Snowmass White Paper: prospects for measurements of the bottom quark mass

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In this white paper for the Snowmass '21 community planning exercise we provide quantitative prospects for bottom quark mass measurements in high-energy collisions at future colliders that can provide a precise test of the scale evolution, or "running" of quark masses predicted by QCD.

Keywords: bottom quark mass, scale evolution, Higgs boson

INTRODUCTION

The masses of quarks are free parameters in the Standard Model of particle physics, whose values must be determined experimentally. Precise measurements are performed through the comparison of measurements of physical observables sensitive to the mass to SM predictions in Quantum Chromodynamics (QCD) beyond leading order accuracy. Like the strong coupling constant, quark masses depend on the renormalization scheme and are renormalization-scale-dependent parameters or "running constants". In this study, we adopt the most popular renormalization scheme, the modified minimal subtraction scheme or $\overline{\text{MS}}$ scheme, where the strong coupling $\alpha_s(\mu)$ and the quark masses $m_q(\mu)$ depend on the dimensionful renormalization scale μ that is identified with the energy scale of the scattering process [1]. The scale evolution is predicted by QCD. Given a measurement at one scale, the value at any other scale is predicted by the renormalization group equation (RGE). RGE calculations have reached 5-loop $(\mathcal{O}(\alpha_s^5))$ accuracy [2-4] and software packages such as RunDec [5] and REvolver [6] provide access to state-of-the-art renormalization evolution and scheme conversions. The predicted evolution can be tested experimentally by performing measurements at different scales. The evolution of the strong coupling has been tested over a broad range of energies [7] and experiments have studied the "running" of MS quark masses for the the charm quark at HERA [8] and the top quark at the LHC [9]. We discuss the case of the bottom

quark mass in this White Paper.

The most precise extractions of the bottom quark mass [10–20] rely on the measurement of the mass of bottomonium bound states and the $e^+e^- \rightarrow$ hadrons cross section as experimental input, in combination with QCD sum rules and perturbative QCD calculations. Several lattice QCD groups have also published results, the most recent of which reaches a precision of approximately 0.3% [21–25] (see also the FLAG report [26]). The world average provided by the Particle Data Group (PDG) [7] also includes inputs from HERA [27] and the BaBar and Belle experiments at the B-factories [28, 29]. This world average of low-scale bottom-quark mass measurements has a relative precision better than 1%:

$$m_b(m_b) = 4.18^{+0.03}_{-0.02} \text{ GeV},$$
 (1)

where the reference value of the bottom mass is quoted in the $\overline{\text{MS}}$ scheme, at a scale given by the mass itself.

Bottom quark mass measurements at a much higher scale became possible at LEP and SLC, where jet rates and event shapes are sensitive to subleading mass effects. A practical method to extract the bottom-quark mass from Z-pole data was proposed in Ref. [30]. Three independent groups completed the necessary next-to-leading order (NLO) theoretical calculation of the three-jet rate for massive quarks [31–37] (an NNLO calculation for the three-jet rate in e^+e^- collisions, without bottom-quarkmass effects, is available in Ref. [38]). The first measurement of this type was performed by the DELPHI collaboration [39] using the LEP Z-pole data. Similar measurements were also performed with SLD [40, 41] data, and by ALEPH [42], OPAL [43] and DELPHI [44, 45]. The combination of the most precise determinations from three-jet rates of each experiment yields the following average for $m_b(m_Z)$:

$$m_b(m_Z) = 2.82 \pm 0.28 \text{ GeV}.$$
 (2)

This value is in good agreement with the average of Ref. [46] that is based on a slightly different sub-set of measurements.

Recently, a new measurement was published of the bottom quark mass at the scale of the Higgs boson mass [47]. The value of $m_b(m_H)$ is inferred from measurements by the ATLAS [48] and CMS [49] experiments of the bottom quark decay width to bottom quarks $\Gamma(H \to b\bar{b})$. The width is normalized to $\Gamma(H \to ZZ)$, the decay width for the ZZ decay mode. The average of the $m_b(m_H)$ results obtained from both measurements yields:

$$m_b(m_H) = 2.60^{+0.36}_{-0.30} \text{ GeV}.$$
 (3)

This result reinforces the experimental evidence for the "running" of the $\overline{\text{MS}}$ bottom quark mass, definitively excluding the no-running scenario with a statistical significance greater than 7 standard deviations.

