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A short overview on low mass scalars at future lepton colliders - Snowmass White Paper

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Abstract

In this whitepaper, I give a short summary on possible channels of low-mass scenarios and their discovery potential at future e^+e^- colliders. This is a summary of talks I recently gave at the CEPC workshop, FCC week and ECFA future collider workshop.

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I. INTRODUCTION

The discovery of a scalar which so far largely agrees with predictions for the Higgs boson of the Standard Model (SM) has by now been established by the LHC experiments, with analyses of Run II LHC data further confirming this. In the European Strategy Report [1, 2], a large focus was put on future e^+e^- colliders, especially so-called Higgs factories with center-of-mass (com) energies around $240 - 250\,\text{GeV}$. While these will on the on hand further help to determine properties of the scalar discovered at the LHC, and especially will help to determine in detail the parameters and shape of the scalar potential, it is also interesting to investigate theis potential to search for additional scalar states. Many new physics models still allow for extra scalar states, including those which have masses $\lesssim 125\,\text{GeV}$.

In this whitepaper, I give a short summary of presentations I gave at various recent meetings and workshops. I give an overview on some models that allow for such light states, and point to phenomenological studies investigating such models. This should be viewed as an encouragement for further detailed studies in this direction.

II. MODELS

A. Singlet extensions

In singlet extensions, the SM scalar potential is enhanced by additional scalar states that are singlets under the SM gauge group. In such scenarios, the coupling of the novel scalar to SM particles is typically inherited via mixing, i.e. mass-eigenstates are related to gauge eigenstates via a unitary mixing matrix. The corresponding couplings and interactions are mediated via a simple mixing angle.

In [3], the authors present the status of current searches for the process

$$p\,p\,\rightarrow\,h_{125}\,\rightarrow\,s\,s\,\rightarrow\,X\,X\,Y\,Y \tag{1}$$

which for such models can be read as a bound in

$$\sin^2\theta\,\times\,\text{BR}_{h_{125}\rightarrow s\,s\rightarrow X\,X\,Y\,Y}.$$

We display these results in figure 1.

We show an example of the allowed parameter space in a model with two additional singlets, the two real scalar extension studied in [4]. In this model, three CP-even neutral scalars exist that relate the gauge and mass eigenstates $h_{1,2,3}$ via mixing. One of these states has to have couplings and mass complying with current measurements of the SM-like scalar, the other two can have higher or lower masses. In figure 2, we show two cases where either one (high-low) or two (low-low) scalar masses are smaller than $125\,\text{GeV}$. On the y-axis, the

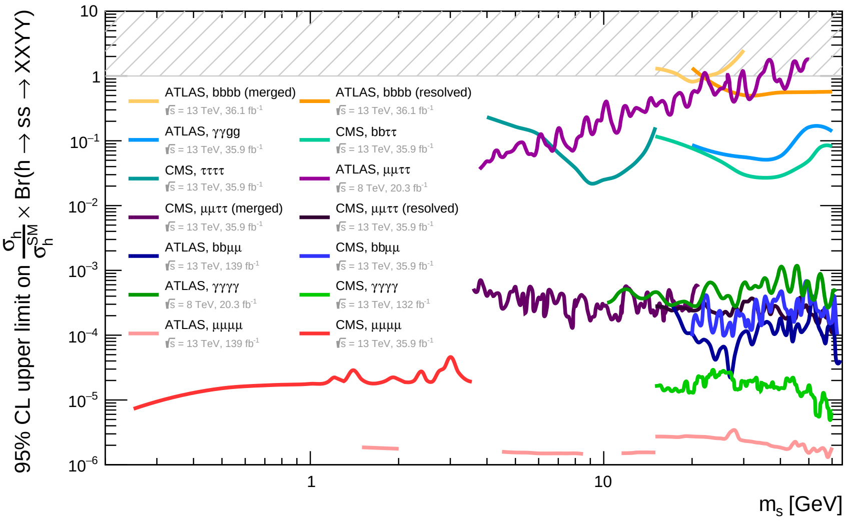

FIG. 1. Limits on the process in eqn (1, taken from [3]. This displays current constraints which can especially be easily reinterpreted in extended scalar sector models, in particular models where couplings are inherited via a simple mixing angle.

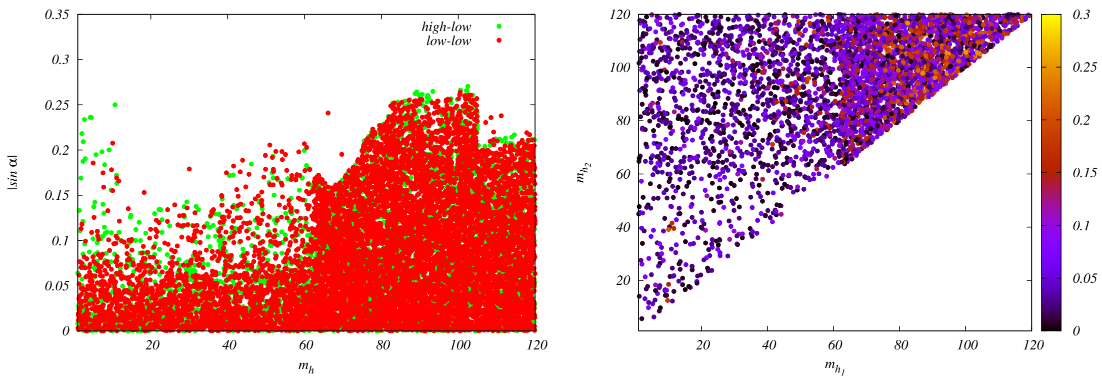

FIG. 2. Available parameter space in the TRSM, with one (high-low) or two (low-low) masses lighter than 125 GeV. *Left*: light scalar mass and mixing angle, with $\sin\alpha = 0$ corresponding to complete decoupling. *Right:* available parameter space in the (m_{h_1}, m_{h_2}) plane, with color coding denoting the rescaling parameter $\sin\alpha$ for the lighter scalar h_1.

respective mixing angle is shown. Complete decoupling would be designated by $\sin\alpha = 0$ in the notation used in this figure.

The points were generated using ScannerS [5, 6], interfaced to HiggsBounds-5.10.2 [7–10] and HiggsSignals-2.6.2 [11, 12], with constraints as implemented in these versions.

