Abstract

Direct levelling is performed extensively in the CERN surveying processes for accelerator elements’ alignment and positioning. LGC software (Logiciel Général de Compensation) computes results and associated statistics for observations used by surveyors. Traditionally, direct levelling campaigns are computed separately from the planimetry, using simple differences of height called in LGC *DVER observation between two measured points. LGC provides another observation model, called *DLEV, using offsets to a horizontal plane at the position of the station. The level planimetric position must therefore be known or be computable by additional observations. This more rigorous model allows a better integration in 3D computation involving other instruments, such as laser trackers or total stations.

This article reviews the traditional method and analyses the use of geo-referenced levelling stations for accelerator element surveying. The influence of the station planimetric position precision and of the geoid model used are studied and illustrated with a practical case.

INTRODUCTION

Commonly, surveyors perform the accelerator alignment using numerous sources of geometrical observations and dedicated processes. It ensures the accelerator components are well enough placed with respect to their nominal positions. At CERN, the standard procedure to determine the position of an accelerator requires total station, ecartometry system and direct levelling observations. Traditionally, the computations are performed in a 2D+1 fashion. The final component altitudes are given by a dedicated 1D computation using only the direct levelling observations.

Nowadays, new instruments, such as non leveled laser trackers, arise as standards for large-scale metrology. The alignment requirements are more demanding and mixing several geometrical observation sources has become crucial. 3D Monitoring systems are also in vogue and require the development of new sensors, both on the hardware and software sides. Those factors contribute to the need to combine different types of observations in one 3D computation. For example, around the ATLAS, CMS, ALICE and LHCb experiments, the low beta quadrupoles are monitored in 3D using a mix of Wire Positioning Systems (WPS), Hydrostatic Levelling Systems (HLS) and Distance Offset Measurement Sensors (DOMS). The low beta quadrupoles are also measured using total stations, direct levelling and stretched wire offset measurements, by the surveyors in charge of the Large Hadron Collider (LHC) machine alignment. The computation performed by the machine surveyors is typically done in 2D+1. Computation result differences can emerge in the colliding area between 2D+1 and 3D measurement. Mixing the two processes is essential for the quality of the LHC element’s positioning.

One missing piece to transform those 2D+1 computations to 3D is to handle the direct levelling observation correctly in LGC software. LGC is a CERN in-house developed software maintained and upgraded since the mid-90s. LGC computes results and associated statistics using a least-square model for all the observations used by surveyors. The computations are done in 3D in the CERN Coordinate System (CCS) [6], an XYZ cartesian reference system. Due to the size of the CERN accelerator complex, geoid models are needed and LGC handles the three used nowadays. Those 3 models are the SPHERE, dating from the SPS construction in the ’70s, the CG1985, dating from the LEP construction in the ’80s and the CG2000, dating from the LHC construction in the 2000s. The geoid models are, for instance, used in the observation made by a total station to establish its vertical vector in the CCS reference system. New observation models are continuously added in LGC following the demands of the surveyors and the development of innovative sensors or instruments.

Two different mathematical models are available for direct levelling observations. The first one is called *DVER (for “Distance VERTicale”). *DVER is a simple difference of height between two points in the least square adjustment. LGC provides another observation model, called *DLEV, using offsets to a horizontal plane at the position of the station. The levelling station planimetric position must therefore be known or be computable by additional observations. This more rigorous model allows a better integration in 3D computation involving other instruments, such as laser trackers or total stations.

This article reviews the traditional method and analyses the use of geo-referenced levelling stations for accelerator element surveying. The influence of the station planimetric position precision and of the geoid model are studied and illustrated with a practical case.

OBSERVATION MODEL DEFINITIONS

*DVER

The current computing process for pure levelling operations uses the *DVER observation (Fig. 1) [7]. Its associated observation model is:

\[ dH = H_{TS} - H_{ST} - C_dH \] (1)
where \( dH \) is the height difference between the level station and a target, typically the position on which the grade rode is placed; \( H_T \) and \( H_S \) are the heights, respectively of the station and of the target; and \( C_{dH} \) is a potential calibrated constant value to add to the observation.

The stochastic model attached to the *DVER is:

\[
\sigma^2_{dH} = \sigma^2_m (2)
\]

where \( \sigma_m \) represents the standard deviation value of the observed height difference. It is not directly the observation standard deviation on the grade rod but a combination of several parameters.