The three sets of measurements, of $m_b(m_b)$, $m_b(m_Z)$ and $m_b(m_H)$, are shown in Fig. 1 and compared to the evolution of the PDG world average from $m_b(m_b)$ to a higher scale using the RGE calculation included in the REvolver code [6] at five-loop precision.

We argue that in the next decade the study of the "running" of the bottom quark mass will turn into a precise test of QCD. Precise measurements at several energy scales can be used to rule out or confirm the presence of massive new coloured states that may contribute to the quark mass evolution. New collider facilities can further enhance this potential in several ways. In this White Paper we provide a quantitative assessment of the potential of the High-Luminosity phase of the LHC (HL-LHC) and a future "Higgs factory" electron-positron collider to test the scale evolution of the bottom quark mass predicted by QCD.

THE BOTTOM-QUARK MASS FROM LOW-ENERGY MEASUREMENTS

Both from the theoretical and experimental points of view, the ideal observable to determine m_b is the bottomonium spectrum. The masses of $b\bar{b}$ bound states are very sensitive to the bottom quark mass and have nearly vanishing experimental uncertainties. On the other hand, we can compute the masses of the low-lying narrow bottominum resonances only through perturbative expansions supplemented with non-perturbative corrections. The typical dynamical scale of these narrow



FIG. 1. The scale evolution of the bottom quark $\overline{\text{MS}}$ mass. The measurements include the PDG world average for $m_b(m_b)$ from low-scale measurements, the measurements of $m_b(m_Z)$ from jet rates at the Z-pole at LEP and SLC and the measurement of $m_b(m_H)$ from Higgs boson branching fractions. The prediction of the evolution of the mass is calculated at five-loop precision with REvolver [6]. The inner dark grey error band includes the effect of missing higher orders and the parametric uncertainties from $m_b(m_b)$ and α_s from the PDG averages. The outer band with a lighter shading includes additionally the effect of a ± 0.004 variation of $\alpha_s(m_Z)$. Figure from Ref. [47].

bottominum masses are of the order of the inverse Bohr radius ~ $C_F \alpha_s m_b \gg \Lambda_{\rm QCD}$, such that perturbative expansions can be used reliably and non-perturbative corrections remain small. Quarkonium masses have been computed to N³LO for arbitrary quantum numbers and ultrasoft resummation is known to N^2LL . The most up to date analyses have been carried out in Refs. [11] and [50] (earlier analyses use the $\overline{\text{MS}}$ mass and are not discussed). They share some features, like employing lowscale short-distance masses (RS and MSR, respectively), using a 3-flavor scheme plus finite charm mass corrections and varying two renormalization scales, whose variation range is inferred from the perturbative logarithms. There are small differences as well: while [11] determines the mass from a single bound state, [50] performs global fits with correlated scale variation using a χ^2 function. In this latter analysis non-perturbative effects are estimated by comparing the m_b results for different sets of quarkonia states. The analyses in Ref. [11] and [50] both use PDG [7] data and find compatible results. Ref. [11] finds:

$$m_b(m_b) = 4.186 \pm 0.037 \,\text{GeV},$$
 (4)

while Ref. [50] finds:

$$m_b(m_b) = 4.216 \pm 0.039 \,\text{GeV}.$$
 (5)

Since experimental data is already extremely accurate, more precision in the future could only come from the theory side. If a new perturbative order becomes available the theoretical uncertainty (that clearly dominates) could go down to 0.026 GeV. This estimate is obtained with a quadratic extrapolation (assuming that the error from N³LO to N⁴LO goes down by the same factor as it does for going from N²LO to N³LO yields a slightly smaller error). This should be taken as a rough estimate only. On the other hand, if N³LL resummation becomes available for all bound states (so far it is only known for P-wave states), an additional error reduction should be expected as well.