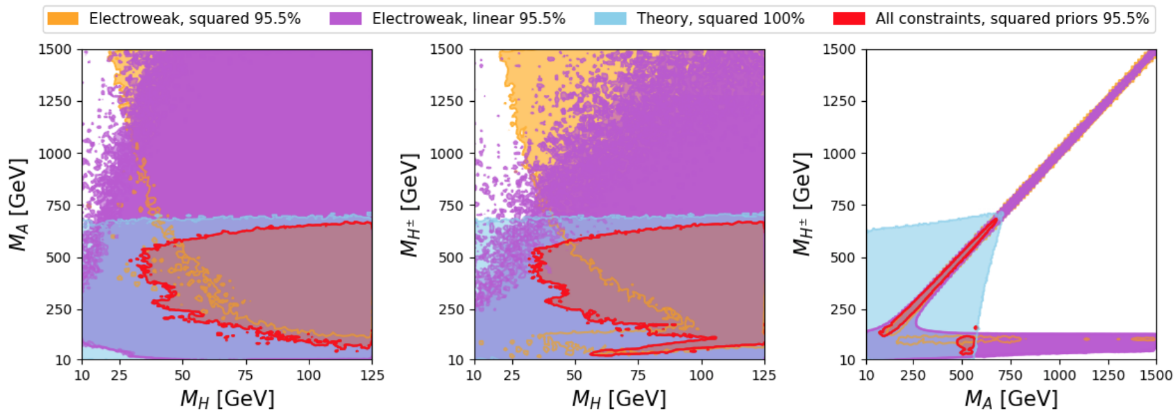

FIG. 3. Allowed regions in the 2HDM, from a scan presented in [14].

B. Two Higgs Doublet Models

Two Higgs doublet models (2HDMs) constitute another example of new physics models allowing for low mass scalar states. A general discussion of such models is e.g. given in [13] and will not be repeated here. In general, such models contain, besides the SM candidate, two additional neutral scalars which differ in CP properties as well as a charged scalar, so the particle content is given by h, H, A, $H^{\pm}$, where one of the two CP-even neutral scalars h, H needs to be identified with the 125 GeV resonance discovered at the LHC. Couplings to the fermions in the Yukawa sector distinguish different types of 2HDMs.

In [14], the authors perform a scan including bounds from theory, experimental searches and constraints, as e.g. electroweak observables, as well as B-physics. Examples for these scan results are shown in figure 3, taken from that reference. We see that for all regions solutions for either one or several low mass scenarios exist and are viable for the constraints discussed in that reference. Unfortunately, the information about the $\cos(\beta - \alpha)$ regions in these scenarios is not available in that reference. Depending on the Yukawa couplings considered, the limits on the absolute value of this rescaling angle vary between 0.05 and 0.25 [15].

C. Other extensions

The scalar sector of the SM can be extended by an arbitrary number of additional scalar fields, such as singlets, doublets, etc. One option which is also often consider is the extension of this sector by both singlets and doublets.

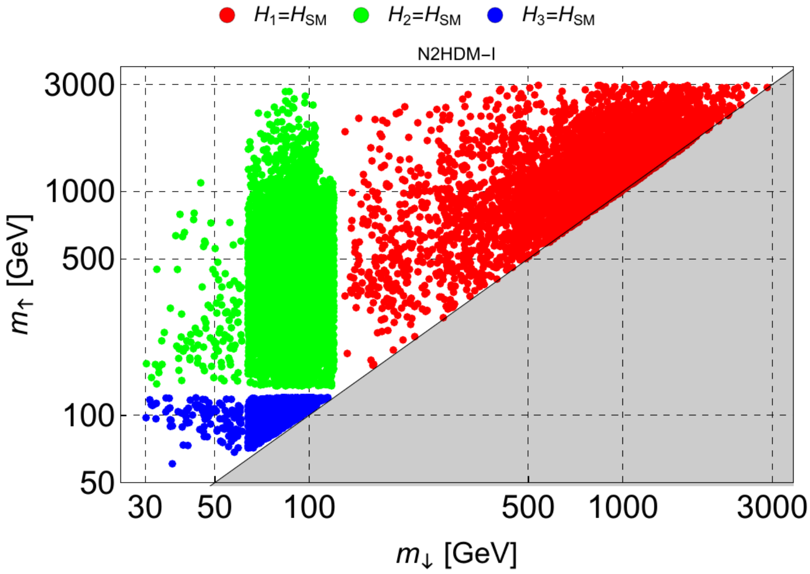

FIG. 4. Scan results in the N2HDM, taken from [16]. There are regions in the models parameter space where either one or two of the additional scalars have masses $\lesssim$ 125 GeV.

1. N2HDM

In [16], the authors consider a model where the SM scalar sector is extended by an additional doublet as well as a real singlet. This model has 3 CP even neutral scalar particles, out of which one needs to have the properties in compliance with LHC measurements of the 125 GeV scalar. The authors perform an extensive scan and find regions in parameter space where either one or both of the additional scalars have masses below 125 GeV. We show an example of the allowed parameter space in figure 4

2. Lepton-specific IDM

In [17], the authors consider a model where the SM scalar sector is augmented by an additional doublet, where they impose an exact $\mathbb{Z}_2$ symmetry. This symmetry is then broken by a specific coupling to the fermionic sector. The authors identify regions in the models parameter space that agree with current searches as well as anomalous magnetic momenta of electron and muon. They identify regions in the models parameter space where the second CP-even scalar can have a mass $\lesssim$ 30 GeV. We display these regions in figure 5.

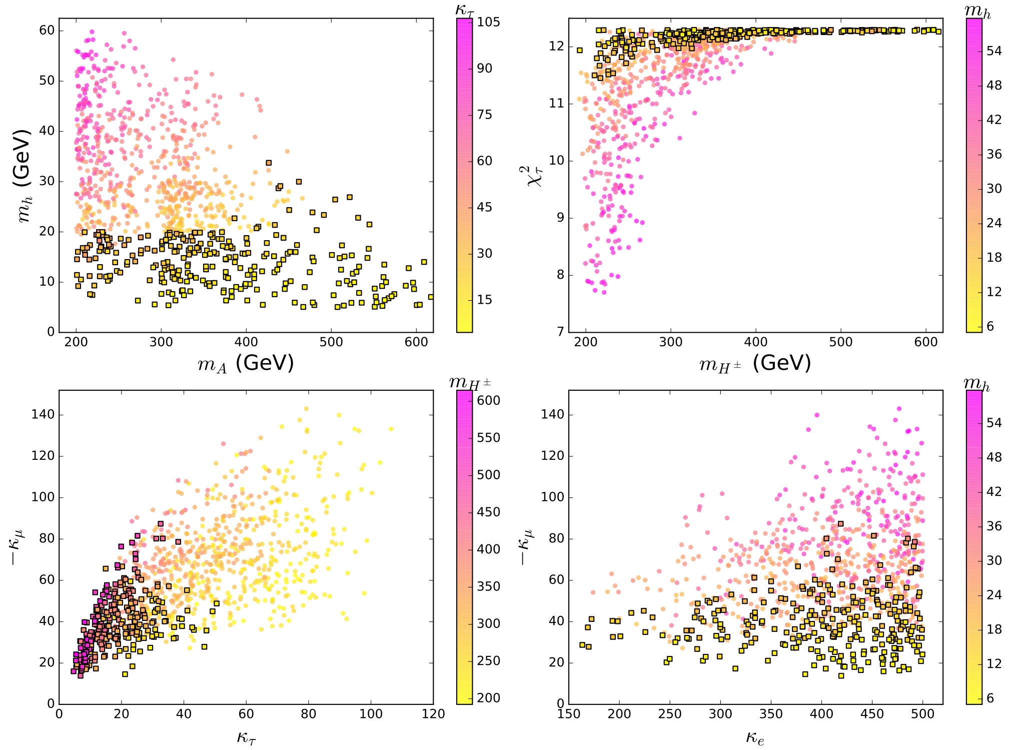

FIG. 5. Allowed regions in the parameter space of the model discussed in [17], taken from that reference, where squares denote allowed and bullets excluded regions in the models parameter space. CP-even neutral scalars with low masses are viable within this model.

