

Theory Techniques for Precision Physics Snowmass 2021 TF06 Topical Group Report

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Submitted to the Proceedings of the US Community Study
on the Future of Particle Physics (Snowmass 2021)

Abstract

The wealth of experimental data collected at laboratory experiments suggests that there is some scale separation between the Standard Model (SM) and phenomena beyond the SM (BSM). New phenomena can manifest itself as small corrections to SM predictions, or as signals in processes where the SM predictions vanish or are exceedingly small. This makes precise calculations of the SM expectations essential, in order to maximize the sensitivity of current and forthcoming experiments to BSM physics. This topical group report highlights some past and forthcoming theory developments critical for maximizing the sensitivity of the experimental program to understanding Nature at the shortest distances.

1 Executive summary

Theoretical techniques for precision physics are the backbone of a successful program in particle physics. When combined with advances at current colliders such as the LHC or Belle II, and with planned future experiments, they allow determinations of fundamental parameters of the Standard Model (SM) to unprecedented precision and probe beyond SM (BSM) physics to very high scales. Potential dark matter candidates, explanations of the hierarchy puzzle, and many other discoveries could be made, even if the mechanisms underlying them are at the tens of TeV scale or even beyond.

To reach the precision level needed to maximize the discovery potential for new physics, a plethora of challenges in theoretical physics must be overcome. Calculations beyond leading order in the perturbative expansions of QCD and the electroweak theory are needed. These require the investigation of new mathematical structures that describe the multi-loop integrals with several mass scales that occur when considering the Higgs, W and Z bosons, and top and bottom quarks. A common theme between collider and flavor physics is the need for rigorous predictions that combine fixed-order perturbative calculations, resummations, and nonperturbative ingredients. Subtraction schemes must be extended to handle the cancellation of infrared singularities to the $N^3\text{LO}$ level in perturbation theory, the order that is expected to become the precision standard for colliders after the LHC. Resummation of large logarithms involving kinematic or phase space parameters requires understanding effective field theories (EFTs) at subleading order in the power expansions. The parton distribution functions for both nucleons and B mesons must be determined to a precision that matches calculations of the parton-level cross sections. Given that current experimental data are consistent with the SM except for a few anomalies under intense investigation, the understanding of experimental results will inevitably include interpreting them as constraints in the framework of the SM effective field theory (SMEFT). Precision in the SMEFT expansion is needed to properly derive bounds on new physics, and investigations into higher-order corrections in both the loop and EFT expansions are ongoing. Improvements in Monte Carlo event generators and parton showers are also critical to maximize the impact of high-precision perturbative calculations, and provide a closer description of experimental data in terms of the hadronic degrees of freedom measured in experiments.

This topical group addresses the challenges that arise whenever precision theoretical predictions are required. Its focus lies close to experiment, since many questions of precision are driven by experimental opportunities, and is also close to more formal theory and mathematics due to

the issues confronted when going to new orders of perturbation theory or extending EFTs to new domains. During the Snowmass process the TF06 working group investigated the current state-of-the-art in several fields, including higher-order corrections in both fixed-order and resummed perturbation theory, the incorporation of a more faithful description of QCD into parton shower simulations, and calculations within the SMEFT beyond the leading dimension-6 order. Several whitepapers summarized the status of critical areas relevant to this working group and outlined where future advances are needed. In this report the main activities and findings of the TF06 working group are presented, with attempts to put each topic in the broader context of how they are important to the high energy physics community.

2 Precision collider phenomenology

2.1 Introduction

More than 150 inverse femtobarns of integrated luminosity have been delivered by the LHC to the ATLAS and CMS experiments. A high luminosity run of the LHC is expected to deliver integrated luminosity reaching inverse attobarns, as are several future collider experiments under discussion. Detectors at these experiments excel at reconstruction, capable of distinguishing signatures overwhelmed by background such as Higgs boson decays to photons and muons. The systematic and statistical errors at the LHC and other modern experiments now often reach the percent level or below thanks to these experimental advances. With such exquisite data coming now, and with future machines such as an electron-ion collider expected to continue this trend, probes of new physics to very high scales become possible. Potential explanations for puzzling aspects of the SM can be found even if the mechanisms responsible are at the tens of TeV level and possibly even higher.

This experimental data challenges the theoretical community to predict observables within and beyond the Standard Model (SM) precisely enough to maximize the discovery potential of new physics. The technical difficulties associated with calculations to the N³LO level in QCD in the presence of multiple mass scales, which lead to new mathematical structures such as iterated elliptic integrals, must be confronted. Resummation of large logarithms, including for novel observables such as jet substructure measurements, will require improved understanding of effective field theory (EFT) beyond the usual leading-power approximation. Extractions of parton distributions will need to be upgraded to the N³LO level in order to match the expected precision of partonic scattering cross sections. Precision calculations within the SM effective field theory (SMEFT) framework will inevitably be part of the future HEP program given that no new particles beyond the SM have yet been discovered. Even in the case of discovery the interpretation of the new states within the SMEFT can help guide future experimental searches by indicating which measurements can help unravel the new state's identity. Improvements in Monte Carlo event generators and parton showers are also critical to fully exploit the impact of high precision perturbative calculations and provide a closer realization of the actual events in terms of the hadronic degrees of freedom measured in experiments.

During the Snowmass process the TF06 working group studied numerous challenges that arise in the process of deriving precise predictions for high- p_T physics. These include the path to N³LO calculations in QCD beyond $2 \rightarrow 1$ processes and the required technical advances,

the study of factorization theorems beyond the leading power, improvements in parton showers, and studies at the dimension-8 level in the SMEFT. The work of TF06 is incorporated into several whitepapers that explain these issues, as well as describe the current progress toward their solutions and what will be needed by the future HEP program. The main activities and findings of the precision collider phenomenology thrust are summarized below, drawing upon the white papers prepared for the Snowmass process.

2.2 Predictions at NNLO and beyond

The experimental precision for a host of benchmark processes is approaching the few-percent level. For some measurements in particular clean channels such as Drell-Yan, the claimed experimental uncertainties are below one percent. Such a precision imposes enormous demands on theory. A wide variety of effects must be brought under control to claim a theoretical precision at this level, ranging from exquisite control over parton distribution functions [1] to the possibility of non-perturbative corrections. One critical component of this program is the calculation of the perturbative hard scattering cross sections to the requisite order in perturbation theory. A famous example of the importance of such higher-order calculations is the cross section for inclusive Higgs boson production at the LHC, for which the higher-order QCD corrections increase the result by nearly a factor of three.

The current standard for cross section predictions at the LHC is next-to-next-to-leading order (NNLO) in perturbative QCD for $2 \rightarrow 2$ scattering processes without internal mass scales in the contributing loops. For a host of processes the $N^3\text{LO}$ corrections in perturbative QCD are available [2–8]. The challenge facing the theory community is to extend these computations of hard scattering cross sections at $N^3\text{LO}$ to include jet processes and more differential observables. Numerous issues must be addressed to achieve this challenge: the computation of the relevant three-loop integrals, understanding the basis of functions needed to describe these corrections, the extension of infrared subtraction schemes to handle triply-unresolved limits, to name only a few. Together with this challenge will also come enormous progress in our understanding of quantum field theory. New understanding of amplitudes, a systematic study of elliptic functions, and novel uses of effective field theory to facilitate such computations have already resulted from this quest.

A roadmap highlighting the current status of the field, and discussing what future progress is needed, was written as part of TF06 working group activities [9]. A brief description of the highlights is given below. The reader is referred to the white paper [9] for more details and complete lists of references.

- Detailed studies of the renormalization and factorization scale dependences of the $N^3\text{LO}$ Higgs boson and Drell-Yan cross sections have been performed. One hope of this study is to gain intuition more generally into the size and impact of scale variations at this order in perturbation theory, since these are often used to estimate uncalculated higher-order corrections. An interesting result of this study in the Drell-Yan neutral-current cross sections is that the NNLO scale variation band does not contain the $N^3\text{LO}$ central value for a range of invariant masses, due to an intricate interplay between the contributing partonic channels [6–8]. This behavior is not observed for inclusive Higgs boson production [2, 10].