Weighted averages of the bottom-tagged hadronic R-ratio in e^+e^- annihilation are alternative physical observables to measure m_b . They are very sensitive to the bottom quark mass, and precise theoretical calculations can be achieved. This method goes under the name of QCD sum rules, and it can take different forms depending on the specific weight function and on whether the integration is cut off at some finite energy or not. For relativistic sum rules it was shown in Refs. [16, 51] that in order to properly estimate perturbative uncertainties it is mandatory to vary the renormalization scales of the strong coupling α_s and bottom quark $\overline{\text{MS}}$ mass independently, in order to avoid a biased dependence on the expansion prescription. On the experimental side one has to account for contributions from narrow resonances (whose masses and electronic widths can be taken from the PDG [7]) and continuum data (so far only BaBar [52] has released data), which only exists for energies below 11.21 GeV. This limits the experimental precision of this method if sum rules without an energy cut are considered (relativistic sum rules). These are very clean, theoretically, since their intrinsic dynamical scale is of order m_b . Alternatively, one can consider sum rules with an energy cut at the last experimental data point (finite-energy sum rules), or use moments with specific weight functions that strongly suppress the contributions from high-energies (non-relativistic sum rules). These approaches, in turn, have additional theoretical drawbacks, related to either a higher sensitivity to non-perturbative effects or to somewhat lower renormalization scales (as is also the case for the bottomonium mass method mentioned above).

We show results from two representative analyses for relativistic sum rules. Ref. [20] uses correlated scale variation for the bottom quark mass and α_s and finds:

$$m_b(m_b) = 4.163 \pm 0.016 \,\text{GeV}.$$
 (6)

Ref. [16] with the independent variation mentioned above, finding:

$$m_b(m_b) = 4.176 \pm 0.023 \,\text{GeV}.$$
 (7)

where the experimental systematic uncertainty by itself amounts to 19 MeV and the theoretical uncertainty to 9 MeV for the more conservative analysis of Ref. [16]. This uncertainty can be reduced substantially if additional data in the continuum region around and above 11.2 GeV becomes available. This would potentially open up the possibility for using the n = 1 moment, which is theoretically the cleanest and most precise, but has the strongest sensitivity to high-energy data. Assuming that this new data decreases the uncertainty on the continuum by a factor of 2, the new experimental error on m_b — obtained from the n = 2 moment — would be reduced to 13 MeV, making for a total uncertainty of 16 MeV if the theoretical error is estimated as in Ref. [16]. If the next perturbative order becomes available, the theoretical uncertainty may be reduced by 30% independent of the way how the perturbative truncation error is estimated.

Concerning non-relativistic sum rules (for which nonperturbative effects are smaller than in quarkonium masses) we discuss the following two analyses: Ref. [15] in which (potential) pNRQCD is used (which provided N³LO fixed-order non-relativistic calculations) together with the PS short-distance mass, and Ref. [53] which uses (velocity) vNRQCD (which provided NNLL renormalization group improved non-relativistic calculations) and employs the 1S short-distance mass. The result from Ref. [15] is:

$$m_b(m_b) = 4.193^{+0.022}_{-0.035} \,\text{GeV},$$
 (8)

and Ref. [53] yields:

$$m_b(m_b) = 4.235 \pm 0.055 \,\text{GeV}.$$
 (9)

where the theoretical uncertainties are an order of magnitude larger than the experimental ones. Improvements on this kind of bottom mass measurements can only come from the theoretical side: additional perturbative orders may become available (either through fixed-order corrections or anomalous dimensions for the summation of logarithms) or an improved determination on the effects of the finite charm quark mass could be reached. In either case an error reduction will require a significant theoretical effort.

In the light of the available results as of 2022 one can envisage that the most precise determination in the future will come from relativistic sum rules (although it is not inconceivable that on the time scale of some of the future collider facilities discussed in the Snowmass study lattice simulations with dynamical bottom quarks are feasible). Assuming new data becomes available, together with one extra perturbative coefficient, and hopefully new theoretical strategies based on a better understanding of the perturbative series (see e.g. Ref. [54], where optimized moments were tentatively introduced), it is not unrealistic to assume that the uncertainty on $m_b(m_b)$ is reduced below 10 MeV.