3. *Scalar triplet model*

Finally, we want to discuss a model containing scalar triplets, leading to a rich particle content as well as the possibility of CP violating terms. The model has been presented in [18]. This model contains 5 neutral, 3 charged, and 2 doubly charged mass eigenstates. The authors present regions in parameter space where masses for some of these can be $\lesssim 125\,\text{GeV}$. We display these results in figure 6.

III. STUDIES AT 90 GEV

For this center-of-mass energy, several searches exist which have already been performed at LEP and are summarized in [19, 20], concentrating on $Z\,h$, $h_1\,h_2$, and $h_1\,h_1\,h_1$ final states, where h_i signifies novel scalars. Possible new studies could build upon these searches. We want to note that the luminosity at FCC-ee and CEPC at this center of mass energy is exceeding LEP luminosity by several orders of magnitude [21, 22].

We also want to present one specific study which investigates several composite models at

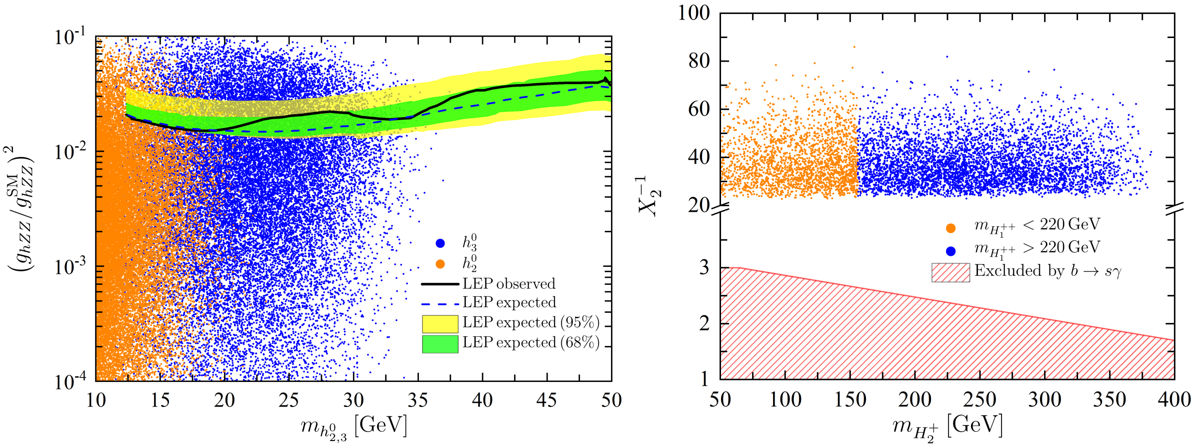

FIG. 6. Allowed regions in the parameter space of the model discussed in [18], taken from that reference. For neutral and charged new scalars, masses $\lesssim$ 125 GeV are achievable.

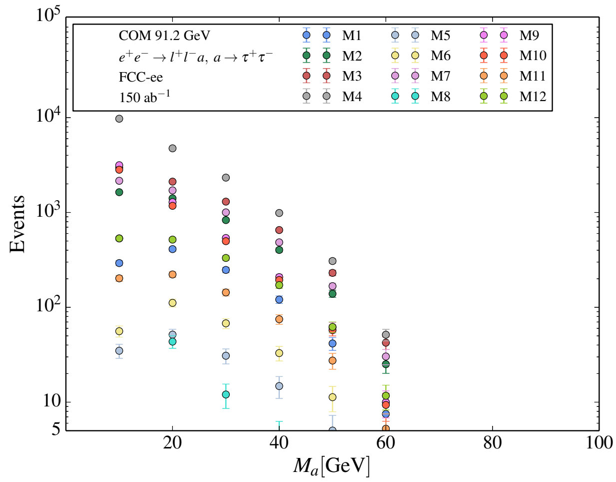

FIG. 7. Rates at a 91 GeVFCC-ee for various models discussed in [23], for $\ell^+\ell^-\tau^+\tau^-$ final states. M7 and M10 reach a 3 σ significance using ML techniques. Figure taken from [23].

a com energy of 91 GeV [23]. The authors consider the process $e^+e^- \to \ell^+\ell^-\tau^+\tau^-$, where the tau-pair stems from an additional pseudoscalar a radiated off one of the fermion lines in the $\ell\ell$ pair-production. They apply a cut-based study as well as an improved analysis using machine learning techniques; for the latter, the authors are able to achieve a 3 σ exclusion for benchmarks with masses $M_a \sim 20\,\text{GeV}$. We display event rates for the various benchmark scenarios in figure 7.

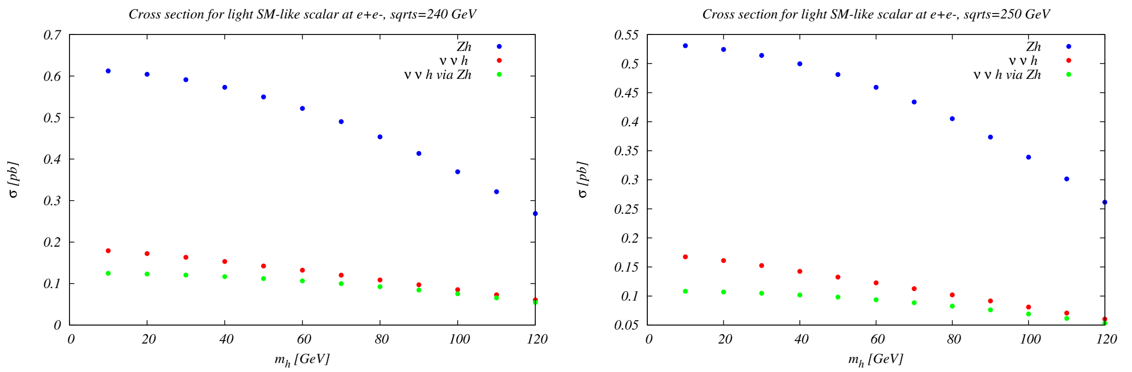

FIG. 8. Leading order production cross sections for $Z\,h$ and $h\,\nu_\ell\,\bar{\nu}_\ell$ production at an $e^+\,e^-$ collider with a com energy of 240 GeV *(left)* and 250 GeV *(right)* using Madgraph5 for an SM-like scalar h. Shown is also the contribution of $Z\,h$ to $\nu_\ell\,\bar{\nu}_\ell\,h$ using a factorized approach for the Z decay.

