# Scalar-mediated dark matter model at colliders and gravitational wave detectors – A White paper for Snowmass 2021

Jia Liu<sup>a,b</sup>, Xiao-Ping Wang<sup>c</sup> and Ke-Pan Xie<sup>d</sup>

<sup>a</sup> School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

<sup>b</sup> Center for High Energy Physics, Peking University, Beijing 100871, China

<sup>c</sup> School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100083, China

<sup>d</sup> Department of Physics and Astronomy, University of Nebraska, Lincoln, NE 68588, USA

#### Abstract

The weakly interacting massive particles (WIMPs) have been the most popular particle dark matter (DM) candidate for the last several decades, and it is well known that WIMP can be probed via the direct, indirect and collider experiments. However, the direct and indirect signals are highly suppressed in some scalar-mediated DM models, e.g. the lepton portal model with a Majorana DM candidate. As a result, collider searches are considered as the only hope to probe such models. In this white paper, we propose that the gravitational wave (GW) astronomy also serves as a powerful tool to probe such scalar mediated WIMP models via the potential first-order phase transition GW signals. An example for the lepton portal dark matter is provided, showing the complementarity between collider and GW probes.

### 1 Motivation

Although dark matter (DM) contributes as large as ~ 85% to the matter of the Universe [1], its origin remains as a long-standing mystery in particle physics [2]. Over the past several decades, the weakly interacting massive particle (WIMP) paradigm [3] has been the most popular explanation for particle DM. The "WIMP miracle" allows a natural explanation for electroweak scale DM particles and interactions. What's more, the WIMP models can be typically probed via the direct [4], indirect [5] and collider [6] searches. However, in many WIMP models, the direct and indirect signals are accidentally suppressed that collider searches are usually expected as the only hope to probe the DM parameter space.

One such example is the lepton portal DM model with Majorana DM candidate [7] (also see Refs. [8–46]). In this model, the DM candidate  $\chi$  couples to the Standard Model (SM) particles via a complex scalar S which is charged -1 under the hypercharge group. This model has negligible direct and indirect search signals: the nuclear recoil cross section in direct detection comes from loop diagrams; while the annihilation cross section suffers from helicity suppression [7]. Therefore, collider experiments are crucial in probing the model.<sup>1</sup> In Ref. [49], we have studied the hadron and electron-positron collider phenomenology of this model, and pointed out that the first-order

<sup>&</sup>lt;sup>1</sup>See Refs. [47,48] for the gamma ray signals for this model.

electroweak phase transition gravitational wave (GW) signals can also probe a considerable fraction of the parameter space. The idea in that paper can be generalized into other WIMP models which are difficult to probe via the direct and indirect experiments.

# 2 Probing the lepton portal DM model with collider and GWs

In this section, we summarize the study in Ref. [49]. The relevant Lagrangian for the lepton portal DM with a Majorana DM candidate is

$$\mathcal{L}_{\chi} = \frac{1}{2} \bar{\chi} i \partial \!\!\!/ \chi - \frac{1}{2} m_{\chi} \bar{\chi} \chi + y_{\ell} \left( \bar{\chi}_L S^{\dagger} \ell_R + \text{H.c.} \right), \tag{1}$$

$$\mathcal{L}_S = \left(D^{\mu}S\right)^{\dagger} D_{\mu}S - V(H,S), \tag{2}$$

$$V(H,S) = \mu_H^2 |H|^2 + \mu_S^2 |S|^2 + \lambda_H |H|^4 + \lambda_S |S|^4 + 2\lambda_{HS} |H|^2 |S|^2,$$
(3)

where  $\chi$  is the DM candidate (gauge singlet), while S is the mediator  $(SU(2)_L \text{ singlet but charged} -1 \text{ under } U(1)_Y)$ . H and  $\ell$  are respectively the SM Higgs doublet and lepton ( $\ell = e, \mu, \tau$ ). The annihilation  $\chi\chi \to \ell^+\ell^-$  via the exchange of a t-channel mediator dominates the DM relic abundance after freeze-out. Although both the direct and indirect signals are suppressed, the model can be probed via collider signals.

The lepton portal coupling  $y_{\ell}$  and scalar portal coupling  $\lambda_{HS}$  can be efficiently probed by the following channels.