THE BOTTOM-QUARK MASS FROM THREE-JET RATES

Future e^+e^- colliders can improve the precision of the $m_b(m_Z)$ measurement. A dedicated high-luminosity run at the Z-pole, i.e. the "GigaZ" programme of a linear collider or the "TeraZ" run at the circular colliders, yields a sample of Z-bosons that exceeds that of the LEP experiments and SLD by orders of magnitude. We adopt the extrapolation of LEP/SLD results in Ref. [55] that assumes that the extraction of $m_b(m_Z)$ from the three-jet rates will be limited by the theory uncertainty and hadronization uncertainties. Both sources of uncertainty are assumed to be reduced by a factor 2. This requires fixed-order calculations at NNLO accuracy, with full consideration of mass effects, which is available for Higgs decays [56].

The Higgs factory program itself, with several inverse attobarn at a center-of-mass energy of 240-250 GeV, can take advantage of radiative-return events. The Lorentzboost of the Z-bosons complicates the selection, reconstruction and interpretation. A dedicated full-simulation study is therefore required to provide a reliable, quantitative projection. However, it is clear that the radiativereturn data has the potential to significantly improve the precision of existing LEP/SLC analysis.

Finally, a high-energy electron-positron collider operated at a center-of-mass energy of 250 GeV or above can extend the analysis to higher energies and thus probe the effect of coloured states with masses heavier than that the Higgs boson on the running of the bottom quark mass. The potential of the three-jet rate measurement to determine $m_b(\mu)$ for $\mu = 250$ GeV has been studied in Ref. [55]. The mass dependence of the observable is found to drop rapidly with increasing μ , since the bottom quark mass dependence is a power-suppressed correction. A measurement with a precision of 1 GeV is feasible for $\mu = 250$ GeV.

THE BOTTOM-QUARK MASS FROM Z–BOSON DECAY

The bottom quark mass at the scale of the Z-boson mass can also be inferred from the $Z \rightarrow b\bar{b}$ decay width. Currently, this method does not offer a competitive precision. Using $R_{0,b} = \Gamma(Z \rightarrow b\bar{b})/\Gamma_{\text{total}} = 0.21582\pm0.00066$, as reported by the LEP/SLC Electro-weak Working Group [57], Ref. [58] finds an uncertainty greater than 1 GeV.

A future high-statistics Z-pole run, together with theory improvements, can significantly enhance the potential of this approach. Following the FCCee Conceptual Design Report [59, 60], that predicts a ten-fold increase of the precision of $R_{0,b}$, one can expect a precision of 140 MeV (5%) on $m_b(m_Z)$ after the "TeraZ" program. This requires considerable improvements in the modelling of B- and D-hadron decays, compared to the reference analysis performed by SLC that forms the basis for the extrapolation by the FCCee study.

THE BOTTOM-QUARK MASS FROM HIGGS DECAY

The measurement of $m_b(m_H)$ from the Higgs decay width to a bottom-antibottom quark pair is expected to increase rapidly in precision as the precision of Higgs coupling measurement improves. The method of Ref. [47] provides a very clean theoretical basis that allows for steady progress as the experimental precision improves. The key aspect of this method is that the Higgs boson is a color-less spin-0 state with a relatively small decay width, such that the analysis is essentially insensitive to the theoretical knowledge of the Higgs production rate. For the same reason very precise theoretical predictions can be made for the Higgs partial width into a bottomantibottom quark pair.

At the relevant dynamical scale, m_H , the QCD corrections are very well under control using $\mu \sim m_H$ as the renormalization scales of α_s and m_b . The partial width $\Gamma(H \to b\bar{b}$ is proportional to the squared of the bottom quark mass. This dependence arises because the decay is governed the bottom Yukawa coupling. For these reasons we expect the determination of $m_b(m_H)$ in $H \to b\bar{b}$ decay to become the "golden" measurement among the high-energy determinations.

The current theory uncertainty from missing higher orders and parametric uncertainties from α_s and m_H is estimated to be 60 MeV [47], well below the current experimental precision. The theory uncertainty is dominated by the parametric uncertainty from the Higgs boson mass. The current uncertainty on the Higgs mass of 240 MeV leads to an uncertainty of ~40 MeV on $m_b(m_H)$ and is expected to come down considerably as more precise determinations of m_H appear. Future prospects for Higgs mass measurements are summarized in Ref. [61]. Both the HL-LHC [62] and the Higgs factory [63] are expected to provide a measurement of the Higgs boson mass to 10-20 MeV precision, which is sufficient to reduce the impact of this source of uncertainty on $m_b(m_H)$ to below 10 MeV.