IV. STUDIES AT 240-250 GEV

Throughout this work, we show for reference leading order predictions for Zh production at e^+e^- colliders for low mass scalars which are SM-like. These results were obtained using Madgraph5 [24] and are given for approximate reference only. We also display the VBF-type production of $e^+e^- \rightarrow h\,\nu_\ell\,\bar{\nu}_\ell$. Note that the latter signature also contains contributions from $Z\,h$ production, where $Z \rightarrow \nu_\ell\,\bar{\nu}_\ell$.

Figure 8 shows the production cross sections for these processes for a center-of-mass energy of $\sim\,240-250\,\text{GeV}$. Using these predictions, and taking into account standard rescaling $\sim\,0.1$, around 10^5-10^6 events could be produced with ILC, FCC-ee, and CEPC design luminosities [21, 22, 25].

A. Dedicated studies

1. *Light scalars in Zh production*

Not many dedicated studies exist that investigate low-mass scalars at Higgs factories. We here point to a study [26] that investigates the sensitivity of the ILC for low-mass scalars in $Z\,h$ production, either using pure Z recoil ("recoil method") or taking the light scalar decay into $b\,\bar{b}$ into account. The y-axis shows the $95\,\%$ CL limit for agreement with a background only hypothesis, which can directly be translated into an upper bound on rescaling. The authors validate their method by reproducing LEP results [20, 27] for these channels prior to applying their method to the ILC. Their predictions are shown in figure 9.

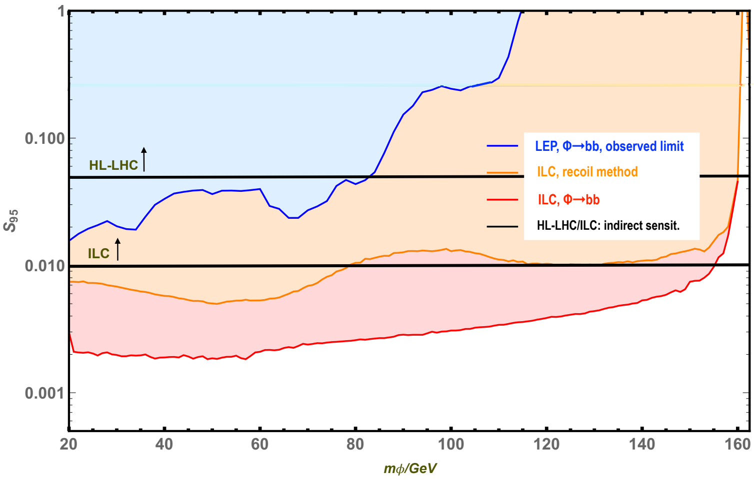

FIG. 9. Sensitivity predictions for an ILC with a com energy of 250 GeV from [26]. See text for details.

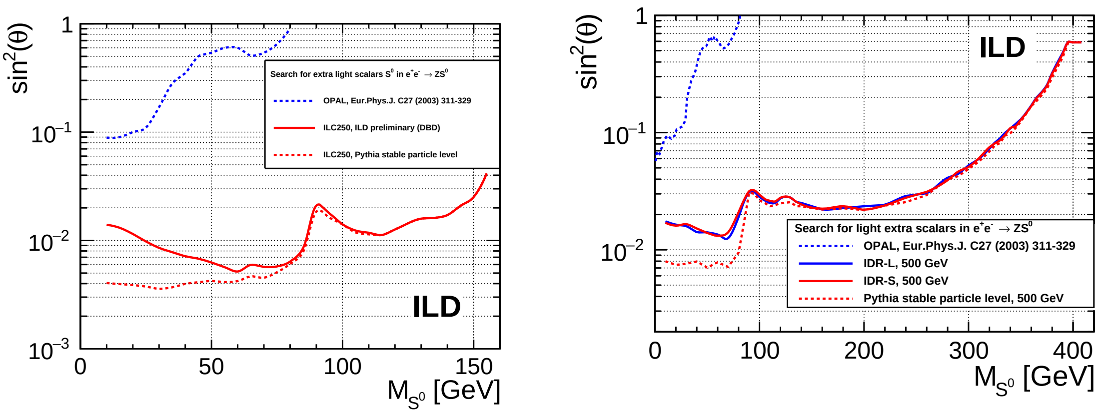

FIG. 10. Upper bounds on the mixing angle for the model discussed in [28], in a comparison of different detector concepts and using the recoil method.

A more detailed study along similar lines using the recoil method only and comparing different detector options has been presented recently in [28]. We display their results in figure 10. The authors perform their analysis in a model where the coupling of the new resonance is rescaled by a mixing angle $\sin\theta$; therefore, their results can directly be compared with the ones presented in [26] and figure 9.

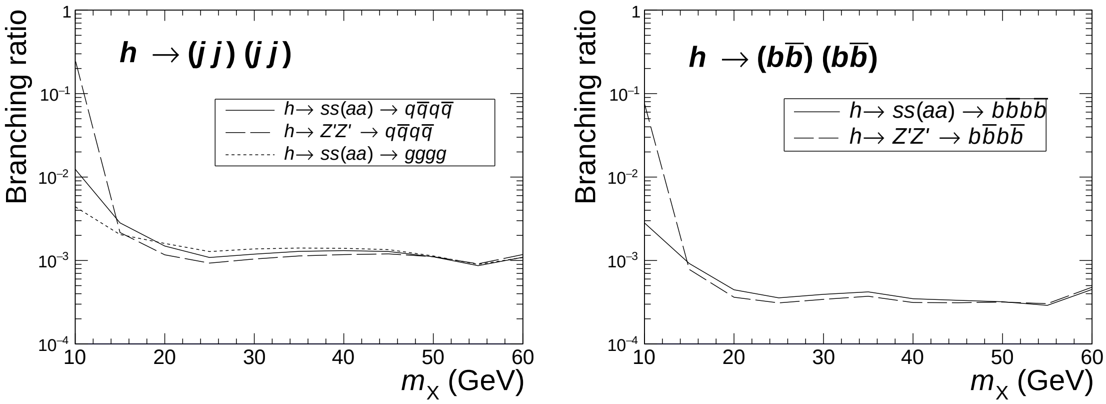

FIG. 11. 95 % confidence bounds on branching ratios for Higgs decay into a pair of lighter particles, for a com energy of 240 GeVand $\int \mathcal{L} = 5\,\mathrm{ab}^{-1}$. Taken from [29].

2. Higgsstrahlung and decay into two light scalars

In [29], the authors consider Higgs-strahlung at a 240 GeVe^+e^- collider, where the Higgs subsequently decays into two light scalar states. The give 95 % confidence level bounds for the branching ratios into the decay productions of the two light scalars as a function of the light scalar masses for an integrated luminosity of $\int \mathcal{L} = 5\,\mathrm{ab}^{-1}$ following a detailed study. Their results are subsequently used by many authors as standard reference. We show their results for various channels in figure 11.