- 1. The pair production and exclusive decays  $S^+S^- \to \ell^+\chi\ell^-\chi$ , leading to a di-lepton plus missing energy final state. At the LHC, the combination of Drell-Yan  $q\bar{q} \to Z^*/\gamma^* \to S^+S^$ channel and the gluon-gluon fusion channel  $gg \to h^* \to S^+S^-$  can probe the scalar portal coupling  $\lambda_{HS}$ ; while at the future lepton collider such as FCC-ee and CEPC, the off-shell production of  $e^+e^- \to Z^*/\gamma^* \to S^{\pm(*)}S^{\mp} \to \ell^+\chi\ell'^-\chi$  offers the opportunity to probe  $y_\ell$ directly.
- 2. Exotic decays of the Higgs or Z boson. The first case is the exotic decay to a pair of leptons and missing energy  $h/Z \to S^{\pm(*)}S^{\mp(*)} \to \ell^+ \chi \ell'^- \chi$ . In the Higgs case, the decay width is  $\propto y_\ell^2 \lambda_{HS}^2$  or  $\propto y_\ell^4 \lambda_{HS}^2$ , providing a new way to probe  $\lambda_{HS}$  and  $y_\ell$ ; while in the Z case, the width is  $\propto y_\ell^4$ . The second case is the Higgs invisible decay  $h \to \chi \chi$ , which can probe the combination  $y_\ell^2 \lambda_{HS}$ .
- 3. One loop corrections to the Higgs couplings, including  $h\ell^+\ell^-$ ,  $h\gamma\gamma$  and hZZ. The leptonic coupling probes the combination  $y_\ell^2\lambda_{HS}$ , while the  $h\gamma\gamma$  and hZZ couplings probe  $\lambda_{HS}$  solely.
- 4. The electron or muon anomalous magnetic moment. Unfortunately, in the minimal model considered in Eq. (1), the contribution is negative that it can not explain the muon g 2 anomaly [50]. While in the future study, if there are pseudo scalars' positive contribution to the muon magnetic moment, we probably can address the muon g 2 anomaly. On the other hand, if Yukawa interactions of scalar mediator induce the effective photon interactions, two loop effects will also need to be considered for muon g 2.

Besides the collider signals, the model might also trigger a first-order phase transition in the early Universe, provided that the mediator bare mass term  $\mu_S^2$  is negative, and the portal coupling



Figure 1: Figure from Ref. [49]. The interplay between gravitational wave detection and LHC searches. The shaded regions are exclusions from LEP [51], LHC Run-I (20.3 fb<sup>-1</sup>) [52] and LHC Run-II (139 fb<sup>-1</sup>) [53,54]. The black dashed lines are projections for the LHC reach at 300 fb<sup>-1</sup> with only the Drell-Yan production of  $S^+S^-$ . The light blue (orange) shaded region corresponds to  $\lambda_{HS} = 2$  (3), with the vertical boxed boundary regions being the LISA-detectable parameter space, while the irregular boundary regions being enhanced part of the LHC projections when including the  $gg \rightarrow h^* \rightarrow S^+S^-$  contribution.

 $\lambda_{HS}$  is large enough. Therefore, the phase transition GWs can also be a probed for  $\lambda_{HS}$ . The interplay between LHC searches and GW probes are shown in Fig. 1, where the LHC and LISA projections are both shown in the  $m_S \cdot m_{\chi}$  plane. The  $pp \to S^+S^- \to \ell^+\chi\ell^-\chi$  reach can be enhanced if  $\lambda_{HS} \neq 0$  such that the off-shell Higgs mediated production  $gg \to h^* \to S^+S^-$  also plays a role. On the other hand, a large enough  $\lambda_{HS}$  can trigger the first-order phase transition and leave signals at the GW detectors like LISA, Taiji, TianQin, BBO and U-DECIGO. The enhancement of LHC reach and the LISA projections are shown in light blue and orange regions for  $\lambda_{HS} = 2$  and 3, respectively. One can see that the LHC and LISA experiments mainly serve as complementary approaches to probe the DM parameter space; while they also have some intersections, which can be used to identify the origin of the excess detected in the future.