The stochastic model attached to the *DLEV is:

\[
\sigma^2_{dz} = \sigma^2_{\text{vert dist}} + D_{Tg} \cdot \sigma_{\text{ppm}}^2 + \sigma^2_{\text{grade rod offset}} \quad (4)
\]

where \( \sigma_{\text{vert dist}} \) is the precision (1\( \sigma \)) of the observation from the levelling instrument on the grade rod; \( D_{Tg} \) is the horizontal distance from the station to the target point; \( \sigma_{\text{ppm}} \) is the part of the observation error due to \( d_{Tg} \) in part per million; \( \sigma_{\text{grade rod offset}} \) is the precision (1\( \sigma \)) of \( c_{dz} \).

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**FROM FIELD OBSERVATION TO FINAL COMPUTATION**

**Current workflow**

For a pure levelling campaign from point A to B, field observations are performed using TSUNAMI [8] or SMART [9] software (“The Survey Unified Notepad for Alignment”; “Smartphone Field Book”). Those solutions register the station and target names, the observation on the grade rod placed on the target point, a possible constant to add to the observation and a potential horizontal distance between the station and the grade rod. The horizontal distance is only recorded if a digital level capable to measure it automatically is used. The software offer control indicators and operational checks to ensure observation correctness. Amongst others, the standard procedure expects a so-called forth and back levelling, a first levelling is made from point A to point B, then from point B to point A. It allows having two independent height differences that can be used as a quality indicator for the levelling campaign. Once the fieldwork is done, the observations are transferred to the CERN Surveying database via its interface GEODE [10]. GEODE corrects the observation of any instrument calibration and of the earth curvature with the formula:

\[
\text{ObsCorrected} = \text{ObsRaw} - \frac{\text{Distance}^2}{2 \cdot R_{\text{Earth}}} \quad (5)
\]

where \( \text{ObsCorrected} \) is the earth curvature corrected observation on the grade rod; \( \text{ObsRaw} \) is the raw observation on the grade rod; \( \text{Distance} \) is the measured distance from the station to the observed point; and \( R_{\text{Earth}} = 6371000 \text{m} \) is a common radius of the earth.

The final computation is done in the office, extracting the needed observations and reference point coordinates via GEODE. This extraction creates a file readable by LGC with the *DVER keyword for each observation made - i.e. transforming corrected observations in height differences between a station and a measured point. The computation is made in a cartesian reference system. The reference points’ provisional values are expressed in altitude over the chosen geoid model. The result gives each point altitude.
Drawbacks

The process is well-proven but has a couple of drawbacks.

In the described computation routine, in a nutshell, the least-square adjustment is only adding and subtracting difference in heights along a common Z-axis. The reference point Z-axis coordinate is given as a height over a geoid model. Therefore, the result of all the survey points is given also as heights in the same geoid model. This process requires a separate computation for the direct levelling observations. It is possible to integrate *DVER observations in 3D computation and using other instruments, in particular, gravity-bound ones. But as shown in the following test case, additional precautions need to be considered.

The horizontal distance from the station to the measured point is not always available or observed by the levelling system. In some operations, non and earth-sphericity-corrected observations are mixed. In those cases, the distances between the station and the different observed points must be reasonably equidistant. The equidistant condition is roughly respected in the standard levelling process defined by CERN surveyors.

The a-priori precision (1σ) on the observations entering in the LGC least-square adjustment are those of a height difference, not of the raw observation measurement. The observation precision determined for the couple instrument-grade rod cannot be used explicitly but needs to be combined in a height difference a-priori precision.

The *DLEV keyword is more rigorous on these points and offers more parameters to adjust during the LGC least-square computation process. But the *DLEV main drawback is that the planimetric position of the station needs to be known or computable with the observations available in the LGC input file.

TEST CASE

A test case is set to highlight the differences between the current process and the use of geo-referenced levelling stations. It also shows the requirements regarding how accurate a station planimetric position should be known to have an acceptable impact on the computation result.

Data Presentation

The data used is a subset of a levelling campaign of the LHC Arc from point 1 (ATLAS) to 8 (LHCb) dating from EYETS 2016-2017 (Extended Year-End Technical Stop). The observed points are fiducials on the accelerator components. The horizontal distances from the station to an observed point are available and uninterrupted during roughly 2.65 km of forth and back levelling. The observation scheme as shown in Fig. 3 is regular with 7 points per station, 2 connecting points per station and with the same observation scheme for the forth and back paths. One reference is fixed in the computation for this study. Using the standard method (*DVER keywords), the forth and back computations taken separately give consistent results. Neither gross errors nor instrument biases are detected in the data.