- One is inexorably led to the issue of masses in loops when calculating predictions at higher orders due to the presence of numerous mass scales in the SM, most notable the W , Z , Higgs and top quark masses at the LHC. Even very simple two-loop self-energy diagrams with masses exhibit complicated functional dependences no longer described by the multiple polylogarithms present in the massless case. The elliptic integrals that appear in massive loop diagrams have received significant attention recently [11, 12], and algorithms for their efficient numerical implementation have been proposed [13]. The functional dependences appearing at higher loop order has been reviewed extensively in the following Snowmass whitepaper [14].
- Each of the individual components of a higher-order calculation is separately divergent due to infrared singularities (IR). In virtual corrections the IR singularities are always manifest, but in real-radiation corrections they only appear explicitly after integrating over QCD radiation. Since one is usually interested in fully exclusive results, a method to extract the IR singularities before integration is required. Techniques used in LHC applications for processes with final-state jets included antennae subtraction [15, 16], sector-improved residue subtraction [17–20], and N -jettiness subtraction [21, 22]. Other important techniques with numerous important LHC applications include q_T -subtraction [23] and projection-to-Born [24]. Significant effort has been devoted to extending these techniques to the N^3 LO level, and in the case of q_T and N -jettiness subtractions many of the ingredients are now known to the required level [25–28].
- A wide variety of other issues must be confronted when attempting to push the precision frontier to N^3 LO, ranging from practical issues such as getting codes to run within the available computational resources [29], to the possibility of collinear factorization violations [30] and enhanced power corrections for observables not inclusive over QCD radiation [31]. These and other issues are surveyed in a TF06 white paper [9]. In particular, it is important to investigate whether the standard collinear factorization approach still applies at this order in perturbation theory. The complication in establishing collinear factorization for hadron collider processes is to show the cancellation of phases arising from soft gluon exchange between the different hard scattering directions. Despite the complications surveyed in [9], it is expected that for a host of hadron collider processes (annihilation into electroweak particles, inclusive jet production, and heavy-quark production), the cancellation of these contributions does occur, and therefore that the standard collinear factorization does hold [32]. A more intricate issue that arises is the non-cancellation of these phases in the presence of disparate energy cuts in different regions of the final-state phase space. Such cuts can lead to “super-leading” logarithmic enhancements and can complicate the standard factorization picture [32].

Achieving N^3 LO precision for collider observables requires a determination of the PDFs at the same order. Already the estimated uncertainty coming from missing N^3 LO effects from PDFs is a significant component of the Higgs cross section error budget [2]. The progress needed to advance PDFs to the requisite level, including a summary of what pieces of the full N^3 LO DGLAP evolution are still missing, was extensively reviewed in the following Snowmass whitepaper [1].

2.3 Resummation for future colliders

Computations of the hard scattering cross section at fixed orders in perturbation theory are sufficient for many applications, but not for all situations. In multi-scale problems, when one mass scale is very different than the others, large logarithms of the scale ratios can appear. Denoting these scales generically as Q_B and Q_S with $Q_S \ll Q_B$, if $\alpha_s \ln^2(Q_B/Q_S) \sim 1$, then fixed-order perturbation theory doesn't converge. To get a reliable prediction one must resum these large logarithms to all orders. These logarithms can be thought of as an incomplete cancellation between real and virtual corrections. In virtual corrections one integrates over all loop momenta. The real radiation phase space can be restricted by experimental cuts. We can schematically write this situation as

$$\frac{1}{\epsilon} + \int_0^{Q_S} \frac{dQ}{Q} \left(\frac{Q}{Q_B} \right)^{-\epsilon} = \ln \frac{Q_B}{Q_S} + \mathcal{O}(\epsilon) \quad (1)$$

where ϵ is the dimensional-regularization parameter that controls infrared divergences. The first term arises from the virtual corrections, while the second term denotes the integration of the real-emission corrections over some final-state phase space. If the phase space is severely restricted so that $Q_S \ll Q_B$, this logarithm becomes large and must be resummed.

There are numerous examples in particle physics where the above situation occurs, most notably in the threshold limit where little energy is available for emission into partonic radiation [33, 34], or in the low transverse momentum limit where all additional radiation is restricted to be collinear to the beam direction [35–37]. These classic examples of all-orders results in QCD have been reformulated in the recent past using the language of soft-collinear effective field theory (SCET) [38–41]. Other recent applications of these techniques include the small- x limit of QCD and the study of jet substructure. Until recently most applications considered only the leading-power term in the small ratio Q_S/Q_B . Terms suppressed by powers of this ratio were dropped. There has recently been significant interest in understanding these sub-leading power terms. This interest is driven by both phenomenological applications, led by the increased precision of the LHC and the expectation of a future electron ion collider, and also by intrinsic theoretical interest in understanding the structure of the resulting factorization theorems at subleading power. Recent results in this field were surveyed as part of the TF06 activities and summarized in the white paper [42]. The reader is referred to the white paper [42] for more details and complete lists of references.

- The study of threshold corrections beyond the leading-power limit has been considered recently within both a direct diagrammatic approach [43, 44] and using SCET [45–47]. It was shown to be possible to resum the next-to-leading power (NLP) corrections to leading-logarithmic (LL) accuracy for color-singlet production processes such as Drell-Yan or Higgs production as well as deep inelastic scattering. The numerical impact of the NLP-LL resummation for these processes is approximately the same as the NNLL resummation to the leading-power terms, indicating that these corrections are relevant for LHC phenomenology [44, 48].
- Another interesting example where resummation in NLP terms is necessary is the case of mass-suppressed amplitudes, such as the b -quark mediated contribution to gluon-fusion

Higgs production. In this case the amplitude goes like $(\alpha_s/\pi)(m_b^2/m_H^2)\ln^2(m_H^2/m_b^2)$. The expansion parameter in QCD is $(\alpha_s/\pi)\ln^2(m_H^2/m_b^2) \sim 1$. Even though the amplitude is suppressed by the ratio m_b^2/m_H^2 , as the Higgs program enters the precision phase this is becoming a limiting uncertainty in the theoretical prediction. The leading-logarithmic resummation of these corrections has been achieved [49, 50], resulting in a factor of two reduction of the uncertainty estimate coming from these terms. A SCET analysis of this process was also performed, allowing for a NLL resummation of mass-suppressed effects [51, 52].

- Jet substructure techniques have been applied to both SM measurements and beyond-the-SM searches over the past decade, and a host of jet grooming techniques have been developed to remove soft radiation effects that are difficult to account for from first principles in QCD (for reviews please see [53, 54]). More understanding will be needed in the next phase of the LHC, where detectors will be optimized for the first time to perform precision jet substructure measurements, and at an EIC, where precision measurements of jets across a wide range of energies is expected to be possible. Many novel applications of jet substructure at an EIC, including its use to unravel transverse-momentum distributions and nuclear effects in electron-nucleus collisions, have been proposed. To make full use of these opportunities higher accuracy at both leading and sub-leading power will eventually be required.
- The small Bjorken- x limit of QCD gives insight into the behavior of gluons within nucleons. In this small- x region, the gluon density grows dramatically and enters the nonlinear regime. The color-glass condensate effective theory (CGC) properly resums logarithms of x that appear in this region [55, 56]. Higher-order cross sections in this effective theory are notoriously difficult to compute, with negative results appearing in the physical regions of phase space. An approach to solve these issues by coupling SCET to the CGC formalism was recently developed [57], which will be important when the gluon saturation regime is more thoroughly studied at the EIC.

2.4 The need for precision PDFs

All hadron collider predictions require understanding of the parton distribution functions that describe how to take a parton of a given momentum fraction from a hadron. Specializing to the collinear parton distribution functions most commonly needed for high-energy collider physics, the PDFs have functional dependence on both the energy scale μ characteristic of the process under consideration, and on the parton momentum fraction parameterized in terms of the Bjorken momentum fraction x . The functional dependence of the PDFs on μ is perturbative, and is governed by the DGLAP evolution equations. The dependence on x is non-perturbative. Although there has been significant recent activity in attempting to calculate PDFs using lattice techniques [58], for the practical purpose of predicting collider physics cross sections they are extracted from experimental data. This leads to the following three issues that must be addressed in order to have PDFs computed to the needed level for LHC and future collider predictions: the perturbative DGLAP evolution of the PDFs must be calculated to match the precision of the hard scattering cross section; the hard scattering cross sections from which the PDFs are

extracted from data must be known to NNLO or N³LO depending on the desired accuracy; the experimental data used in the extraction must have small enough uncertainties to match the above theoretical uncertainties, and must also be sufficiently broad enough to fix the functional dependences of all PDFs on x .