The interplay between GW detections and future  $e^+e^-$  collider searches are shown in Fig. 2, where we show the LISA projections and the CEPC Higgs measurement projections for comparison. The first-order phase transition and LISA-detectable GW parameter space are determined by  $m_S$ and  $\lambda_{HS}$ , as shown in gray shaded regions. The CEPC projections for Higgs invisible decay,  $h\mu\mu$ and  $h\tau\tau$  couplings depend an extra parameter, the DM mass  $m_{\chi}$ . Given an  $m_{\chi}$ , the region above the colored lines in Fig. 2 can be probed by the CEPC with a collision energy of 240 GeV and an integrated luminosity of 5 ab<sup>-1</sup>. The intersections between the LISA and CEPC projections can be used for crosschecking the excess obtained in either approach; while in other parameter space the two approaches are complementary.



Figure 2: Figure from Ref. [49]. The interplay between GW detection and future  $e^+e^-$  collider searches. The gray shaded region is the LISA detectable parameter space, varying  $\lambda_S$  from 0 to  $4\pi$ . From left to right, we show the sensitivities for  $\lambda_{HS}$  from future FCC-ee and CEPC precision measurements, based on invisible Higgs decay branching ratio  $Br(h \to inv) = 0.3\%$ , Higgs leptonic coupling precision reaches  $\delta \kappa_{\mu} < 8.7\%$  and  $\delta \kappa_{\tau} < 1.5\%$ .

# 3 Summary and outlook

We have shown in Ref. [49] that collider searches can probe the lepton portal DM model efficiently, and the GW detection experiments can be an important complementary crosscheck. In general, such complementarity between collider searches and GW experiments exist in many WIMP models with scalar DM and/or scalar mediators, dubbed dark scalars S, which are usually oddly charged under a  $\mathbb{Z}_2$  symmetry. This is because the joint potential between Higgs and the dark scalars, especially the Higgs portal coupling  $|S|^2|H|^2$ , is inevitable in the model, and the Higgs portal coupling could serve as the source of the potential barrier that trigger the first-order phase transition. Therefore, the WIMP model might manifest itself both via collider and GW signals in the future, and a correlation/complementarity study would be extremely useful.

In the future, more studies can be done along this line, for example in Ref. [49] the charged singlet scalar mediator is considered while doublet scalar as mediator is also possible. Moreover, the study shows the example for slepton-like scalars and one can extend to squark-like scalar as mediator as well. In addition, DM itself as scalar can also be possible to show its relevance at colliders and GW probes. In this aspect, single scalar DM model has been extensively studied already [55–60], while higher multiplets scalar DM is less studied and worth probing.

#### Acknowledgments

The work of J.L. is supported by National Science Foundation of China under Grant No. 12075005 and by Peking University under startup Grant No. 7101502597. The work of X.P.W. is supported by National Science Foundation of China under Grant No. 12005009. K.P.X. is supported by the University of Nebraska-Lincoln.

## References

[1] Planck Collaboration, N. Aghanim et al., "Planck 2018 results. VI. Cosmological

parameters," *Astron. Astrophys.* **641** (2020) A6, arXiv:1807.06209 [astro-ph.CO]. [Erratum: Astron.Astrophys. 652, C4 (2021)].

