Review of Neutral Naturalness

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Abstract

The hierarchy between the mass of the Higgs boson and larger mass scales becomes ever more puzzling as experiments push to higher energies. Neutral naturalness is the umbrella term for symmetry-based explanations for these hierarchies whose quark symmetry partners are not charged under the $SU(3)_c$ color gauge group of the Standard Model. Though the first manifestations of this idea predate the physics runs of the Large Hadron Collider, since the Higgs discovery this paradigm has grown and developed to include a wide variety of models, connections to intriguing collider signals, various dark matter candidates, intersections with astrophysics and cosmology, motivations for lattice studies, and ties to neutrinos and flavor. In this review we orient the reader within this growing literature and reveal interesting directions for further study. Submitted to the Proceedings of the US Community Study on the Future of Particle Physics (Snowmass 2021)

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I. EXECUTIVE SUMMARY

As experiments have reached ever higher energies without clear evidence of deviations from the Standard Model (SM), the so-called hierarchy problem between the electroweak scale and higher physics energy scales has become increasingly acute. A particularly interesting explanation of Higgs naturalness is positing a new symmetry which relates the SM quarks to colorless partners, which has become known as neutral naturalness. These are primarily bottom-up constructions that characterize the new partner particles and interactions only up to the few TeV scale and require completion at high energies. Typically, these completions include new colored states with masses of order a few TeV, motivating future high-energy colliders to thoroughly test Higgs naturalness due to new symmetries.

Realizations of neutral naturalness typically include a hidden, dark, or secluded sector of particles that are related to at least some of the SM fields by a discrete symmetry. Candidates for the cosmological dark matter, either as a single particle species or from a collection of stable particles, are often obvious consequences of this structure. This discrete symmetry also relates the SM's $SU(3)_c$ color group to a distinct hidden-sector SU(3) gauge group. Consequently, neutral naturalness generically leads to rich dark sectors, with phenomenological features such as dark showers, semivisible jets, quirks, and long-lived particles. Useful nonperturbative inputs for studying this strongly-coupled hidden SU(3) sector can be obtained from lattice gauge theory calculations of the spectrum of bound states, matrix elements, and other observables. Because at least some of the hidden-sector particles are essential to explaining the mass of the Higgs boson, the Higgs is a robust link between the sectors. Signals of these connections can appear in deviations from the SM Higgs couplings and exotic Higgs decay modes that can be probed at future Higgs factories.

While the number of novel realizations of the idea of neutral naturalness continues to grow, the majority of past work has focused on the twin Higgs framework. This pertains to higherenergy (UV) completions, collider phenomenology, dark matter candidates, cosmological and astrophysical signals, and intersections with neutrino and flavor physics. That this narrower focus has led to such a rich variety of signals and overlaps with other aspects of beyond the SM physics is impressive, but also points the way to further exploration through other neutral naturalness models. Such explorations may lead to new connections to higher-scale physics that applies to the SM flavor structure, the strong CP puzzle, or gauge unification. New dark matter candidates, including those associated with UV completions as well as lighter, non-WIMP, candidates may also be discovered.

Neutral naturalness connections across the range of new phenomena are particularly fascinating due to the discrete symmetries that relate hidden-sector particles to SM counterparts. This assortment of concrete models accommodates a variety of new signals at the intensity frontier, energy frontier, and cosmic frontier which are correlated with each other and, through discrete symmetry, with the known properties of the SM. This makes models of neutral naturalness compelling benchmarks for organizing simplified model signals in addition to suggesting new strategies for discovering new physics.

II. INTRODUCTION

Despite its phenomenal experimental successes, the SM can only be considered as a lowenergy, effective field theory (EFT) description of microphysics. As typically accepted, the SM does not specify how the neutrinos obtain mass nor explain the observed asymmetry between matter and antimatter in the Universe. The cosmological dark matter cannot be composed of SM fields and any structure that explains the structure of quark and lepton flavor or the absence of measurable CP violation in QCD must include physics beyond the SM. In addition to these known unknowns, other particles and interactions may be essential to Nature's structure, even if we have not yet recognized what role they play.