A more recent study [30] investigates the same process, but for $4\,\tau$ final states, for the same center-of-mass energy and integrated luminosity. The results, for varying values of tracking efficiency, are shown in figure 12. Note that curent constraints on the invisible branching ratio of the Higgs, the signal strength, as well as SM-like decays of the light scalars currently render a bound $\lesssim 10^{-3}$.

Several works make use of the bounds derived in [29]. In [31], the authors investigate the allowed parameter space in the scNMSSM, an NMSSM extension that relaxes unification requirements at the GUT scale [32–35], also known as NUHM, which contains in total 5 scalar particles; if CP is conserved, 3 are CP-even and 2 CP-odd. The authors investigate various bounds on the models parameter space, and show the allowed scan points and predictions for the above channels for various scalar combinations. We show their results in figure 13.

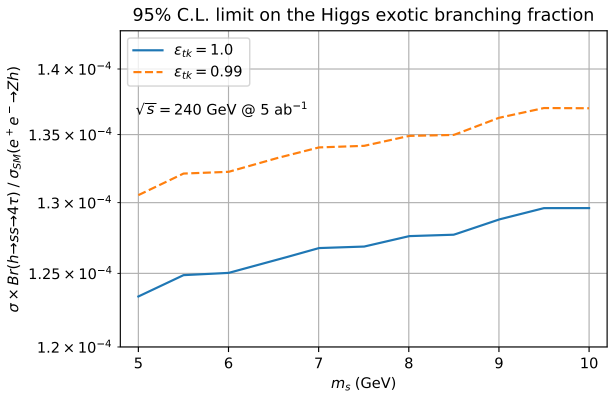

FIG. 12. Bounds on decay of the SM Higgs boson into two light scalars, with a $4\,\tau$ final state, at an e^+e^- collider with a com energy of 240 GeV, with different assumptions on tracking efficiencies. Taken from [30].

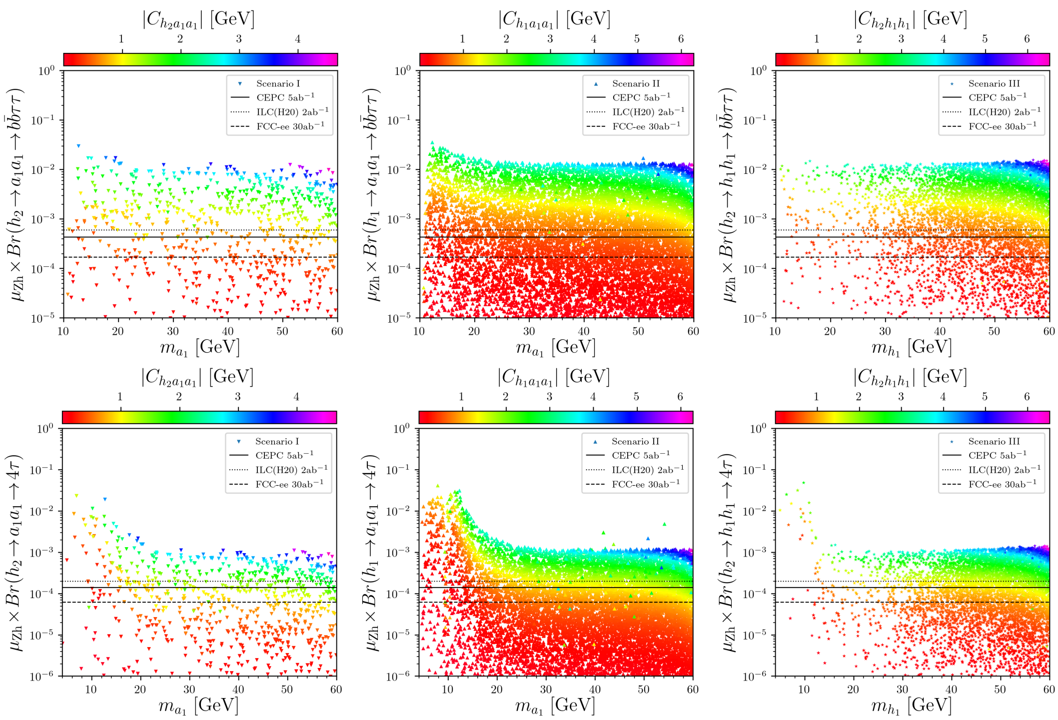

FIG. 13. Allowed rates for various Higgsstrahlung processes with successive decays into two light scalars. *Top:* $2\,b\,2\,\tau$ final state. *Bottom:* 4τ final state. Also shown are expected upper bounds for various collider machines, with projections from [29]. Figure taken from [31].

Finally, in simple singlet extensions it is possible to test regions in the models parameter space which can lead to a strong first-order electroweak phase transition. Several authors have worked on this; we here show results from [36], where in addition several collider sensitivity projections are shown, including the bounds derived in [29]. From figure 14, it

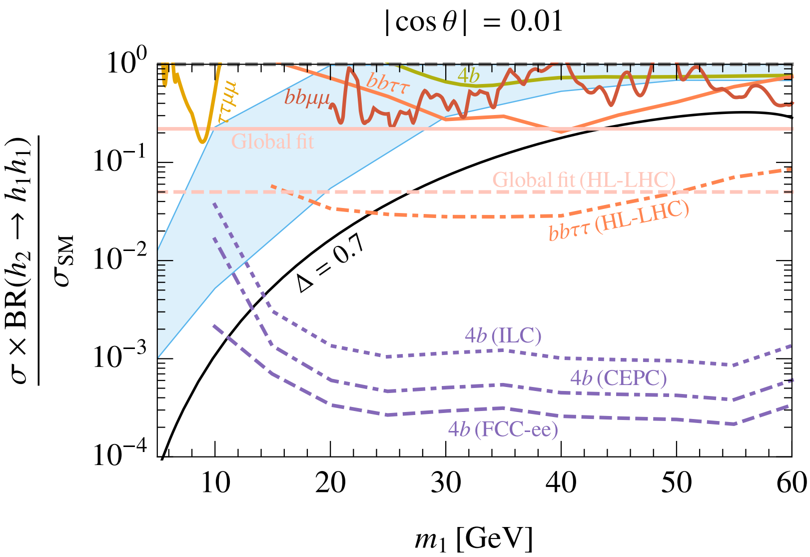

FIG. 14. Expected bounds on Higgs production via Higgs strahlung and subsequent decay into two light scalars, in the singlet extension scenario discussed in [36]. The blue band denotes the region where a strong first-order electroweak phase transition is possible. We see that e^+e^- Higgs factories are required on order to confirm or exclude such scenarios. Figure taken from [36].

becomes obvious that e^+e^- Higgs factories would be an ideal environment to confirm or rule out such scenarios.

Related work, with a spontaneous breaking of the imposed $\mathbb{Z}_2$ symmetry, has been presented in [37].