- [2] G. Bertone, D. Hooper, and J. Silk, "Particle dark matter: Evidence, candidates and constraints," *Phys. Rept.* 405 (2005) 279–390, arXiv:hep-ph/0404175.
- [3] B. W. Lee and S. Weinberg, "Cosmological Lower Bound on Heavy Neutrino Masses," *Phys. Rev. Lett.* **39** (1977) 165–168.
- [4] M. Schumann, "Direct Detection of WIMP Dark Matter: Concepts and Status," J. Phys. G 46 no. 10, (2019) 103003, arXiv:1903.03026 [astro-ph.CO].
- J. M. Gaskins, "A review of indirect searches for particle dark matter," Contemp. Phys. 57 no. 4, (2016) 496-525, arXiv:1604.00014 [astro-ph.HE].
- [6] A. Boveia and C. Doglioni, "Dark Matter Searches at Colliders," Ann. Rev. Nucl. Part. Sci. 68 (2018) 429-459, arXiv:1810.12238 [hep-ex].
- [7] Y. Bai and J. Berger, "Lepton Portal Dark Matter," JHEP 08 (2014) 153, arXiv:1402.6696
  [hep-ph].
- [8] Z.-H. Yu, X.-J. Bi, Q.-S. Yan, and P.-F. Yin, "Tau Portal Dark Matter models at the LHC," *Phys. Rev. D* 91 no. 3, (2015) 035008, arXiv:1410.3347 [hep-ph].
- [9] W. Altmannshofer, P. J. Fox, R. Harnik, G. D. Kribs, and N. Raj, "Dark Matter Signals in Dilepton Production at Hadron Colliders," *Phys. Rev. D* 91 no. 11, (2015) 115006, arXiv:1411.6743 [hep-ph].
- [10] J.-H. Yu, "Vector Fermion-Portal Dark Matter: Direct Detection and Galactic Center Gamma-Ray Excess," Phys. Rev. D 90 no. 9, (2014) 095010, arXiv:1409.3227 [hep-ph].
- [11] M. Garny, A. Ibarra, and S. Vogl, "Signatures of Majorana dark matter with t-channel mediators," Int. J. Mod. Phys. D 24 no. 07, (2015) 1530019, arXiv:1503.01500 [hep-ph].
- [12] P. Agrawal, Z. Chacko, C. Kilic, and C. B. Verhaaren, "A Couplet from Flavored Dark Matter," JHEP 08 (2015) 072, arXiv:1503.03057 [hep-ph].
- [13] A. Ibarra and S. Wild, "Dirac dark matter with a charged mediator: a comprehensive one-loop analysis of the direct detection phenomenology," JCAP 05 (2015) 047, arXiv:1503.03382 [hep-ph].
- [14] Y. Cai and A. P. Spray, "Fermionic Semi-Annihilating Dark Matter," JHEP 01 (2016) 087, arXiv:1509.08481 [hep-ph].
- [15] S. Baek and Z.-F. Kang, "Naturally Large Radiative Lepton Flavor Violating Higgs Decay Mediated by Lepton-flavored Dark Matter," JHEP 03 (2016) 106, arXiv:1510.00100 [hep-ph].
- [16] M.-C. Chen, J. Huang, and V. Takhistov, "Beyond Minimal Lepton Flavored Dark Matter," JHEP 02 (2016) 060, arXiv:1510.04694 [hep-ph].

- [17] A. Berlin, D. S. Robertson, M. P. Solon, and K. M. Zurek, "Bino variations: Effective field theory methods for dark matter direct detection," *Phys. Rev. D* 93 no. 9, (2016) 095008, arXiv:1511.05964 [hep-ph].
- [18] P. Agrawal, Z. Chacko, E. C. F. S. Fortes, and C. Kilic, "Skew-Flavored Dark Matter," Phys. Rev. D 93 no. 10, (2016) 103510, arXiv:1511.06293 [hep-ph].
- [19] A. Mukherjee and M. K. Das, "Neutrino phenomenology and scalar Dark Matter with A<sub>4</sub> flavor symmetry in Inverse and type II seesaw," Nucl. Phys. B **913** (2016) 643–663, arXiv:1512.02384 [hep-ph].
- [20] J. A. Evans and J. Shelton, "Long-Lived Staus and Displaced Leptons at the LHC," JHEP 04 (2016) 056, arXiv:1601.01326 [hep-ph].
- [21] W. Chao, H.-K. Guo, and H.-L. Li, "Tau flavored dark matter and its impact on tau Yukawa coupling," JCAP 02 (2017) 002, arXiv:1606.07174 [hep-ph].
- [22] D. Borah, S. Sadhukhan, and S. Sahoo, "Lepton Portal Limit of Inert Higgs Doublet Dark Matter with Radiative Neutrino Mass," *Phys. Lett. B* 771 (2017) 624-632, arXiv:1703.08674 [hep-ph].
- [23] K. Kowalska and E. M. Sessolo, "Expectations for the muon g-2 in simplified models with dark matter," JHEP 09 (2017) 112, arXiv:1707.00753 [hep-ph].
- [24] G. H. Duan, L. Feng, F. Wang, L. Wu, J. M. Yang, and R. Zheng, "Simplified TeV leptophilic dark matter in light of DAMPE data," JHEP 02 (2018) 107, arXiv:1711.11012 [hep-ph].
- [25] Q. Yuan et al., "Interpretations of the DAMPE electron data," arXiv:1711.10989 [astro-ph.HE].
- [26] Y.-L. Tang, L. Wu, M. Zhang, and R. Zheng, "Lepton-portal Dark Matter in Hidden Valley model and the DAMPE recent results," *Sci. China Phys. Mech. Astron.* 61 no. 10, (2018) 101003, arXiv:1711.11058 [hep-ph].
- [27] S.-F. Ge, H.-J. He, and Y.-C. Wang, "Flavor Structure of the Cosmic-Ray Electron/Positron Excesses at DAMPE," Phys. Lett. B 781 (2018) 88-94, arXiv:1712.02744 [astro-ph.HE].
- [28] R. Ding, Z.-L. Han, L. Feng, and B. Zhu, "Confronting the DAMPE Excess with the Scotogenic Type-II Seesaw Model," *Chin. Phys. C* 42 no. 8, (2018) 083104, arXiv:1712.02021 [hep-ph].
- [29] M. J. Baker and A. Thamm, "Leptonic WIMP Coannihilation and the Current Dark Matter Search Strategy," JHEP 10 (2018) 187, arXiv:1806.07896 [hep-ph].
- [30] J. Hisano, R. Nagai, and N. Nagata, "Singlet Dirac Fermion Dark Matter with Mediators at Loop," JHEP 12 (2018) 059, arXiv:1808.06301 [hep-ph].
- [31] A. Gaviria, R. Longas, and O. Zapata, "Charged lepton flavor violation and electric dipole moments in the inert Zee model," *JHEP* **10** (2018) 188, arXiv:1809.00655 [hep-ph].