Differences *DVER/*DLEV

Two separate computations are made using *DVER and *DLEV. Only one fixed reference height is chosen close to point 8. The levelling station positions are determined by
multilateration using the horizontal measured distances on the fiducials. They are reasonably well known in 3D for the multilateration routine. A planimetric precision (1\sigma) between 5 and 11 mm is reached. The *DLEV computation requires a geoid model to compute the local vertical for each station. Figure 4 shows negligible differences between the *DVER computation and the SPHERE/CG1985 *DLEV computation. Those models are simple and can be equated to an ellipsoid. With the use of the more complex and precise CG2000 geoid, differences up to 0.5 mm appear after 2.65 km of levelling.

![Figure 4: Differences in height of geo-referenced SPHERE, CG1985 and CG2000 *DLEV computations to a reference *DVER computation, along the 2.65 km of the levelling operation](image)

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A planimetric precision (1\sigma) between 5 and 11 mm is thus reached. The *DLEV computation requires a geoid model to compute the local vertical for each station. Figure 4 shows negligible differences between the *DVER computation and the SPHERE/CG1985 *DLEV computation. Those models are simple and can be equated to an ellipsoid. With the use of the more complex and precise CG2000 geoid, differences up to 0.5 mm appear after 2.65 km of levelling.

The *DLEV computation with geoid undulation corrections on the observations is equivalent to the *DLEV result. As the station position should anyway be known or computable, the *DLEV takes into account natively the vertical vector variations in the LGC least-square process. In contrast, *DVER computation observations should be corrected (Fig. 6) at the level of the GEODE extraction where the earth-curvature and calibration rectifications are applied.

![Figure 6: Geoid model dependant corrections to apply to every observation to integrate to *DVER computation with respect to the levelling path](image)

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The point 8 to 1 area (Fig. 5) is not where the biggest and noisiest undulations are happening along the LHC tunnel. The primary consideration for the test case selection was to have uninterrupted measured horizontal distances station-to-point available for a long stretch of forth and back levelling. The area between point 2 and 4 is a good candidate for further studies.

**Station position determination**

The station position should be known in any case to improve the direct levelling process. The station position sets the orientation of the horizontal plane (see Fig. 2) in the CCS. This paragraph covers the influence of position precision (1\sigma) on the height determination in the same test case.

**Error propagation model**

In the conditions of an arc levelling (i.e. forth/back observations, two connecting points per station, almost regular distances from stations to connecting points), a simplified model can be used following the Fig. 3. It evaluates the influence of the knowledge (at a 1\sigma precision) of the levelling station position on the height determination along the 2.65 km levelling path of the test case. The simplified model combines the two stations forth/back (placed roughly at the same position and observing the same connecting points) into one levelling “pseudo-station” (called “Cell”) having only one virtual connecting point. The error propagation contribution for each Cell on
the connecting point height difference can be modelled as follow:

\[ \sigma_H^2 = N_{\text{Cell}} \cdot \left( \frac{\sigma_{\text{Obs}}^2}{2} + D_{eq} \cdot \sigma_{\text{Vertical Deviation}}^2 \right) \]  

(6)

\[ D_{eq} = \frac{D_1^2 + D_2^2}{2} \]  

(7)

\[ \sigma_{\text{Vertical Deviation}} = \arctan \frac{\sigma_{\text{Station}}}{R_{\text{Earth}}} \]  

(8)

where \( N_{\text{Cell}} \) is the number of the cell, \( \sigma_{\text{Obs}} \) is the precision (1\( \sigma \)) of the observation; \( D_1 \) and \( D_2 \) are the distances from the station to respectively the first and the second connecting point; \( R_{\text{Earth}} \) is the radius of the earth and \( \sigma_{\text{Station}} \) is the horizontal distance accuracy (1\( \sigma \)) with respect to the station true position.