The status of PDFs for collider physics, both the current status and desired future improvements, was addressed during the Snowmass process as a joint effort of both TF06 and the energy frontier [1]. Several findings that address the precision issues raised in the previous paragraph are discussed below.

- The DGLAP evolution of PDFs is currently known to the NNLO level. To match the desired N³LO level for hard scattering cross sections the calculation of the evolution kernels must be pushed to one order higher. This is currently under active investigation [59–61], and in fact the evolution of the flavor-singlet PDF combination at this order is now possible [62, 63]. The incorporation of heavy quarks into this framework through the requisite matching calculations, needed to properly evolve the PDFs through mass thresholds where heavy flavors become active, is almost complete to N³LO [64].
- All modern PDF sets come with uncertainty estimates. These uncertainties account only for the propagation of experimental errors from the data sets used, and do not account for any theoretical uncertainties on the hard scattering cross sections that enter the global fits. There has been recent attempts to incorporate theoretical uncertainties from scale variations into PDF fits by dividing the processes that enter into categories based on their underlying structure and assuming correlated errors for processes with similar structures, allowing a theory covariance matrix to be formed [65, 66]. The initial findings indicate NLO PDF determinations are not shifted much by theory uncertainties, but more detailed investigations are expected in future global fits.
- Much of our current detailed understanding of PDFs comes from the deep-inelastic scattering experiments at HERA. In the coming decade new DIS data from an electron-ion collider (EIC) is expected, with integrated luminosities reaching an order of magnitude higher than HERA, and with the possibility of polarizing both electron and proton beams [67]. The EIC is expected to cover larger- x and smaller- Q^2 values than HERA, providing new probes of higher-twist effects that could affect PDF determinations. The anticipated statistical and systematic errors are expected to reach the percent-level or lower. Initial simulations indicate that uncertainties in the valence quark sector could decrease up to 80% at high- x , while the small- x sea quark region could see uncertainty reductions up to 50% [67, 68].

2.5 Future prospects for parton showers

Both fixed-order and resummed predictions describe collider observables that are inclusive, or differential in a few variables. They are formulated in terms of partonic degrees of freedom. Parton-shower event generators are needed to provide a closer realization of the actual events in terms of hadronic degrees of freedom measured in experiment. Parton showers use the infrared properties of QCD to generate multiple emissions of softer partons starting from a given hard process. At scales $\mu \sim \Lambda_{QCD}$ these partons are combined into hadrons using string or cluster

models. Parton-shower event generators such as HERWIG [69], PYTHIA [70] and SHERPA [71] are heavily used by experimental collaborations in their analyses and form an indispensable tool for understanding events at high-energy colliders.

There has been numerous theoretical improvements in parton showers over the past years, resulting in programs more faithful to the underlying QCD theory. Matching parton showers to higher-order fixed-order QCD calculations is now available for color-singlet production processes at NNLO, and for arbitrary processes at NLO. Merging of processes with differing numbers of final-state jets is now available at NLO. Algorithms to go beyond the leading-color approximation, as well as to include full spin correlations in order to properly describe azimuthal distributions, are becoming available. One of the advantages of parton-shower simulations is that they inherently resum the singular emissions of QCD, and therefore do not suffer from divergences at kinematic endpoints that often plague fixed-order results, while simultaneously being more flexible than the bespoke resummation calculations described in the previous section. There is an ongoing effort to quantify to exactly what order different parton shower algorithms provide resummation for different observables. At future very high energy colliders it is expected that multiple emissions of electroweak gauge bosons will become important, and there is ongoing work to incorporate electroweak radiation into existing showers.

Parton showers were reviewed as part of the Snowmass process within both the TF06 working group [72], focusing on theoretical issues needed for future developments, and very extensively as well within the energy frontier working group EF05 [73]. More details regarding current status and necessary future developments can be found within those whitepapers.

2.6 Theoretical developments in the SMEFT beyond dim-6

The main goal of the precision program in particle physics is to uncover signatures of new physics to reveal a more fundamental theory beyond the SM. One way to guide experimental searches for new phenomena is to propose complete models of new physics with ultraviolet completions. Examples of this approach include supersymmetric versions of the SM with various mechanisms for soft SUSY breaking, or technicolor theories. Another approach, which is becoming increasingly used as LHC searches continue to return null results, is to write down an effective theory that incorporates a wide variety of possible UV models. One advantage of this method is that it is agnostic to high-energy details of UV completions and instead focuses on the dynamics at lower-energies that are relevant for current experimental searches. These EFTs may contain new light degrees of freedom that solve issues of the SM such as the need for a dark matter candidate, or they may contain only the observed SM states, implying a mass gap between the SM states and any new physics. If the discovered Higgs particle arises completely from an underlying SU(2) doublet, the resulting EFT is known as the Standard Model EFT (SMEFT). When the Higgs boson does not necessarily arise completely from an SU(2) doublet, the resulting theory is known as the Higgs EFT (HEFT) or the electroweak chiral Lagrangian. The HEFT is extensively reviewed in a number of papers [74–76]. Since there is currently no evidence for a deviation of Higgs properties from those predicted for an SU(2) doublet, we focus here on the SMEFT.

The SMEFT Lagrangian is constructed containing only the SM degrees of freedom, and assuming that all operators satisfy the SM gauge symmetries. This leads to a result that differs

from the SM Lagrangian by a series of higher-dimensional operators:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \sum_i \frac{1}{\Lambda^{d-4}} C_{d,i} \mathcal{O}_{d,i}. \quad (2)$$

Here, Λ denotes a high energy scale at which the EFT description breaks down. d denotes the dimension of the operator $\mathcal{O}_{d,i}$. The Wilson coefficients $C_{d,i}$ encode the dynamics of the UV completion. The index i runs over all operators that appear at a given dimension. Predictions in this EFT require not only expansions in the usual SM couplings, but also in the operator dimensions that appear in Eq. (2). The EFT contains two expansion parameters: the ratio v/Λ , where v denotes the vacuum expectation value of the Higgs field and is representative of the SM particle masses, and E/Λ , where E is the characteristic energy scale of the experimental processes under consideration. As experimental precision increases, and as E increases, higher operator powers must be considered for reliable predictions. The leading dimension-5 correction in Eq. (2) contains the Majorana neutrino mass term that violates lepton number. The UV scale associated with this operator is usually taken to be approximately 10^{13} GeV to reproduce the observed neutrino masses, and is therefore not relevant for current or planned future collider experiments. The same statement holds for all odd-dimensional operators. The complete, non-redundant basis of dimension-6 operators has been known for over a decade now [77–79], and is extensively used as a framework for global fits of LHC and other experimental data. It is the study of the dimension-eight and higher terms that go beyond the first order in the SMEFT expansion, needed for precision studies, that is a natural topic for the TF06 working group.

The theoretical advances needed to understand dimension-8 and beyond terms in the SMEFT was summarized as part of the TF06 working group in the white paper [80]. Only a brief outline of the relevant work is presented here. The reader is referred to the white paper [80] for more details and complete lists of references.