- [32] B. J. Kavanagh, P. Panci, and R. Ziegler, "Faint Light from Dark Matter: Classifying and Constraining Dark Matter-Photon Effective Operators," JHEP 04 (2019) 089, arXiv:1810.00033 [hep-ph].
- [33] J. Kawamura, S. Okawa, and Y. Omura, "Current status and muon g 2 explanation of lepton portal dark matter," JHEP 08 (2020) 042, arXiv:2002.12534 [hep-ph].
- [34] H. Okada and Y. Shoji, "Dirac dark matter in a radiative neutrino model," *Phys. Dark Univ.* 31 (2021) 100742, arXiv:2003.11396 [hep-ph].
- [35] S.-F. Ge, H.-J. He, Y.-C. Wang, and Q. Yuan, "Probing flavor structure of cosmic ray e<sup>∓</sup> spectrum and implications for dark matter indirect searches," *Nucl. Phys. B* 959 (2020) 115140, arXiv:2004.10683 [astro-ph.HE].
- [36] C. Bœhm, X. Chu, J.-L. Kuo, and J. Pradler, "Scalar Dark Matter Candidates Revisited," arXiv:2010.02954 [hep-ph].
- [37] S. Okawa and Y. Omura, "Light mass window of lepton portal dark matter," arXiv:2011.04788 [hep-ph].
- [38] K. Kowalska and E. M. Sessolo, "Minimal models for g 2 and dark matter confront asymptotic safety," arXiv:2012.15200 [hep-ph].
- [39] R. Verma, M. Kashav, S. Verma, and B. C. Chauhan, "Scalar Dark Matter in an Inverse Seesaw Model with A\_4 Discrete Flavor Symmetry," arXiv:2102.03074 [hep-ph].
- [40] C. Alvarado, C. Bonilla, J. Leite, and J. W. F. Valle, "Phenomenology of fermion dark matter as neutrino mass mediator with gauged B-L," arXiv:2102.07216 [hep-ph].
- [41] S.-i. Horigome, T. Katayose, S. Matsumoto, and I. Saha, "Leptophilic fermion WIMP ~ Role of future lepton colliders," arXiv:2102.08645 [hep-ph].
- [42] Y. Bai and J. Berger, "Muon g-2 in Lepton Portal Dark Matter," arXiv:2104.03301 [hep-ph].
- [43] A. Jueid, S. Nasri, and R. Soualah, "Searching for GeV-scale Majorana Dark Matter: inter spem et metum," JHEP 04 (2021) 012, arXiv:2006.01348 [hep-ph].
- [44] G. Arcadi, L. Calibbi, M. Fedele, and F. Mescia, "Systematic approach to B-physics anomalies and t-channel dark matter," arXiv:2103.09835 [hep-ph].
- [45] G. Arcadi, L. Calibbi, M. Fedele, and F. Mescia, "Muon g 2 and B-anomalies from Dark Matter," arXiv:2104.03228 [hep-ph].
- [46] L. Calibbi, R. Ziegler, and J. Zupan, "Minimal models for dark matter and the muon g-2 anomaly," JHEP 07 (2018) 046, arXiv:1804.00009 [hep-ph].
- [47] M. Cermeño, C. Degrande, and L. Mantani, "Signatures of leptophilic t-channel dark matter from active galactic nuclei," arXiv:2201.07247 [hep-ph].