This model supposes that \( \sigma_{\text{Station}} \) is directed along the station-to-point observation vector. The \( D_{eq} \) value supposes that distances from the two back and forth connecting points are equivalent for the whole levelling path. Both of those conditions are mainly fulfilled in this observation scheme resulting in the following parameters:

- \( N_{\text{Cell}} \): From 1 to 113 cells, each cell distant by 24.9 m from the previous \( \sigma_{\text{Obs}} = 0.02 \text{mm} \)
- \( D_1 \): 16 m and \( D_2 \): 11 m
- \( R_{\text{Earth}} \): 6.371 000 m
- \( \sigma_{\text{Obs}} = 0.02 \text{mm} \)
- \( \sigma_{\text{Station}} \): Variable from 0 to 5 m for comparison purposes

With the station positions known perfectly, the height of a point should be determined with a 0.15 mm precision after 2.65 km of levelling. As an example, if the station position is only known with a 2m precision (1\( \sigma \)), the altitude determination after 2.65 km of levelling goes up to 0.16 mm.

**Test Method**  
Simulations are performed to assess the influence of the altitude with regard to different station planimetric position precisions (1\( \sigma \)). A *DLEV* computation serves as the baseline for the analysis. This reference computation uses the test case observation scheme, station fixed at their true value determined by multilateraion and perfect simulated observations. Errors are then added to the direct levelling observations respecting a gaussian distribution of mean = 0 and standard deviation = 0.02 mm. Variations are then introduced on the station positions: either radially, longitudinally (using an accelerator pseudo bearing, see Fig. 3) or directionless. Two types of variations are inserted: gaussian distribution of mean = 0 and of standard deviation varying from 0 m to 5 m; systematic errors ranging from 0 m to 5 m. All results are presented as an average and standard deviation on the basis of 300 simulations.

**Result**  
Directionless gaussian distribution errors are introduced in the station coordinates. A station can move freely around its true value. The noise added has no defined direction. Figure 8a shows that the altitudes are almost not influenced by the precision of the station. The standard deviation on the altitudes is depreciated. For this configuration, the decrease in the altitude precision (1\( \sigma \)) is of about 0.04 mm after 2.65 km of levelling if the station is known within a standard deviation of 5 m.

![Figure 7: Height precision (1\( \sigma \)) with respect to the levelling path for several station planimetric precisions (1\( \sigma \))](image)

![Figure 8: (a) Average and (b) standard deviations of the fiducials altitudes (300 simulations) with several directionless variations on the station positions and \( \sigma_{\text{Obs}} = 0.02 \text{mm} \)](image)

The same behaviour is observed with the introduction of only longitudinal gaussian distributed errors: limited influence on the altitude values but the precision is affected (Fig. 9). For this configuration: the decrease in the altitude precision (1\( \sigma \)) is about 0.06 mm after 2.65 km of levelling if the station is known within a standard deviation of 5 m.
The decrease is slightly more than in the directionless case because of the pure longitudinal factor.

In contrast, longitudinal systematic variations applied on the station coordinates affect immensely the fiducials altitude (Fig. 10). Longitudinal systematic errors must be avoided. Radial gaussian distributed and systematic errors do not alter the altitudes and the precision of the fiducials.

CONCLUSION AND PERSPECTIVE

For the integration of levelling operations within a 3D computing environment, the use of a geo-referenced levelling station is advised. The keyword *DLEV in LGC takes into account natively the geoid model considerations. The study focused on a test case where the vertical vector deviation has limited variations along the levelling path in the CG2000 geoid model. Even in those conditions, the datum influences long direct levelling operations. Additional studies should be performed where the vertical vector variations are more radical in the LHC tunnel.

The precision at which the station should be known depends on the observation scheme. For a long fairly straight direct levelling campaign presented as a test case: the radial position determination is of second order; the longitudinal position should be reasonably known; any longitudinal systematism should be strongly avoided. Even in simpler cases with less redundancy and a less robust observation scheme, a station position known within a 2 m radius is transparent to the levelling result.

Nowadays, this position is not registered by surveyors in the field. The operation in an accelerator environment is always time sensitive so an easy, fast and robust solution must be implemented. A multilateration of the position with distances observed by digital level and introduced as a horizontal distance in the LGC least square would be enough. But, the use of a digital levelling system is not always possible, so alternatives have to be provided too. For instance, in some areas where the observation scheme can be standardized and reproduced roughly every year, the position of the station on the ground could be marked. The mark is then surveyed in 3D and used over the years. Another solution could be to assess the position of the station using the neighbouring element known, for example with the help of a geographic information system. Additional tests will be performed to choose the more adapted solutions on the practical, software and statistical sides.

REFERENCES