- Predictions at dimension-8 and higher require an understanding of the relevant operator basis, a non-trivial task whose difficulty is highlighted by the fact that the independent operator basis for dimension-6 was not understood until decades after the first attempt at writing it down. The problem of counting operators at a given dimension was solved using Hilbert series techniques, and an understanding of the role played by the conformal group in incorporating constraints from integration-by-parts identities and field redefinitions [81–84]. The explicit dimension-8 operator basis was recently derived [85, 86].
- For $1 \rightarrow 2$ scattering processes, the structure of the SMEFT expansion is simple enough to allow for predictions to all orders in the $1/\Lambda$ expansion. This all-orders solution has a geometric interpretation, and is titled “geoSMEFT [87].” All-orders results for certain amplitudes can also be derived within the on-shell approach to SMEFT [88]. A main goal of this program is to learn from the available exact results how to better estimate truncation uncertainties in EFT analyses when higher-order corrections in $1/\Lambda$ are not derived [89].
- The renormalization group equations governing the running of the dimension-6 SMEFT Wilson coefficients have been completely determined [90–92]. Accounting for these effects

is important when combining results from experiments that span energy scales. The determination of these effects at dimension-8 is still in its infancy. A study of the effects from pairs of dimension-6 insertions in loops, which contribute at the same order as dimension-8 effects, was recently performed [93]. The running effects at $\mathcal{O}(v^4/\Lambda^4)$ have a significant impact on positivity bounds on Wilson coefficients in the SMEFT [94].

- The phenomenological impact of $\mathcal{O}(v^4/\Lambda^4)$ effects can be significant, especially when the measurement is extremely precise or the experimental energy is large. Fits to the electroweak precision data can shift significantly upon inclusion of quadratic insertions of certain dimension-6 operators [95]. Inclusion of genuine dimension-8 effects can lead to novel angular dependence in Drell-Yan production [96], as well as to shifts of the transverse momentum spectrum not possible at dimension-6 [97], and can significantly change the interpretation of dimension-6 bounds obtained using LHC data [98]. Novel low-energy phenomena such as toroidal quadrupole moments of the deuteron or positronium can be induced by dimension-8 operators [99], and low-energy experiments can generally help disentangle dimension-6 from dimension-8 effects in fits [100]. There have also been recent studies of the explicit form that dimension-8 corrections take in UV models [101, 102].

3 Precision flavor physics

3.1 Introduction

A key feature of flavor physics — the study of interactions that distinguish between the three generations, i.e., break the global $[SU(3)]^5$ symmetry on the standard model — is the plethora of observables that probe very high mass scales, well beyond the center-of-mass energy of the LHC or future colliders. This is because quantum effects allow virtual particles to modify the results of precision measurements. Flavor physics can teach us about (multi-)TeV-scale new physics, which cannot be learned from the direct production of new particles. The high mass-scale sensitivity arises because the SM flavor structure implies strong suppressions of flavor-changing neutral-current (FCNC) processes (by the GIM mechanism, loop factors, and CKM elements). Even as the LHC continues to directly probe the TeV scale, ongoing and planned flavor physics experiments are sensitive to beyond standard model (BSM) flavor-changing interactions at much higher mass scales. These experiments provide essential constraints and complementary information on models proposed to explain any discoveries at the LHC or future colliders, and they have the potential to reveal new physics that is inaccessible at the energy frontier.

Throughout the history of particle physics, studies of rare processes, especially flavor-changing neutral currents (FCNCs) and CP violation, have led to new and deeper understanding of Nature. Weak interactions foretold the electroweak scale. Kaon decay experiments were crucial for the development of the standard model: the discovery of CP violation in $K_L^0 \rightarrow \pi^+\pi^-$ decay ultimately pointed toward the three-generation CKM model [103, 104]; the absence of strangeness-changing neutral current decays (i.e., the suppression of $K_L^0 \rightarrow \mu^+\mu^-$ with respect to $K^+ \rightarrow \mu^+\nu$) led to the prediction of the fourth (charm) quark [105], and the measured value of the $K_L - K_S$ mass difference made it possible to predict the charm quark mass [106, 107] before charm particles were detected. More recently, the larger than expected $B_H - B_L$ mass

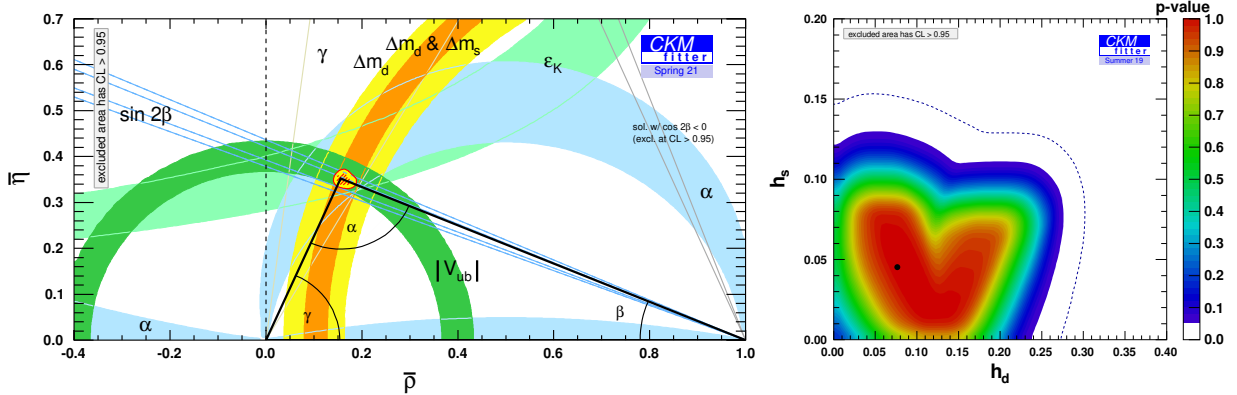


Figure 1: Left: Constraints on the CKM parameters $\bar{\rho}$ and $\bar{\eta}$ in the SM (at 95% CL) [116]. Right: The allowed range of h_d and h_s , the magnitudes of the BSM contributions to B^0 and B_s^0 mixing relative to the SM (see text) [117]. The black dot shows the best-fit point.

difference foretold the high mass of the top quark. Precision measurements of time-dependent CP asymmetries in B -meson decays in the BaBar and Belle experiments established the SM as the leading source of CP violation observed to date in flavor-changing processes, leading to the 2008 Nobel Prize for Kobayashi and Maskawa. Nevertheless, corrections to the SM in FCNC processes at the tens of percent level are still allowed, and extensions of the SM are strongly constrained by flavor physics measurements and may have observable signals in the next generation of experiments. (See Refs. [108–111] for recent reviews.)

In the next one to two decades, the LHC experiments [112, 113], Belle II [114], BES III [115] and their planned and possible upgrades will increase b -, c -, and τ -decay data sets by about two orders of magnitude. As in the past, theory will be crucial for the interpretation of the measurements and for maximizing their sensitivity to BSM physics, and experimental results will be essential inputs of theory considerations and triggers for new developments.

3.2 Flavor probes of new physics

The measurements of dozens of CP -violating and FCNC processes at e^+e^- colliders and at the LHC are consistent with the SM predictions, with ever-increasing precision (see Fig. 1, left plot). (A few exceptions, where significant anomalies occur, are discussed below.) This strengthened the “new physics flavor problem”, which is the tension between the hierarchy puzzle motivating BSM physics near the electroweak scale, and the high scale that is seemingly required to suppress BSM contributions to flavor-changing processes. The higher the scale of new physics, the more general its flavor structure may be, and the less clearly it can help with the hierarchy puzzle. Improving constraints on the deviations from the SM predictions both in the properties of the Higgs particle and in the flavor sector, only makes the puzzle stronger; TeV-scale NP entering even at the same loop order and with SM-like CKM couplings is being constrained.

As a simple example, relevant for a large class of models, assume that the dominant effect of BSM physics is to modify the mixing amplitudes of neutral mesons, and leave tree-level decays unaffected. This can be parameterized by two real parameters for each neutral meson system.

The mixing of B_q^0 mesons (where $q = d, s$) are simplest to analyze, as they are dominated by short-distance physics. Writing the mixing amplitude as $M_{12}^q = M_{12(\text{SM})}^q (1 + h_q e^{2i\sigma_q})$, the constraints on h_d and h_s , the magnitudes of the BSM contributions relative to the SM, are shown in the right plot in Fig. 1. It shows that order 10 – 20% corrections to M_{12} are still allowed (evidence for $h_q \neq 0$ would rule out the SM). Similar conclusions apply to other neutral meson mixings, as well as many other $\Delta F = 1$ FCNC decays (e.g., $B \rightarrow X\gamma$, $B \rightarrow X\ell^+\ell^-$, $B_{d,s} \rightarrow \ell^+\ell^-$, $K \rightarrow \pi\nu\bar{\nu}$, etc.).