- [48] M. Cermeño, C. Degrande, and L. Mantani, "Circular polarisation of gamma rays as a probe of dark matter interactions with cosmic ray electrons," *Phys. Dark Univ.* 34 (2021) 100909, arXiv:2103.14658 [hep-ph].
- [49] J. Liu, X.-P. Wang, and K.-P. Xie, "Searching for lepton portal dark matter with colliders and gravitational waves," JHEP 06 (2021) 149, arXiv:2104.06421 [hep-ph].
- [50] Muon g-2 Collaboration, B. Abi et al., "Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm," Phys. Rev. Lett. 126 (2021) 141801, arXiv:2104.03281 [hep-ex].
- [51] ALEPH, DELPHI, L3, OPAL Experiments, "Combined LEP Chargino Results, up to 208 GeV for low DM," 2002. http: //lepsusy.web.cern.ch/lepsusy/www/inoslowdmsummer02/charginolowdm\_pub.html. LEPSUSYWG/02-04.1.
- [52] **ATLAS** Collaboration, G. Aad *et al.*, "Search for direct production of charginos, neutralinos and sleptons in final states with two leptons and missing transverse momentum in pp collisions at  $\sqrt{s} = 8$  TeV with the ATLAS detector," *JHEP* **05** (2014) 071, arXiv:1403.5294 [hep-ex].
- [53] **ATLAS** Collaboration, G. Aad *et al.*, "Search for electroweak production of charginos and sleptons decaying into final states with two leptons and missing transverse momentum in  $\sqrt{s} = 13$  TeV *pp* collisions using the ATLAS detector," *Eur. Phys. J.* **C80** no. 2, (2020) 123, arXiv:1908.08215 [hep-ex].
- [54] **ATLAS** Collaboration, G. Aad *et al.*, "Searches for electroweak production of supersymmetric particles with compressed mass spectra in  $\sqrt{s} = 13$  TeV *pp* collisions with the ATLAS detector," *Phys. Rev.* **D101** no. 5, (2020) 052005, arXiv:1911.12606 [hep-ex].
- [55] M. Jiang, L. Bian, W. Huang, and J. Shu, "Impact of a complex singlet: Electroweak baryogenesis and dark matter," *Phys. Rev. D* 93 no. 6, (2016) 065032, arXiv:1502.07574 [hep-ph].
- [56] V. R. Shajiee and A. Tofighi, "Electroweak Phase Transition, Gravitational Waves and Dark Matter in Two Scalar Singlet Extension of The Standard Model," *Eur. Phys. J. C* 79 no. 4, (2019) 360, arXiv:1811.09807 [hep-ph].
- [57] X.-F. Han, L. Wang, and Y. Zhang, "Dark matter, electroweak phase transition, and gravitational waves in the type II two-Higgs-doublet model with a singlet scalar field," *Phys. Rev. D* 103 no. 3, (2021) 035012, arXiv:2010.03730 [hep-ph].
- [58] Z. Zhang, C. Cai, X.-M. Jiang, Y.-L. Tang, Z.-H. Yu, and H.-H. Zhang, "Phase transition gravitational waves from pseudo-Nambu-Goldstone dark matter and two Higgs doublets," *JHEP* 05 (2021) 160, arXiv:2102.01588 [hep-ph].
- [59] D. Borah, A. Dasgupta, and S. K. Kang, "Gravitational waves from a dark U(1)D phase transition in light of NANOGrav 12.5 yr data," *Phys. Rev. D* 104 no. 6, (2021) 063501, arXiv:2105.01007 [hep-ph].

[60] F. Costa, S. Khan, and J. Kim, "A Two-Component Dark Matter Model and its Associated Gravitational Waves," arXiv:2202.13126 [hep-ph].