}\right)Q(T(x,t))\sin(\lambda_n x)\,dx. \tag{12}$$

A solution of the system of equations (9) may be written in the form

$$T(x,t)=q(x,t)+\int\limits_0^t\int\limits_0^l\left(1+\tau_r\frac{\partial}{\partial\tau}\right)Q(T(\xi,\tau))K(\xi,x;\ t-\tau)\,d\xi d\tau, \tag{13}$$

where

$$q(x,t)=\frac{2}{l}\sum_{n=1}^{\infty}\exp(-\lambda_n^2 t)\left(1+\tau_r\left(\frac{\beta_n}{\alpha_n}+\lambda_n^2\right)\left(1-e^{-\frac{t}{\tau_r}}\right)\right)\alpha_n\sin(\lambda_n x), \tag{14}$$

$$K(\xi,x;t-\tau)=\frac{\tau_r}{l}\left(1-\exp\left(-\frac{t-\tau}{\tau_r}\right)\right)\sum_{n=1}^{\infty}\exp\left(-\lambda_n^2(t-\tau)\right)\sin(\lambda_n x)\sin(\lambda_n\xi). \tag{15}$$

In order to construct solution (13), we can use the projective method [5]. Finally, we have the system of nonlinear algebraic equations

$$T_{j\mu} = F_{j\mu} + \sum_{i=1}^{N}\sum_{k=1}^{M} c_{ikj\mu} T_{ik}^{m}, \qquad j = \overline{1,N};\ \mu = \overline{1,M}, \tag{16}$$

where

$$F_{j\mu} = \frac{1}{\Delta x \Delta t}\int\limits_{t_{\mu-1}}^{t_\mu} dt \int\limits_{x_{j-1}}^{x_j} \mu(x,t)dx, \qquad c_{ikj\mu} = \frac{1}{\Delta t}\int\limits_{t_{\mu-1}}^{t_\mu} dt \int\limits_{t_{k-1}}^{t_k} G_{ij}(t-\tau)d\tau,$$

$$\mu(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} \exp(-\lambda_n^2 t)\sin(\lambda_n x)\left(1 + \tau_r\left(1 - e^{-\frac{t}{\tau_r}}\right)\left(\frac{\beta_n}{\alpha_n} + \lambda_n^2 - \frac{\gamma_n}{2}\right)\right),$$

$$\gamma_n = \int\limits_0^l \theta_i^m(\xi)\sin(\lambda_n \xi)d\xi,$$

$$G_{ij}(t-\tau) = \frac{\tau_r}{l}\int\limits_{x_{j-1}}^{x_j} dx \int\limits_{x_{i-1}}^{x_i} \sum_{n=1}^{\infty}\left(1 - \lambda_n^2 + \left(\frac{1}{\tau_r} - 1 + \lambda_n^2\right)\exp\left(-\frac{t-\tau}{\tau_r}\right)\right) \times$$
$$\times \exp(-\lambda_n^2(t-\tau))\sin(\lambda_n x)\sin(\lambda_n \xi)d\xi.$$

After having solved the system of equations (16), we get the solution of the problem by (5)–(7) . The results of calculations show that the values of temperature determined by (5)–(7) essentially differ from values determined by (2), (6), (7). In so doing, a regular change of temperature is similar in both modells. In particular, the blow–up regime [6] exists.

References

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