The bounds from the consideration of a greater variety of CP -violating and flavor-changing observables are shown in Fig. 2, encompassing the quark-, lepton-, and scalar sectors [118]. The scale of dimension-6 operators are shown, assuming $\mathcal{O}(1)$ coupling strength at present (light shading) and with anticipated mid-term improvements (dark shading; including HL-LHC, Belle II, MEG II, Mu3e, Mu2e, COMET, ACME, PIK, and SNS). The greatest improvements in mass-scale sensitivity in the next 10 – 20 years, by a whole order of magnitude, are expected in μ to e conversion ($\mu N \rightarrow eN$) and electric dipole moment (EDM) experiments. The bounds require that new physics is either at a very high scale or involves tiny couplings; the hatched bars show how they are weaker in models with minimal flavor violation, where the loop- and CKM-suppressions of the SM also apply.

Some of these bounds, especially from the kaon sector, have been known since the 1960s (most significantly Δm_K and ϵ , and later ϵ'), thus flavor constraints have always been an input and rarely an output or prediction of BSM model building. With the development of new classes of BSM models, numerous mechanisms were invented to suppress new contributions to flavor-changing processes, and thereby render TeV-scale BSM scenarios not yet ruled out.

Regarding the impact on model building, in supersymmetric (SUSY) extensions of the SM, box diagrams with winos and squarks contribute to meson mixing, and the structure of their couplings depends on SUSY breaking. Some of the mechanisms proposed to suppress the SUSY contributions include, degeneracy [119], quark-squark alignment [120], heavy (3rd generation) squarks [121], Dirac gauginos with an extra R symmetry [122], split SUSY [123], and many more [124]. There are specific models to address the new physics flavor problem in extra-dimensional models as well [125]. It was the severe FCNC constraints in technicolor theories that inspired the ansatz that BSM sources of the breaking of the global $[U(3)]^5$ symmetry of the SM, without Yukawa couplings, may be proportional to the same Yukawas — minimal flavor violation (MFV) — which has since been widely applied to BSM model building [126–128]. Since the SM already breaks the $[U(3)]^5$ flavor symmetry, MFV gives a framework to characterize “minimal reasonable” deviations from the SM. The corresponding constraints are indicated by the lower hatched parts of the sensitivity-bars in Fig 2, and show that BSM scenarios with MFV flavor structures have no signals in many of these observables, and reduced sensitivity in the one to tens of TeV range in others. Many other BSM scenarios are reviewed in the whitepaper [129]. Finding other paradigms that enable flavor-safe BSM model building would be influential.

3.3 Flavor developments broadly impacting theory

The richness of B physics and the large b -quark mass ($m_b \gg \Lambda_{\text{QCD}}$) enables many complementary tests of the SM, and has been a driving force to develop new perturbative multi-loop and nonperturbative effective field theory techniques since the 1980s (see, e.g., Ref. [130]). An

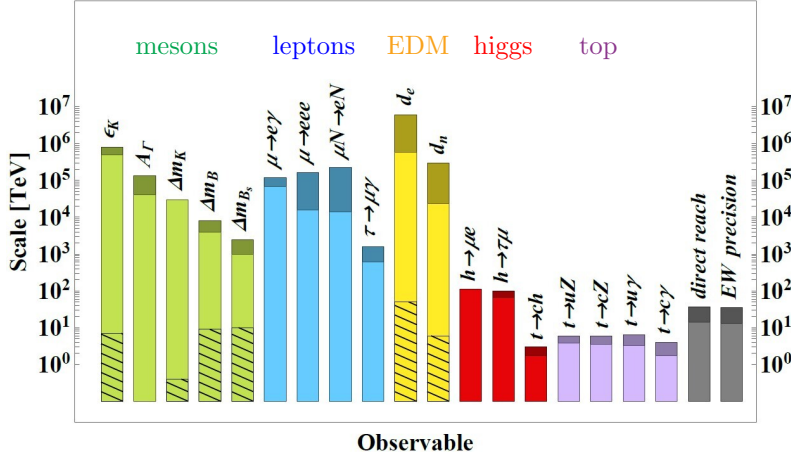


Figure 2: Scales of dimension-6 operators probed by various observables assuming unit couplings. The shaded regions show estimated future improvements, and the hatched regions show the scales probed in models with MFV flavor structure. (From Ref. [118].)

iconic example is $B \rightarrow X_s \gamma$, which receives large and calculable QCD corrections [131, 132] and is also sensitive to BSM contributions [133, 134]. It has become one of the most complex SM calculations, with some parts of it involving the evaluation of 3-loop matching at the electroweak scale, 4-loop running, and 3-loop matrix elements [135, 136]. In addition, only a restricted part of the photon energy phase space is accessible experimentally, and the well-measured part is subject to nonperturbative effects related to the b -quark distribution function in the B meson [137, 138]. This makes it essential to consistently combine fixed-order perturbative calculations, resummations of large logs (of various kinds) in endpoint regions, and nonperturbative ingredients [139, 140]. Addressing each of these issues has grown into significant areas of research.

The desire for (hadronic) model-independent understanding of semileptonic $B \rightarrow D^{(*)} \ell \bar{\nu}$ decays led to the development of the heavy quark effective theory (HQET) [141–143]. The techniques developed in that context were in turn instrumental in the development of many other effective field theories. For example, HQET had a significant impact on the development of NRGR [144], although instead of an HQET-like formulation with time-dependent velocity labels for the fields in a $g_{\mu\nu}$ background, it was simpler to formulate the EFT using worldlines [145] (as in an early formulation of HQET [146]). Studying inclusive $B \rightarrow X \ell \bar{\nu}$ decays led to the development of the heavy quark expansion [147–150], which has served as a model for other operator product expansions. The soft-collinear effective theory SCET [38, 39] was developed initially motivated by summing Sudakov logs in $B \rightarrow X_s \gamma$ in an effective theory framework. First applications included form factor relations in exclusive heavy to light decays in the phase space region $q^2 \ll m_B^2$ [151, 152]. Systematically exploring the previously found vanishing of the forward-backward asymmetry in $B \rightarrow K^* \ell^+ \ell^-$ at a particular value of q^2 [153] led to an explosion in theoretical and experimental studies of angular distributions in $B \rightarrow K^* \ell^+ \ell^-$ and related decays. QCD factorization [154, 155], and its systematic understanding using SCET, has been and will remain crucial to understand nonleptonic decays and many CP violating observables. SCET has also become part of standard theory tool-kit for higher order collider

physics calculations.

Heavy quark physics, particularly the renormalon problem and the unsuitability of the $\overline{\text{MS}}$ mass below the scale \overline{m}_Q , prompted the development of various short-distance quark mass definitions [156–161], critical for precise calculations of inclusive B decays and threshold problems. They will also be crucial for the determination of the top quark mass with the smallest possible uncertainty, should an e^+e^- collider run at the $t\bar{t}$ threshold in the future.

In the area of multi-loop calculations, flavor physics instigated developments of numerous technical aspects [162]. These include: (i) The Laporta algorithm [163], a generic way to solve integration-by-parts (IBP), that eventually became a workhorse behind many multi-loop computations, was developed in the context of the analytic computation of the electron $g-2$ at order α^3 [164]; (ii) Applications of IBPs beyond the traditional `Mincer` code was to a large extent due to flavor problems (such as the $\overline{\text{MS}}$ to on-shell mass relations, zero recoil $b \rightarrow c\ell\bar{\nu}$ calculations, etc.); (iii) Loop calculations for processes with massive particles in initial and/or final state, including integrals and IBPs, were driven by flavor physics problems ($B \rightarrow X_s\gamma$, semileptonic decays, etc.); (iv) The expansions “by regions” applied to Feynman diagrams as a tool to obtain physical results (in HQET- and NRQCD-like expansions) were also developed and driven initially by computations for flavor problems.