Extensions of the SM are likely tied to an energy scale above which new particles and

interactions are manifest. Gravity is the most famous example, whose detailed quantum interactions are tied to the Planck scale, which is of order 10¹⁹ GeV. Within the SM the Higgs boson is unique: it is the only elementary scalar field. As such, the generic EFT expectation is that the vacuum expectation value (VEV) of the Higgs field (and hence the mass of the Higgs boson) is set by the largest scale it interacts with, including the Planck scale. Therefore, within the SM the hierarchy between the Planck scale and the weak scale is mysterious. Indeed, concrete calculations of the Higgs mass within SM extensions have borne out the EFT expectation.

The hierarchy between the Higgs mass and other scales may emerge from a symmetry. If the Higgs is a pseudo-Nambu-Goldstone boson (pNGB) of an approximate global symmetry, then its mass is controlled by the symmetry breaking parameter. If nature manifests supersymmetry (SUSY), then the chiral symmetry that protects fermion masses can be transferred to bosons like the Higgs. In general, any symmetry that makes the Higgs insensitive to higher mass scales is only as effective as the largest parameter that breaks the symmetry. Therefore, to meaningfully address the hierarchy problem the top quark, whose coupling to the Higgs is largest, must transform under the new symmetry. The other particles that, along with the top quark, fill out a representation of this symmetry are referred to as top partners.

The Large Hadron Collider (LHC) has provided a wealth of data with which the ATLAS and CMS collaborations have sought for various types of top partners. As a hadron machine, the LHC is well equipped to discover the new colored particles predicted by many symmetrybased solutions to the hierarchy problem. The LHC sensitivity to such partners varies from model to model, but in general colored top partners have been excluded at energies below a couple of TeV. While the gap between the top-partner bounds and the Higgs mass is small compared to its separation from the Planck scale, it is still large enough that these models must be tuned somewhat to produce the correct Higgs mass. This is sometimes referred to as the little hierarchy problem.

Beginning in 2005 [1], symmetries were found that explain these little hierarchies up to a few TeV *without* colored top partners. Direct production of colorless top partners at a hadron collider is typically challenging, so the limits on such particles are weaker. Because the color-neutral top-partners masses can be lower, the tuning is reduced. In other words, the models are more natural. Consequently, experimental probes of Higgs naturalness require even higher-energy colliders [2].

All known realizations of (color) neutral naturalness feature a discrete symmetry that relates SM fields to colorless counterparts. Though these partner fields are neutral under SM color, the quark partners are charged under an SU(3) gauge group that is related to SM color by the discrete symmetry. Consequently, neutral naturalness motivates certain classes of rich hidden sectors, with a variety of particles and forces. What is more, the rich structure of these hidden sectors is related (at least in part) to the SM by the discrete symmetry. This leads to correlations between precision SM measurements and dark sector structure.

The Higgs always links the SM to these new sectors. The colorless partners must couple to the Higgs with SM field-like strength in order to protect the Higgs mass. This ensures the Higgs sector acts as a portal between the SM and the new sector. However, the manifestations of this connection can vary from one concrete model to another, as do other phenomenological consequences. The purpose of our review is to map this rich diversity so that new researchers can more easily investigate and extend them.

We note that there are many interesting explanations of the hierarchy between electroweak and Planck scales that do not predict new SM colored states at low energy which we do not review in this article. This review focuses on new internal symmetries that make the Higgs less sensitive to high energy scales. Other color neutral constructions include cosmological selection [3–10] and UV/IR mixing [11–14], including symmetries in string theory that may protect the Higgs mass [15–18]. These and other intriguing ideas are reviewed in [19].

This review is organized by topic so that readers may easily navigate to the aspects of neutral naturalness they are most interested in. This leads to some redundancy, as many works are relevant to multiple sections. We begin, in Sec. III, with an overview of the various specific realizations of neutral naturalness. The twin Higgs scenario is emphasized due to the growing body of modifications that have been made to its basic structure as well as the several UV completions that have been suggested. We then broadly classify models as being related to SUSY or to pNGB Higgs constructions.

The following section, IV, considers the various collider signals that have been associated with these models. In this section the organization is by collider signature rather than by model. We then, in Sec. V, discuss results from lattice gauge theory that have been important to understanding the phenomenology of these models. As yet unknown quantities that can improve the predictive power of phenomenological studies are also mentioned.