3. *Other channels*

In [38], the authors investigate a slightly different channel, i.e. tau pair-production where a light pseudoscalar is radiated off one on the outgoing fermion lines and decays again into $\tau^+\tau^-$, leading to a 4 τ final state. They are investigating this within a type X 2HDM, which in addition allows them to explain the current discrepacy between theoretical prediction and experiment for the anomalous magnetic momentum of the muon. They perform a detailed study including background and determine 2 and 5 σ countours in the m_A, $\tan\beta$ plane, where $\tan\beta$ denotes the ratio of the vevs of the two doublets. Their results are shown in figure 15.

It is also interesting to investigate models with give the possibility of light charged scalars. A corresponding study has been performed in [39], where the authors consider charged scalar

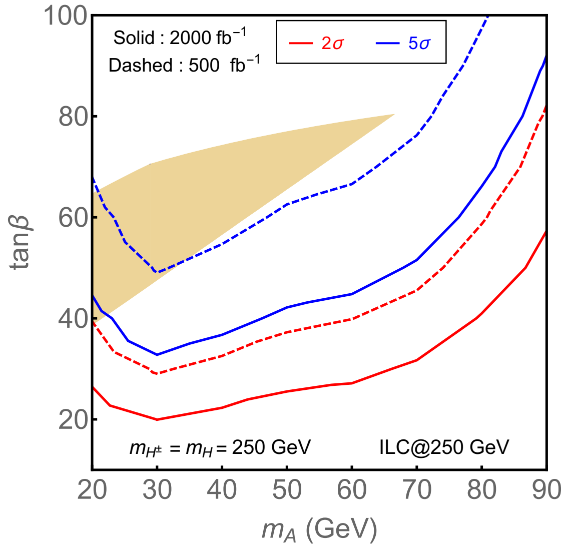

FIG. 15. Exclusion and discovery regions in the 2HDM type X model, in the m_A, $\tan\beta$ plane. The color region additionally explains the current $g_\mu - 2$ discrepancy. Taken from [38].

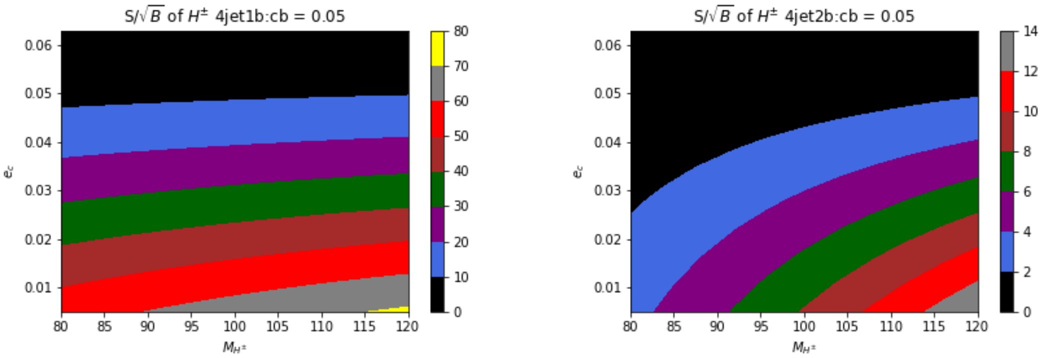

FIG. 16. Significances as a function of charged scalar mass and charm tagging efficiency at an 240 GeV CEPC, at an integrated luminosity of $1\,\text{ab}^{-1}$, within a 3HDM as presented in [39], considering a $c\bar{c}b\bar{b}$ final state. Figures taken from that reference.

pair-production within a 3HDM, with successive decays into $c\,\bar{c}\,b\,\bar{b}$ final states. The authors perform a detailed study and present their results in the 1 and 2 b-jet tagged category, as a function of light scalar mass and charm tagging efficiency. We show the corresponding significances in figure 16, for a com energy of 240 GeVand an integrated luminosity of $1\,\text{ab}^{-1}$.

B. Cross section predictions

Inspired by possible low-mass excesses in at LEP [20] and CMS [40], in [41] several models are fitted to these excesses that contain singlet and doublet extensions of the SM

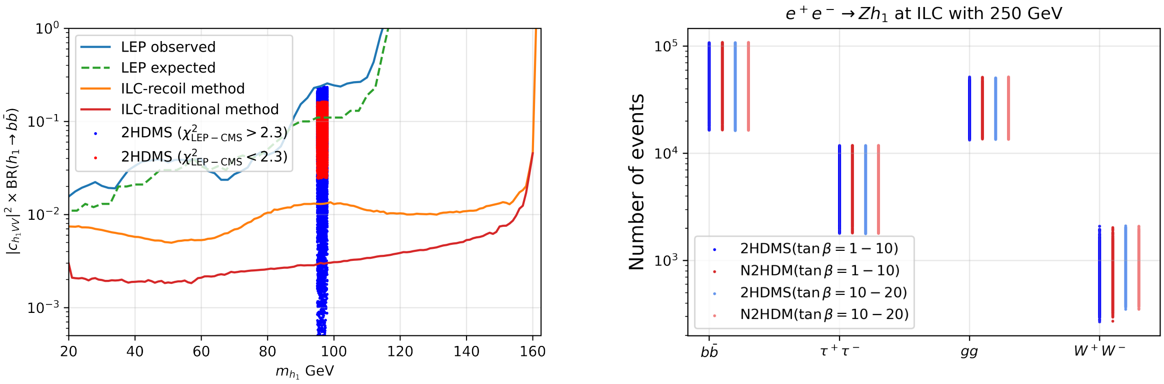

FIG. 17. *Left:* Points in the 2HDMs that agree with both CMS and LEP excess and which can be probed at the ILC. *Right:* predicted rates in the 2HDMS and N2HDM at 250 GeVusing full target luminosity.

scalar sector; in particular, they consider models with an additional doublet as well as a (complex) singlet, labelled N2HDM and 2HDMs, respectively. For both models, as well as varying $\tan\beta$ ranges (where $\tan\beta$ denotes the ratio of the vevs in the 2HDM part of the models), the authors consider the possibility to explain the observed accesses and give rate predictions for a 250 GeV collider with a total luminosity of $\mathcal{L} = 2\,\text{ab}^{-1}$. We display their results in figure 17. We see that also other final states for the h decay, as e.g. $\tau^+\tau^-$, gg, or W^+W^- can render sizeable rates.

V. OTHER CENTER OF MASS ENERGIES

The FCC-ee and CEPC colliders are supposed to also run with a center-of-mass energy of $\sim$ 160 GeV, already tested at LEP. In analogy to figure 8, in figure 18 we show again cross section predictions for the process $Z\,h$ in dependence of the mass of h, assuming a SM-like scalar. Note we here assume onshell production of $Z\,h$, which leads to a hard cutoff for $M_h \sim 70\,\text{GeV}$. Detailed studies should in turn assume contributions from offshell Zs and hs as well.