There are many other examples that EFT tools, initially developed for flavor physics, find uses in other areas. Precision calculations of WIMP-nucleon scattering rates were made more systematic and simpler using EFT techniques [165–168] compared to not using those [169]. Methods borrowed from SCET were used to derive precise predictions for the photon spectrum resulting from wino dark matter annihilation [170,171], and resummations all the way up to the Planck scale [172] may have significant effects. In the context of Higgs measurements, predictions for exclusive $H \rightarrow J/\psi\gamma$ and related decays use techniques developed in flavor physics, and may be a promising way to eventually probe the charm Yukawa coupling.

3.4 Semileptonic decays

Semileptonic decays are important both for more precise determinations of CKM elements in the SM, and because of their sensitivity to BSM physics. The uncertainty in the determination of $|V_{cb}|$ is one of the largest parametric uncertainties in the SM predictions for FCNC rates involving V_{ts} and/or V_{td} , such as $K \rightarrow \pi\nu\bar{\nu}$, ϵ_K , Δm_B , $B \rightarrow X\gamma$, $B_{d,s} \rightarrow \mu^+\mu^-$, $B \rightarrow X\ell^+\ell^-$. The uncertainty of $|V_{ub}|$ is of central importance to BSM searches, too, since together with the angle γ they form a tree-level “reference” determination of the unitarity triangle, with which results of other measurements impacted by loop processes can be compared. For inclusive decays, the operator product expansion will likely remain the main tool, while exclusive decays will likely rely on lattice QCD calculations, and extending those for heavy-to-light decays to the full ranges of q^2 would make big impacts. Since this Topical Group is about the role of precision theory, while the lepton flavor universality violating (LFUV) anomalies motivate both theory and experiment at present, we only describe the most-often discussed observables for context. We focus on what will make their SM predictions the most precise, and refer to the Snowmass whitepaper on flavor model building [129] for a review of models that can accommodate them.

Most prominent among the anomalies are the hints for LFUV in two set of processes. One is in neutral current $b \rightarrow s\ell^+\ell^-$ transitions, $R_{K^{(*)}} = \mathcal{B}(B \rightarrow K^{(*)}\mu^+\mu^-)/\mathcal{B}(B \rightarrow K^{(*)}e^+e^-)$,

where the most recent LHCb measurement shows a 3.1σ deviation from the SM, $R_K(1.1 < q^2 < 6.0 \text{ GeV}^2) = 0.846_{-0.039-0.012}^{+0.042+0.013}$ [173]. The R_{K^*} results with the full Run 1–2 data sets are not yet available, and neither is an average by experimentalists for the significance of the deviation from the SM. Recent fits by theorists [174–176] quote the significance as above 4σ . The other presently most significant anomaly is that in charged current $b \rightarrow c\ell\bar{\nu}$ transitions, $R(D^{(*)}) = \mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu})/\mathcal{B}(B \rightarrow D^{(*)}l\bar{\nu})$, where $l = e, \mu$ (BaBar and Belle measurements use the e, μ average, whereas LHCb uses the μ mode). In this case the significance for the deviation from the SM is quoted as 3.1σ [177] to 3.6σ [178], depending on the treatment of correlations. In both of these cases, the present data hint at about 15 – 20% corrections to the SM predictions.

While at the current central values, more precise measurements of $R_{K^{(*)}}$ could establish a breakdown of the SM without theory input, for the $R(D^{(*)})$ measurements theory is critical in assessing as to whether there is a deviation from the SM. In many models inspired by these hints of LFUV, there are correlated deviations from the SM predictions in transitions mediated by operators with flavor structures $b\bar{s}\ell^+\ell^-$ and $b\bar{s}\nu\bar{\nu}$, which makes, for example, searching for $B \rightarrow K^{(*)}\nu\bar{\nu}$ particularly interesting. The BSM model building spurred by these anomalies has also led to many new ATLAS and CMS searches [129].

The role of lattice QCD in exclusive semileptonic B decay calculations will be crucial, especially for the determination of CKM elements and refining predictions for $R(D^{(*)})$. It will take some time before LQCD can make a big impact on R_{K^*} , as in heavy-to-light decays the small- q^2 region corresponds to large $K^{(*)}$ momenta in the B restframe, which is hard to compute on the lattice. Recently new ideas emerged for lattice calculations of inclusive B decays [179]. These and other expected developments are summarized in the whitepaper [180] and the TF05 report.

A common challenge in both $b \rightarrow q\ell\bar{\nu}$ and $b \rightarrow q\ell\bar{\ell}$ mediated decays is that going forward, the role of electromagnetic corrections needs to be better understood, and the related uncertainties assessed. Recent work has focused on $b \rightarrow q\ell\bar{\ell}$ [181] inspired by R_{K^*} , and there are also ongoing efforts to understand the roles of photon radiation [182–184] and how accurate PHOTOS [185–187] is in describing it. More broadly, in both semileptonic and nonleptonic decays, a better understanding of electromagnetic radiative corrections, as well as all sources of isospin violation, are expected to become increasingly important. Initial steps toward systematically including isospin violation and electromagnetic corrections in lattice QCD calculations have also started and will be increasingly relevant in the future [188].

$b \rightarrow q\ell\bar{\nu}$ mediated decays Semileptonic B decays have continued to receive significant attention due to the importance of the determinations of $|V_{cb}|$ and $|V_{ub}|$, and also because there have been persistent tensions between their measurements from inclusive and exclusive semileptonic decays. Experimental improvements will come from cleaner measurements with large Belle II data sets, and measurements of the ratio $|V_{cb}|/|V_{ub}|$ at LHCb from exclusive decays.

There is a lot of ongoing work to improve our theoretical knowledge. Recently the order α_s^3 corrections to the inclusive semileptonic $B \rightarrow X_c\ell\bar{\nu}$ decay rate have been computed [189], as well as the 3-loop corrections to the relations between the kinetic heavy quark mass and other quark mass definitions [190]. The impact of these on $|V_{cb}|$ have started to be assessed [191]. Already in the early 2000s, the $\Lambda_{\text{QCD}}^2/m_b^2$ corrections to measurable differential spectra were known, and (its moments) fitted to experimental measurements to determine simultaneously $|V_{cb}|$ and the hadronic matrix elements. The nonperturbative corrections to the inclusive $B \rightarrow X_c\ell\bar{\nu}$

rate have been worked out to $1/m_b^5$ [192], and recently some differential distributions have also been studied at order $1/m_b^3$ and α_s [193]. In the future, such perturbative and nonperturbative corrections to the OPE may be calculated for differential rates as well, which, when combined with future data, could constrain higher-order terms in the heavy quark expansion more than ever done before, and simultaneously determine $|V_{cb}|$. In the meanwhile, other strategies also emerge, such as the observation that due to reparametrization invariance the q^2 spectrum has simpler structure at higher orders in the $\Lambda_{\text{QCD}}/m_{c,b}$ expansion than other distributions [194], allowing for complementary determinations of $|V_{cb}|$ [195].

Recently, with the first publications of “unfolded” measurements of differential $B \rightarrow D^{(*)}\ell\bar{\nu}$ decay distributions from Belle, it became possible for theorists to perform fits to the data using different assumptions. This has allowed testing different implementations of constraints on the shapes of form factors from analyticity and unitarity, and assessing the roles of model-dependent QCD sum rule inputs on prior measurements of $|V_{cb}|$ from these exclusive decays. It also allows theorists to find possible new manifestations of LFUV, such as the recent 4σ claim [196] of an e vs. μ LFUV in an angular distribution.

Regarding the $R(D^{(*)})$ measurements and their tension with the SM, a lot of effort has gone into refining the SM predictions. This is a prime example of observables for which no useful statement could be made about the (in)consistency of the data with the SM without theory input. If new physics contributes to these observables, then it also affects the measurements made using SM expectations for decay distributions and experimental efficiencies. Thus, fitting the measurements based on the SM to new physics models containing different operators is of limited reliability. The `Hammer` [197] tool allows reweighting event samples to arbitrary NP scenarios or to any hadronic matrix elements.