The effect of the strong coupling α_s , which amounts to an 0.2% uncertainty in the ratio of branching ratios for an uncertainty of 0.001 in the value of $\alpha_s(m_Z)$, is relatively small. A much larger parametric α_s uncertainty was reported in Ref. [64], which stems mainly from the parametric dependence of the evolution of the bottom quark mass from $\mu = m_b$ to $\mu = m_H$ on the value of α_s . This source of uncertainty does not exist in the measurement of $m_b(m_H)$. The parametric uncertainty from the value of α_s is expected to remain sub-dominant even with only a very modest improvement of its world average.

The dominant theory uncertainty is due to missing higher order electroweak corrections for the branching ratios, that are currently known with NLO precision. These electroweak uncertainties are about a factor of two larger than the current uncertainties from QCD. Thus the knowledge of the leading NNLO EW correction is expected to be sufficient to take full advantage of the power of the Higgs factory data.

Ref. [62] provides the projections for the LHC and its luminosity upgrade, extrapolating the partial run 2 results under the following assumptions: both statistical and systematic uncertainties are envisaged to scale with integrated luminosity L as $1/\sqrt{L}$ up to certain limits, while theory uncertainties are expected to improve by a factor two. This "S2 scenario" leads to a projected uncertainty on the Higgs branching ratio to bottom quarks of 4.4% (1.5% stat., 1.3% exp., 4.0% theo.) and on $\lambda_{bz} = \mu^{bb}/\mu^{ZZ}$ of 3.1% (1.3% stat., 1.3% syst., 2.6% theo.), an improvement by nearly a factor of ten with respect to the first measurement in Ref. [47].

The Higgs boson couplings will be measured to even higher precision at future e^+e^- Higgs factories [61]. The recoil analysis yields a precisely measured production cross section, and normalization of the total width, while the branching fractions can be inferred from a simple counting experiment on the selected events. Sub-% precision is expected for the $Hb\bar{b}$ coupling, based on detailed prospect studies of Ref. [65, 66].

For a numerical estimate of the potential of the Higgs factory projects to improve $m_b(m_H)$ we adopt the projections for the ratio $BR(H \to b\bar{b})/BR(H \to W^+W^-$ from the ILC project. This ratio can be more precisely measured than the ratio $BR(H \to b\bar{b})/BR(H \to ZZ)$, due to the better statistical precision in the $H \to W^+W^$ channel. The 250 GeV stage of the International Linear Collider (ILC) can measure the former ratio to 0.86% precision [67, 68]. The corresponding uncertainty on $m_b(m_H)$ is ±12 MeV. The complete ILC programme with stages at 250 GeV and 500 GeV is expected to improve the precision further, to 0.47% on $BR(H \to b\bar{b})/BR(H \to W^+W^-)$, and yields an experimental uncertainty of 6 MeV on $m_b(m_H)$. Other Higgs factory projects are expected to reach similar precision.

At this stage, the experimental precision of $m_b(m_H)$ is expected to reach a relative precision of 0.2%, a factor two better than the current world average for $m_b(m_b)$. At this point, additional theoretical care will be needed to assess the uncertainties in the theoretical predictions that are used for extraction of $m_b(m_H)$ and in relating it to $m_b(m_b)$.

THE STRONG COUPLING

A thorough and precise test of the scale evolution of the bottom quark mass requires precise values for the strong coupling, both at the scale of the bottom quark mass and at the electro-weak scale $\mu \sim m_Z \sim m_H$. In the traditional approach to the determination of quark masses, one usually assumes that the strong coupling is an external parameter that leads only to parametric uncertainties. In this context, it is deemed sufficient to consider the PDG world average for $\alpha_s(m_Z)$ (or at any other renormalization scale μ) and use the Standard Model evolution equation to determine the strong coupling at the scale needed for the theoretical calculations. This is a reasonable approach for the determination of the bottom quark mass at a given scale, but for a precise test of the QCD evolution one must apply a more conservative view. The running of the strong coupling may be affected by the same new physics effects that alter the running of the quark masses and therefore cannot be assumed to follow the Standard Model RGE evolution. The analysis of the scale evolution of the bottom mass for scales between m_b and m_Z or m_H therefore also requires precise measurements of the strong coupling $\alpha_s(\mu)$ over the interval of scales considered.