We see that for this lower com energy, there is basically no contribution to the $\nu_\ell\,\bar{\nu}_\ell h$ final state that does not originate from $Z\,h$. Using FCC-ee target luminosity for this energy, and again assuming a general suppression factor ~ 0.1 stemming from signal strength, we expect up to 10^6 events depending on the mass of the additional scalar.

For this center-of-mass energy, several searches exist which have already been performed at LEP and are summarized in [19, 20], concentrating on $Z\,h$, $h_1\,h_2$, and $h_1\,h_1\,h_1$ final

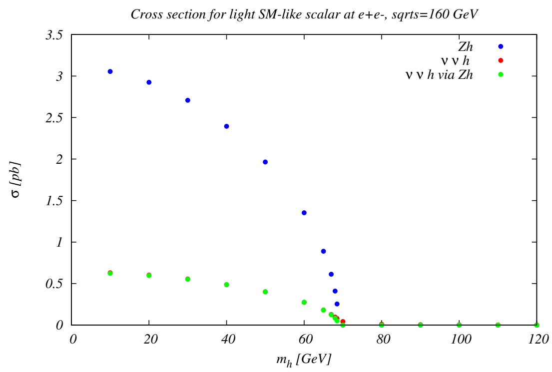

FIG. 18. As figure 18, for a com of 160 GeV. We assume onshell final states.

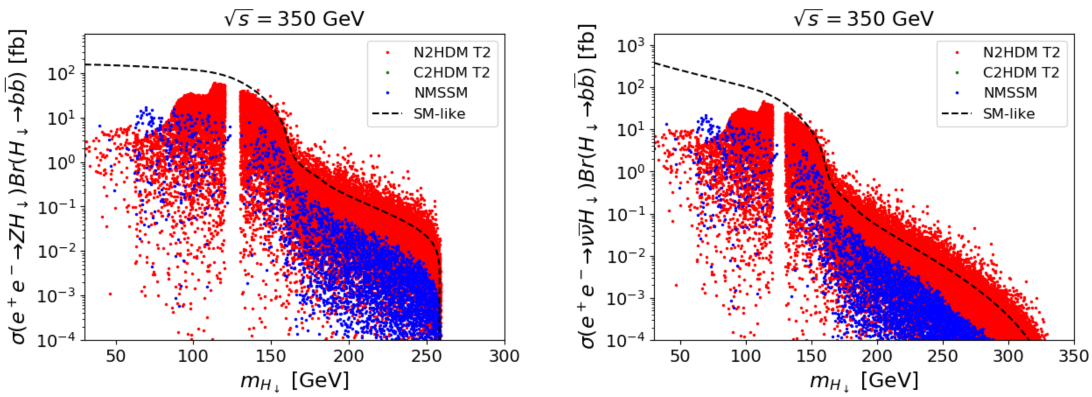

FIG. 19. Achievable rates for various light scalar production modes at an e^+e^- collider with a com energy of 350 GeV, in various 2HDM variant models. Figure taken from [42].

states, which could be further pursued in future collider studies. We want to note that the luminosity at FCC-ee at this center of mass energy is exceeding LEP luminosity by several orders of magnitude.

Finally, we present a study that investigates various types of 2HDMs containing several neutral scalars [42], for a collider energy of 350 GeV. The authors perform a scan of the allowed parameter space and render predictions for the Higgs-strahlung process as well as $\nu_\ell \bar{\nu}_\ell h$ final states with the scalar decaying into $b\bar{b}$ pairs. We show their results in figure 19.

VI. CONCLUSION

In this short note, I have presented several models and searches that investigate the sensitivity of future e^+e^- machines for scalars with masses $\lesssim 125\,\text{GeV}$. This corresponds to a summary of several talks I have recently given and is thereby not meant to be inclusive.

I have pointed to models that allow for such light scalars, as well as several references that either provide rates or pursue dedicated studies. I have also pointed to the connection of low-scalar searches at such colliders and the electroweak phase transition within certain models. My impression is that further detailed studies are called for, with a possible focus on so-called Higgs factories with center-of-mass energies around 240-250 GeV.

ACKNOWLEDGEMENTS

I thank Sven Heinemeyer and the conveners of the CEPC workshop for inspiring me to set up an overview on these models. Several authors of the references cited here were also helpful in answering specific questions regarding their work.

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[1] Richard Keith Ellis et al. Physics Briefing Book: Input for the European Strategy for Particle Physics Update 2020. 10 2019, 1910.11775.

[2] European Strategy Group. 2020 Update of the European Strategy for Particle Physics. Technical report, Geneva, 2020.

[3] Maria Cepeda, Stefania Gori, Verena Martinez Outschoorn, and Jessie Shelton. Exotic Higgs Decays. 11 2021, 2111.12751.

[4] Tania Robens, Tim Stefaniak, and Jonas Wittbrodt. Two-real-scalar-singlet extension of the SM: LHC phenomenology and benchmark scenarios. *Eur. Phys. J. C*, 80(2):151, 2020, 1908.08554.

[5] Rita Coimbra, Marco O. P. Sampaio, and Rui Santos. ScannerS: Constraining the phase diagram of a complex scalar singlet at the LHC. *Eur. Phys. J. C*, 73:2428, 2013, 1301.2599.

[6] Margarete Mühlleitner, Marco O. P. Sampaio, Rui Santos, and Jonas Wittbrodt. ScannerS: Parameter Scans in Extended Scalar Sectors. 7 2020, 2007.02985.

[7] Philip Bechtle, Oliver Brein, Sven Heinemeyer, Georg Weiglein, and Karina E. Williams. HiggsBounds: Confronting Arbitrary Higgs Sectors with Exclusion Bounds from LEP and the Tevatron. *Comput. Phys. Commun.*, 181:138–167, 2010, 0811.4169.

[8] Philip Bechtle, Oliver Brein, Sven Heinemeyer, Georg Weiglein, and Karina E. Williams. HiggsBounds 2.0.0: Confronting Neutral and Charged Higgs Sector Predictions with Exclusion Bounds from LEP and the Tevatron. *Comput. Phys. Commun.*, 182:2605–2631, 2011, 1102.1898.

[9] Philip Bechtle, Oliver Brein, Sven Heinemeyer, Oscar Stål, Tim Stefaniak, Georg Weiglein, and Karina E. Williams. HiggsBounds – 4: Improved Tests of Extended Higgs Sectors against Exclusion Bounds from LEP, the Tevatron and the LHC. *Eur. Phys. J. C*, 74(3):2693, 2014, 1311.0055.

[10] Philip Bechtle, Daniel Dercks, Sven Heinemeyer, Tobias Klingl, Tim Stefaniak, Georg Weiglein, and Jonas Wittbrodt. HiggsBounds-5: Testing Higgs Sectors in the LHC 13 TeV Era. *Eur. Phys. J. C*, 80(12):1211, 2020, 2006.06007.

[11] Philip Bechtle, Sven Heinemeyer, Oscar Stål, Tim Stefaniak, and Georg Weiglein. $HiggsSignals$: Confronting arbitrary Higgs sectors with measurements at the Tevatron and the LHC. *Eur. Phys. J. C*, 74(2):2711, 2014, 1305.1933.