Concerning connections with SMEFT and HEFT, it was pointed out recently that the $R(D^{(*)})$ anomaly may be possible to accommodate via the operator $(\bar{c}\gamma_\mu P_R b)(\bar{\tau}\gamma^\mu P_L \nu)$ in HEFT [198], while this was known to be difficult in SMEFT. The EFT formulation of low-energy observables below the electroweak scale (LEFT) [199, 200] would allow systematic and model independent interpretations of combinations of flavor physics and high- p_T anomalies.

$b \rightarrow q\ell^+\ell^-$ mediated decays Among FCNC B decays with the largest rates, besides $B \rightarrow X_s\gamma$, the $b \rightarrow q\ell^+\ell^-$ mediated decays have also been known for decades to provide complementary sensitivity to new physics, and a richer set of observables in the simplest exclusive decay channels, $B \rightarrow K^{(*)}\ell^+\ell^-$. While the inclusive rate is calculable in an OPE, the comparison of theory and experiment for exclusive decays must rely on form factor calculations. An exception is LFUV ratios of decay rates, which were hardly discussed in the theory literature before the first LHCb measurement of R_K [201].

While the hints of LFUV in $R_{K^{(*)}}$ are theoretically very clean to interpret as a breakdown of the SM at the current level of sensitivity, there are a number of other observables in these and related decays, where significant deviations from the SM predictions have been claimed. A central question for heavy-to-light decays is our ability to predict the form factors parametrizing each decay. For many channels, lattice QCD calculations exist at high q^2 and light-cone QCD sum rules at small q^2 . How reliable these calculations are, and the interpolations to connect them, are somewhat open questions. For example, LHCb measured $\mathcal{B}(B_s \rightarrow \phi\mu^+\mu^-)(1.1 < q^2 < 6.0 \text{ GeV}^2)$ at a rate that is 3.6σ below a SM prediction [202], based on a combination of

light-cone sum rule [203] and lattice QCD [204] inputs. With more data and cross-checks, we will learn if this is a breakdown of these calculations or of the SM.

Concerning more differential observables in $B \rightarrow K^* \ell^+ \ell^-$, there have been attempts since the 1990s to find BSM-sensitive but hadronic physics insensitive observables, such as the q^2 value at which the forward-backward asymmetry vanishes. This became more systematic when SCET developments [205, 206] put the form factor relations, first found in LEET, on more solid footing. The form factors can be written as a nonperturbative part, which obeys symmetry relations, and at leading order depend on only two functions of q^2 , and a hard scattering part, which breaks the symmetry relations, but is computable in an expansion in α_s . When studying observables that describe the full angular distribution in $B \rightarrow K^*(K\pi)\ell^+\ell^-$ [207], of which P'_5 became best known due to tensions between SM predictions and data [208], detailed theory input is needed to assess whether the data are (in)consistent with the SM. Two main sources of theoretical uncertainties are subject to ongoing research and healthy debates. One relates to estimating subleading corrections to the heavy-to-light form factor relations, not fully calculable from first principles at present. The other relates to the role of $c\bar{c}$ loop contributions to the $\ell^+\ell^-$ invariant mass spectrum [209]. Most important is the region $q^2 < 6 \text{ GeV}^2$; while seemingly far below $m_{J/\psi}^2$, given $\Gamma_{J/\psi} \simeq 93 \text{ keV}$, a Breit-Wigner description is not sufficient. In the $q^2 > m_{\psi'}^2$ region, factorization is known to be a poor approximation [210]. Future progress is much desired, and will likely be relevant for a better understanding of nonleptonic decays, too.

Recent fits to (certain subsets of) this data yield combined deviations from the SM that are quoted at least at the 4σ level [174–176]. Prompted by the many measurements relevant for constraining BSM scenarios, several fitting codes have been developed to aid connecting the experimental measurements with BSM models, such as flavio [211], EOS [212], and SuperIso [213].

3.5 Nonleptonic decays and CP violation

To fully utilize the next generation of measurements, a better theoretical understanding of nonleptonic decays is much desired. To date, most CP violation measurements have been performed for such decays, as they not only allow measurements of the CKM unitarity triangle angles, but also provide numerous probes of CP violating BSM interactions.

Consider the well-known and important example, the determination of $\sin(2\beta)$ from the time-dependent CP asymmetry in $B \rightarrow J/\psi K_S$ and related modes, which was the “gold-plated” measurement for BaBar and Belle. The theoretical uncertainty was known to be at the percent level, and was negligible for the past B factories. The reason is that amplitudes with one weak phase dominate the decay, with deviations suppressed by $|V_{ub}V_{us}/(V_{cb}V_{cs})| \approx 0.02$ times a ratio of hadronic matrix elements which is expected to be well below unity. (This “ V_{ub} contamination” is often, and a bit confusingly, called “penguin pollution”.) The current world average is $\sin(2\beta) = 0.699 \pm 0.017$ [214] and since the measurements will improve by a lot, constraining this uncertainty will become important. The same question arises for the analogous mode for B_s decay, the time-dependent CP asymmetry in $B_s \rightarrow J/\psi\phi$, which is sensitive to BSM contributions to B_s mixing.

The contributions of these CKM-suppressed $b \rightarrow u\bar{u}s$ operators to the respective amplitudes can either be attempted to be calculated, or related to other observables by flavor symmetry relations. There is no consensus yet regarding robust estimations of the uncertainties of QCD

factorization based calculations of these quantities [215]. Factorization receives $\mathcal{O}(30\%)$ corrections even in some channels like $B \rightarrow D^{(*)}\pi$, where it is actually proven in the heavy quark limit at leading order in $\Lambda_{\text{QCD}}/m_{c,b}$. Some approaches combine $SU(3)$ flavor symmetry and diagrammatic assumptions. In the absence of a rigorous way to estimate the uncertainties, it is unclear if such approaches can establish whether possible future tensions are due to BSM contributions or underestimated uncertainties. It is possible using only $SU(3)$ flavor symmetry to relate the V_{ub} contamination to other observable decay rates [216], including the isospin violation in $B \rightarrow J/\psi K$ and $J/\psi\pi$, and a challenge is to disentangle that from the rate difference between $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ and B^+B^- [217].

Another important set of processes are the so-called ‘‘penguin modes’’, such as $B \rightarrow \phi K_S$, $\eta^{(\prime)} K_S$, ωK_S , $\pi^0 K_S$, etc. (and the analogous $B_s \rightarrow \phi\phi$). The interest in these modes is due to the fact that the amplitudes are dominated by one-loop contributions in the SM, with weak phases close to the $b \rightarrow c\bar{c}s$ operator, and BSM contributions could alter the SM amplitudes and affect the CP asymmetries (which are expected to be close to $\sin 2\beta$ in the SM). Past works attempted both explicit calculations [218–220] or flavor symmetry based relations [221], and it is unclear what theory approaches can match the uncertainties expected with the full LHCb and Belle II data sets. Developing new model-independent methods to go beyond the current state of the art will be important for the interpretation of future data.

Charmless two-body B decays have received ample attention, and will continue to do so. It is known since the CLEO observation of $B \rightarrow K\pi$ [222], before $B \rightarrow \pi\pi$, that ‘‘penguin’’ amplitudes are not nearly as suppressed relative to ‘‘tree’’ amplitudes as previously expected. Developments in SCET and QCD factorization have shed some light on this, while many open questions remain, not the least is whether there are any hints of BSM contributions. To what extent charm loop contributions to such decays are reliably tractable using perturbative methods remains unresolved [223–225]. The difference of direct CP asymmetries, $A_{K^-\pi^0} - A_{K^-\pi^+} = 0.086 \pm 0.013$ [214] has been puzzling since the early 2000s using factorization-based approaches (but not based on $SU(3)$ flavor symmetry alone). The reason is that the dominant tree and penguin contributions coincide in these two modes, and the color-allowed and color-suppressed tree contributions to $K^-\pi^0$ have a power-suppressed relative strong phase in SCET, hence one is led to require annihilation and electroweak penguin contributions to play a substantial role (which are often assumed to be negligible).