Lattice determinations of the strong coupling have achieved sub-% precision [69]. The Flavour Lattice Averaging Group (FLAG) working group on the strong coupling constant, provides the following projection in Ref. [70]: "a total error clearly below half a percent for $\alpha_s(m_Z)$ seems achievable within the next few years by pushing the step-scaling method further, possibly in combination with the decoupling strategy."

An independent low-energy determination of the strong coupling comes from determinations from τ -decays [70]. Today, these yield a value of $\alpha_s(m_{\tau}) = 0.3077 \pm 0.0065(exp.) \pm 0.0038(theory)$ [71]. The dominant experimental uncertainties are expected to be reduced strongly using Belle II data [72] and eventually new Z-pole data at a future electron-positron collider [73], potentially achieving sub-% precision.

The determination of the strong coupling at the electro-weak scale is currently dominated by LEP measurements. In Ref. [47] an uncertainty of 0.004 was assigned to $\alpha_s(m_Z)$ based solely on electroweak-scale measurements. A future electron-positron collider running at the Z pole provides an ideal environment to reduce this uncertainty. Ref. [74] claims the strong coupling can be measured to better than 0.1% exploiting Z-boson hadronic pseudo-observables, provided the theoretical uncertainties are reduced by incorporating missing higher-order QCD and mixed QCD+EW corrections [75].

Based on these qualitative prospects, we assign an uncertainty of 0.5% to the strong coupling over the range from m_b to m_H .

PROJECTION FOR THE TEST OF THE SCALE EVOLUTION

The projections and extrapolations discussed in the previous sections have been included in Fig. 2. The markers are centered on the current central values for $m_b(m_Z)$ and $m_b(m_H)$ and the error bars indicate the projected precision. The solid line indicates the evolution of the PDG world average from $m_b(m_b)$ to a higher scale using the RGE calculation included in the REvolver code [6] at five-loop precision. The uncertainty band includes the projected uncertainty of 10 MeV on $m_b(m_b)$ (dark grey) and an 0.5% uncertainty on $\alpha_s(m_Z)$.



FIG. 2. Prospects for measurements of the scale evolution of the bottom quark $\overline{\text{MS}}$ mass at future colliders. The markers are projections for $m_b(m_Z)$ from three-jet rates at the Z-pole and for $m_b(m_H)$ from Higgs boson branching fractions. The RGE evolution of the mass is calculated at five-loop precision with REvolver [6].

The independent determinations of the bottom quark mass at different energies yield a precision test of the scale evolution of the bottom quark mass. High-scale determinations can be used to search for the impact of new massive coloured states on the scale evolution, using a similar strategy to studies of α_s [76, 77], and possibly incorporating the analysis of α_s and m_b in a combined fit. The implementation of this programme, and a precise estimate of its sensitivity, is left for future work.

SUMMARY

In the next decades, with the completion of the high luminosity programme of the LHC and the construction of a new "Higgs factory" electron-positron collider, rapid progress is envisaged in the measurement of Higgs coupling measurement. These precise measurements will enable an extraction of the $\overline{\text{MS}}$ bottom quark mass $m_b(\mu)$ at the scale given by the Higgs boson mass, $m_b(m_H)$, with a precision of the order of 10 MeV. With a relative precision of 2 per mille, the high-scale measurement can reach a similar precision as $m_b(m_b)$ based on low-energy measurements.

Together with improved measurements of $m_b(m_b)$ from low-energy data, $m_b(m_Z)$ from three-jet rates in $e^+e^$ collisions (and possibly new measurements at scales smaller than m_Z and larger than m_H), one can expect to map out the scale evolution of the bottom quark mass from m_b to m_H with a precision at the few per mille level. At the same time, improved measurements of the strong coupling at each of these scales reduce the uncertainty in the evolution between the two energies. When all these elements are brought together, they form a powerful test of the "running" of quark masses predicted by the Standard Model and allow for stringent limits on coloured states with mass below the electroweak scale.

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