[12] Philip Bechtle, Sven Heinemeyer, Tobias Klingl, Tim Stefaniak, Georg Weiglein, and Jonas Wittbrodt. HiggsSignals-2: Probing new physics with precision Higgs measurements in the LHC 13 TeV era. *Eur. Phys. J. C*, 81(2):145, 2021, 2012.09197.

[13] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, Marc Sher, and Joao P. Silva. Theory and phenomenology of two-Higgs-doublet models. *Phys. Rept.*, 516:1–102, 2012, 1106.0034.

[14] Otto Eberhardt, Ana Peñuelas Martínez, and Antonio Pich. Global fits in the Aligned Two-Higgs-Doublet model. *JHEP*, 05:005, 2021, 2012.09200.

[15] Combined measurements of Higgs boson production and decay using up to 139 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment. Technical report, CERN, Geneva, Nov 2021. All figures including auxiliary figures are available at https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2021-053.

[16] Hamza Abouabid, Abdesslam Arhrib, Duarte Azevedo, Jaouad El Falaki, Pedro. M. Ferreira, Margarete Mühlleitner, and Rui Santos. Benchmarking Di-Higgs Production in Various Extended Higgs Sector Models. 12 2021, 2112.12515.

[17] Xiao-Fang Han, Tianjun Li, Hong-Xin Wang, Lei Wang, and Yang Zhang. Lepton-specific inert two-Higgs-doublet model confronted with the new results for muon and electron g-2 anomalies and multilepton searches at the LHC. *Phys. Rev. D*, 104(11):115001, 2021, 2104.03227.

[18] P. M. Ferreira, B. L. Gonçalves, and F. R. Joaquim. The hidden side of scalar-triplet models with spontaneous CP violation. 9 2021, 2109.13179.

[19] G. Abbiendi et al. Decay mode independent searches for new scalar bosons with the OPAL detector at LEP. *Eur. Phys. J. C*, 27:311–329, 2003, hep-ex/0206022.

[20] S. Schael et al. Search for neutral MSSM Higgs bosons at LEP. *Eur. Phys. J. C*, 47:547–587, 2006, hep-ex/0602042.

[21] A. Abada et al. FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2. *Eur. Phys. J. ST*, 228(2):261–623, 2019.

[22] Mingyi Dong et al. CEPC Conceptual Design Report: Volume 2 - Physics & Detector. 11 2018, 1811.10545.

[23] Alan S. Cornell, Aldo Deandrea, Benjamin Fuks, and Lara Mason. Future lepton collider prospects for a ubiquitous composite pseudoscalar. *Phys. Rev. D*, 102(3):035030, 2020, 2004.09825.

[24] Johan Alwall, Michel Herquet, Fabio Maltoni, Olivier Mattelaer, and Tim Stelzer. MadGraph 5 : Going Beyond. *JHEP*, 06:128, 2011, 1106.0522.

[25] Philip Bambade et al. The International Linear Collider: A Global Project. 3 2019, 1903.01629.

[26] P. Drechsel, G. Moortgat-Pick, and G. Weiglein. Prospects for direct searches for light Higgs bosons at the ILC with 250 GeV. *Eur. Phys. J. C*, 80(10):922, 2020, 1801.09662.

[27] R. Barate et al. Search for the standard model Higgs boson at LEP. *Phys. Lett. B*, 565:61–75, 2003, hep-ex/0306033.

[28] Yan Wang, Mikael Berggren, and Jenny List. ILD Benchmark: Search for Extra Scalars Produced in Association with a Z boson at $\sqrt{s} = 500$ GeV. 5 2020, 2005.06265.

[29] Zhen Liu, Lian-Tao Wang, and Hao Zhang. Exotic decays of the 125 GeV Higgs boson at future e^+e^- lepton colliders. *Chin. Phys. C*, 41(6):063102, 2017, 1612.09284.

[30] Jessie Shelton and Dong Xu. Exotic Higgs Decays to Four Taus at Future Electron-Positron Colliders. 10 2021, 2110.13225.

[31] Shiquan Ma, Kun Wang, and Jingya Zhu. Higgs decay to light (pseudo)scalars in the semi-constrained NMSSM. *Chin. Phys. C*, 45(2):023113, 2021, 2006.03527.

[32] Debottam Das, Ulrich Ellwanger, and Ana M. Teixeira. LHC constraints on $M_{1/2}$ and m_0 in the semi-constrained NMSSM. *JHEP*, 04:117, 2013, 1301.7584.

[33] Ulrich Ellwanger and Cyril Hugonie. The semi-constrained NMSSM satisfying bounds from the LHC, LUX and Planck. *JHEP*, 08:046, 2014, 1405.6647.

[34] Keisuke Nakamura and Daisuke Nomura. Charged Lepton Flavor Violation in the Semi-Constrained NMSSM with Right-Handed Neutrinos. *Phys. Lett. B*, 746:396–405, 2015, 1501.05058.

[35] Kun Wang, Fei Wang, Jingya Zhu, and Quanlin Jie. The semi-constrained NMSSM in light of muon g-2, LHC, and dark matter constraints. *Chin. Phys. C*, 42(10):103109–103109, 2018, 1811.04435.

[36] Jonathan Kozaczuk, Michael J. Ramsey-Musolf, and Jessie Shelton. Exotic Higgs boson decays and the electroweak phase transition. *Phys. Rev. D*, 101(11):115035, 2020, 1911.10210.

[37] Marcela Carena, Zhen Liu, and Yikun Wang. Electroweak phase transition with spontaneous Z_2-breaking. *JHEP*, 08:107, 2020, 1911.10206.

[38] Eung Jin Chun and Tanmoy Mondal. Searching for a Light Higgs Boson via the Yukawa Process at Lepton Colliders. *Phys. Lett. B*, 802:135190, 2020, 1909.09515.

[39] A. G. Akeroyd, Stefano Moretti, and Muyuan Song. Light charged Higgs boson with dominant decay to a charm quark and a bottom quark and its search at LEP2 and future e^+e^- colliders. *Phys. Rev. D*, 101(3):035021, 2020, 1908.00826.

[40] Albert M Sirunyan et al. Search for a standard model-like Higgs boson in the mass range between 70 and 110 GeV in the diphoton final state in proton-proton collisions at $\sqrt{s} = 8$ and 13 TeV. *Phys. Lett. B*, 793:320–347, 2019, 1811.08459.

[41] S. Heinemeyer, C. Li, F. Lika, G. Moortgat-Pick, and S. Paasch. A 96 GeV Higgs Boson in the 2HDM plus Singlet. 12 2021, 2112.11958.

[42] Duarte Azevedo, Pedro Ferreira, M. Margarete Mühlleitner, Rui Santos, and Jonas Wittbrodt. Models with extended Higgs sectors at future e^+e^- colliders. *Phys. Rev. D*, 99(5):055013, 2019, 1808.00755.