3.6 Charm and Kaon decays

CP violation in both charm mixing and decay are probes of QCD dynamics and BSM physics. While mixing and FCNCs in K , B , and B_s mesons are generated by loop diagrams with intermediate up-type quarks in the SM, in the D sector intermediate down-type quarks are at play. (Or, in SUSY, down-type squarks for K , B , and B_s , and up-type squarks for D meson FCNCs.)

After an earlier hint of a much larger effect, direct CP violation in D decay was established in 2019 by LHCb, $A_{CP}(K^-K^+) - A_{CP}(\pi^-\pi^+) = -(1.54 \pm 0.29) \times 10^{-3}$ [226]. This result can be accommodated in the SM, but it requires hadronic enhancements compared to the available (model dependent) calculations. CP violation in D mixing is thought to be a theoretically cleaner probe of the SM. While the experimental bounds improved recently [227], there is probably still significant room for discovery [228]. Future theory research needs to establish what

the maximal size of CP violation in D mixing is, that could not be attributed to the SM and would signal new physics. While we cannot at present make precision calculations because of hadronic uncertainties, one can explore consequences of flavor $SU(3)$ symmetry. Different observables arise at different orders of $SU(3)$ breaking, thus, by studying relations that hold to higher order in $SU(3)$ breaking, we can test the picture of flavor symmetry breaking and understand a pattern that can be used to probe the SM. The situation can dramatically change if there is a theoretical breakthrough that enables the calculation of some of the hadronic inputs needed for these observables. The prime candidate is lattice QCD, which may be able to address nonleptonic D decays well before addressing nonleptonic B decays

Another active and interesting area is the study of FCNC D decays [229]. The challenge is to limit the role of long-distance contributions, and there are several promising directions. For example, the lack of photon coupling to $\nu\bar{\nu}$ implies that $c \rightarrow u\nu\bar{\nu}$ mediated decays may be sensitive probes of BSM [229]. In the next decade, LHCb, Belle II and BESIII are expected to measure many rare decays and CP asymmetries with good precision (and measure them individually, without having to form differences, as in the LHCb measurement mentioned above).

The rare decays $K^\pm \rightarrow \pi^\pm\nu\bar{\nu}$ and especially $K_L \rightarrow \pi^0\nu\bar{\nu}$ are sensitive to some of the highest scales in flavor physics (in the $\Delta F = 1$ sectors, i.e., outside of neutral meson mixings), as the SM contributions are severely suppressed both by the GIM mechanism and small CKM angles. The $K^\pm \rightarrow \pi^\pm\nu\bar{\nu}$ rate is only known at the $\sim 40\%$ level to agree with the SM [230], and $K_L \rightarrow \pi^0\nu\bar{\nu}$ may still deviate from the SM prediction substantially.

Recently it was proposed that $K \rightarrow \mu^+\mu^-$ can also be a clean probe of short-distance physics [231, 232]. While the total rate has large long-distance contributions, the CP violating part has a theoretically clean interpretation in terms of short distance physics. The new idea is that by doing time-dependent interference measurements, $\mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{\ell=0}$ can be determined, which has clean interpretation in terms of short distance physics. It allows testing the SM and is sensitive to BSM physics. There can be BSM contributions to $K \rightarrow \mu^+\mu^-$ without affecting the $K \rightarrow \pi\nu\bar{\nu}$ decays to a detectable extent [233].

Possible future avenues for the kaon program are outlined in a contributed paper, Ref. [234].

4 Outlook

Precision measurements can unlock energy scales far above the direct reach of collider experiments. For example, CP -violating observables and FCNC processes allow energy scales in excess of tens of TeV to be probed. Along with this opportunity comes a challenge: precision calculations in the SM are needed to tap into the potential of these data sets. The power and lure of such data have proven over and over to stimulate both theorists and experimenters to invent creative groundbreaking methods, both in high- p_T and in flavor physics, which enable new calculations and measurements at previously unexpected precision and sensitivity.

The future of HEP will benefit from large, exquisite data from the high-luminosity LHC and Belle II, as well as future colliders with even higher luminosity. As the opportunity becomes greater so does the challenge. Continued advances in high-precision theory are needed to maximize the discovery potential of these future experiments, including ever-higher order perturbative calculations, advanced Monte Carlo event generators and parton showers for a closer

description of experimental data, and parton distribution functions extracted to a matching precision. The past and expected forthcoming impact of precision calculations on the interpretation of experimental data demands this research to be well supported.

Acknowledgement

We thank all the contributors to the TF06 working group for their input and white paper submissions. We thank Walter Goldberger, Kirill Melnikov, Bernhard Mistlberger, Robert Szafron, and Jure Zupan for helpful discussions. R. B. is supported by the U.S. Department of Energy (DOE) contract DE-AC02-06CH11357. Z. L. was supported in part by the U.S. Department of Energy under contract DE-AC02-05CH11231. W. A. is supported by the U.S. Department of Energy grant number DE-SC0010107. S. D. B. is supported by the projects P18-FR-4314, and A-FQM-467-UGR18 (FEDER). F. C. was partially supported by the ERC Starting Grant 804394 HIPQCD and by the UK Science and Technology Facilities Council (STFC) under grant ST/T000864/1. M. C. is supported by the Ramón y Cajal programme as well as by the SRA grant PID2019-106087GB-C21/C22 (10.13039/501100011033) and the Junta de Andalucía grants FQM 101, A-FQM-211-UGR18 and P18-FR-4314 (FEDER). A. D. C. is supported by MCIN/AEI under the grant PID2019-106087GB-C21 (10.13039/501100011033) and by ESF Investing in your future, as well as by Junta de Andalucía under grant FQM 101. W. C. is supported by the Natural Science Foundation of China (NSFC) under the contract No. 11975200. B. H. is supported by the Swiss National Science Foundation, under grants no. PP00P2-170578 and no. 200020-188671, and through the National Center of Competence in Research SwissMAP. S. J. is supported by the UK Science and Technology Facilities Council under grant ST/T001011/1. T. K. is supported by the JSPS Grant-in-Aid for Early-Career Scientists (Grant No. 19K14706). H.-L. L. is supported by F.R.S.-FNRS through the IISN convention "Theory of Fundamental Interactions" (N: 4.4517.08). X. L. is supported by the National Natural Science Foundation of China under Grant No. 12175016 and by the National Special Support Program for High-level Talents. A. M. is supported by NSF fund no. PHY-2112540. M. R. M. is supported by the UK Science and Technology Facilities Council (grant numbers ST/P001246/1). The work of M. R. M. has also received funding from the European Union's Horizon 2020 research and innovation program as part of the Marie Skłodowska-Curie Innovative Training Network MCnetITN3 (grant agreement no. 722104). T. M. is supported by the World Premier International Research Center Initiative (WPI) MEXT, Japan, and by JSPS KAKENHI grants JP19H05810, JP20H01896, and JP20H00153. E. M. is supported by the U.S. Department of Energy through the Office of Nuclear Physics and the LDRD program at Los Alamos National Laboratory. B. M. is supported by the U.S. Department of Energy, Contract DE-AC02-76SF00515. F. P. is supported by the U.S. Department of Energy (DOE) contract DE-AC02-06CH11357 and DE-FG02-91ER40684. D. S. is supported in part by the U.S. Department of Energy under grant DE-SC0011640. G. S. is supported by the National Science Foundation, awards 1915093 and 2210533. R. S. is supported by the U.S. Department of Energy under Grant Contract DE-SC0012704. L. V. is supported by Fellini, Fellowship for Innovation at INFN, funded by the European Union's Horizon 2020 research programme under the Marie Skłodowska-Curie Cofund Action, grant agreement no. 754496. G. V. is supported by the U.S. Department of Energy, Contract DE-AC02-76SF00515. J.-H.Y. is supported by the Na-

tional Science Foundation of China under Grants No. 12022514, No. 11875003 and No. 12047503, and National Key Research and Development Program of China Grant No. 2020YFC2201501, No. 2021YFA0718304, and CAS Project for Young Scientists in Basic Research YSBR-006, the Key Research Program of the CAS Grant No. XDPB15. J. Z. acknowledges support in part by the DOE grant DE-SC0011784 and NSF OAC-2103889.

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