Towards centimetric geoid over FCC area: application of the Stokes-Helmert method

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Outline

• Introduction
• Basic geoid concepts
• Data acquisition and control dataset
• Evaluation of existing geoid models
• Computation of a gravimetric geoid model
• Conclusion and outlook
The FCC Project

- The goal of the FCC is to push the energy and intensity frontiers of particle colliders, with the aim of reaching collision energies of 100 TeV in the search for new physics.
- ~91km ring-shaped underground tunnel located beneath the French departments of Haute-Savoie and Ain, and the Swiss canton of Geneva.
- An electron-positron collider (FCC-ee) then a proton-proton collider (FCC-hh)
- 5 Years feasibility studies started in 2020
Introduction

The FCC Project

- Extends the geographic area of the CERN facilities from 70 km2 to 1000 km2
- Need new geodetic infrastructure
  - Civil engineering including tunnelling
  - Installation and maintenance of infrastructure above/below ground
- Monitoring crustal deformations
- Connecting facilities to others (locally, nationally, internationally)
- Accelerator alignment
Basic geoid concepts

- The geoid is the equipotential surface of the Earth’s gravity field which best fits global mean sea level extended under the continents.

- Required to transform ellipsoidal heights into orthometric heights.

- The deflection of the vertical (the slope of the geoid) need to be accurately known to align the machine in a Euclidean plane.

- Gravity equipotential surfaces are not parallel inside the Earth: We need to compute a “pseudo-geoid” at the level of the machines.
Basic geoid concepts

• Several different methods of computation
  • Least Squares Modification of Stokes’ Formula; Remove-Compute-Restore; Stokes-Helmert

• Computational approaches differ due...
  • Input data possibilities
  • Treatment of input data errors
  • Selected Stokes integral and modifications
  • Methods and rigorousness on the treatment of topography
  • Selection of integration area and modification parameters
  • Selection and usage of supplementary low and high-frequency data

• Different approaches should produce comparable solutions (and they don’t)

• Different methods will be tested during the feasibility study
Data acquisition and control dataset

Deflection of the vertical

- \( \xi \) (North-South component)
- \( \eta \) (East-West component)

GNSS-levelling
Data acquisition and control dataset

**Geodetic control profile**

- 83 determination of the DoV
- 8.8 km cross-border levelling
- 36 GNSS levelling stations
- 51 relative gravity observations

30 GNSS-levelling stations
Evaluation of existing geoid models

- Significant difference between the models
  - Range of the difference over an enlarged FCC area in the order of 30 to 40 cm (5 cm standard deviation)

- GNSS-levelling
  - RAF20 (normal height): standard deviation of the residuals was 3.5 cm, ranging from -4.4 to +12.6 cm.
  - CHGeo2004 (orthometric height): standard deviation of the residuals was 2.9 cm, ranging from -10.9 to +1.2 cm.

<table>
<thead>
<tr>
<th>Model</th>
<th>Δξ [arcsec] North-South component</th>
<th>Δη [arcsec] East-West component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>EGM2008</td>
<td>EIGEN-6C4</td>
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<tr>
<td>Average</td>
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<tr>
<td>St. Dev</td>
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<td>2.58</td>
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<tr>
<td>Minimum</td>
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<td>-2.47</td>
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<tr>
<td>Maximum</td>
<td>6.73</td>
<td>6.51</td>
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</table>
Computation of a gravimetric geoid

Data evaluation

- 700,000 gravity (free-air anomaly) points available
Computation of a gravimetric geoid

Data evaluation

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- Filtering process to remove wrong points
  - Control between overlapping sources
  - Evaluation against global models
  - Evaluation of the positioning precision
  - Reduction of the density of points apart of more than 1 degree (~111 km)
Computation of a gravimetric geoid

Data evaluation

- 700,000 gravity (free-air anomaly) points available
- Filtering process to remove wrong points
  - Control between overlapping sources
  - Evaluation against global models
  - Evaluation of the positionning precision
  - Reduction of the density of points apart of more than 1 degree (~111 km)
- Filtered dataset 95,316 points
Computation of a gravimetric geoid

External dataset

- Digital terrain model
  - Medium resolution DTM (1" spacing ~ 30 m) SRTM1 (void filled with ACE2 values)
  - Low resolution ACE

- Global gravity field models
  - GECO (up to degree/order 2160 ~ 5’)
  - GOCO 05s (up to degree/order 280)

- Earth density
  - UNB_TopodensT_2v01
Computation of a gravimetric geoid

Methodology and computation

- **Stokes Helmert method as implemented in the SHGeo+ package** (Fugro Geoid, University of New Brunswick)

- **2 approaches**
  - Constant and varying lateral density

- **Find optimal combination of the parameters**
  - Stokes integration radius (Near zone radius)
  - Modification degree (exclude gravity field below the modification degree)
  - Maximum degree/order of the reference field (Far zone effect computation)
Evaluation of the results (varying density)

- Deflection of the vertical

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \xi$ [arcsec]</th>
<th>$\Delta \eta$ [arcsec]</th>
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</thead>
<tbody>
<tr>
<td>Average</td>
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<td>-0.01</td>
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<tr>
<td>St. Dev</td>
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<td>1.38</td>
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<tr>
<td>Minimum</td>
<td>-3.21</td>
<td>-3.24</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.18</td>
<td>4.67</td>
</tr>
</tbody>
</table>

- Geoid heights: $-8.1 \text{ cm} < \Delta n < +2.2 \text{ cm}$; $\sigma = 2.5 \text{ cm}$
Evaluation of the results (constant density)

- Deflection of the vertical

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<th>Δη [arcsec]</th>
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<tbody>
<tr>
<td>Average</td>
<td>1.18</td>
<td>-0.09</td>
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<tr>
<td>St. Dev</td>
<td>1.53</td>
<td>1.18</td>
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<tr>
<td>Minimum</td>
<td>-2.03</td>
<td>-3.01</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.89</td>
<td>2.67</td>
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- GNSS levelling: -3.3 cm < Δn < +4.7 cm; σ = 1.8 cm
Computation of a gravimetric geoid

Comparison with swiss model CHGeo2004

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<tr>
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Conclusion and outlook

• **Conclusion**
  • Final geoid (1’ resolution (1.8 km)) determined under the assumption of a constant mass density
  • Precision of 1.9 cm relative to GNSS/Level points
  • No improvement compared to CHGeo2004 with respect to DoV

• **Outlook**
  • Densifying gravity data (Jura mountain)
  • Evaluation with GNSS-levelling results of the geodetic profile
  • PhD thesis ongoing at ETH Zürich studying other approaches