The rich dark sectors of neutral naturalness are connected to the cosmological dark matter in Sec. VI. Here again the twin Higgs scenario has been most thoroughly investigated, though the possibilities related to other frameworks are intriguing. Other connections to astrophysics and cosmology, including gravitational waves, baryogenesis, and the Hubble tension, are discussed in Sec. VII. Intersections with neutrinos and flavor physics are outlined in Secs. VIII and IX, respectively. These relationships are less explored, but offer interesting correlations between precision SM measurements and hidden-sector observables.

III. MODELS

The broad ideas of neutral naturalness have been realized in several specific ways. Many of them center on the twin Higgs framework. However, the other familiar symmetry-based solutions to the hierarchy problem, supersymmetry and pNGB Higgs models, also have many neutral natural realizations.

A. The Twin Higgs and its Variants

Neutral naturalness began with the mirror twin Higgs [1] model, which also exemplifies many of the characteristics that are common to most constructions.¹ To the SM with its $[SU(3)_c \times SU(2)_L \times U(1)_Y]_A$ gauge structure is added a twin sector with identical structure, that is, a gauge structure of $[SU(3)_c \times SU(2)_L \times U(1)_Y]_B$. The twin character of this sector is manifest in a discrete Z_2 symmetry that relates the couplings of the two sectors. In addition to this mirror set-up the Higgs sectors of the two copies are assumed to satisfy an approximate global symmetry. For instance, one can arrange the visible and twin sector Higgs doublets, H_A and H_B (respectively), in terms of a four-plet of a global SU(4)

$$\mathcal{H} = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \ . \tag{1}$$

¹ There are twin Higgs models that do not exhibit neutral naturalness; the top partners carry color [20–23]. The same is true of other models that are similar in construction to many neutral naturalness models [24].

If this approximate symmetry is spontaneously broken down to SU(3) at some scale f then seven pNGBs are produced.² Because of the $SU(2)_L \times U(1)_Y$ gauging in each sector, six of the pNGBs are eaten by electroweak gauge fields. This leaves one physical pNGB that is associated with the observed Higgs boson. As a pNGB, the Higgs mass is controlled by the approximate symmetry of the potential.

The SM fermions are not extended into representations of SU(4), so one might worry that Yukawa interactions $\lambda_f \overline{f} f H$ would break the symmetry and produce large one-loop corrections to the scalar potential. However, the discrete Z_2 symmetry that relates the SM fermions to the twin fermions ensures that the quadratically divergent contributions to the Higgs mass from the SM and twin fermions cancel. In effect, the SM and twin fermions interact with the scalar sector as part of accidental SU(4) multiplets.

This may be understood schematically as follows. The dominant one-loop contributions to the $|H_A|^2$ and $|H_B|^2$ operators are from the visible-sector and twin-sector top quarks, t_A and t_B , respectively. These contributions are

$$\frac{3\lambda_{t_A}^2}{8\pi^2}\Lambda_{\rm UV}^2 \left|H_A\right|^2 , \quad \text{and} \quad \frac{3\lambda_{t_B}^2}{8\pi^2}\Lambda_{\rm UV}^2 \left|H_B\right|^2 , \qquad (2)$$

where $\Lambda_{\rm UV}$ is the UV cutoff of the effective theory, typically taken to be a few TeV. The Z_2 symmetry between the sectors ensures that $\lambda_{t_A} = \lambda_{t_B} \equiv \lambda_t$. This means that the sum of these two contributions is

$$\frac{3\lambda_t^2}{8\pi^2}\Lambda_{\rm UV}^2\left(|H_A|^2 + |H_B|^2\right) = \frac{3\lambda_t^2}{8\pi^2}\Lambda_{\rm UV}^2\left|\mathcal{H}\right|^2 \ . \tag{3}$$

This result is SU(4) symmetric and, therefore, cannot contribute to the mass of any pNGB of the broken SU(4) symmetry. It is important to note that this one-loop effect does not prevent the NGBs from getting a mass. The $SU(2)_L$ gauge couplings and the Yukawa coupling to fermions break the global SU(4) and generate a $\delta(|H_A|^4 + |H_B|^4)$ potential term. This gives the Higgs a $m_h \sim f\sqrt{\delta}$ mass where $\delta \sim \ln \Lambda_{\rm UV}/f$ depends logarithmically on the cutoff.

In general, we can write the scalar potential as [25]

$$V_{H} = -\mu^{2} |\mathcal{H}|^{2} + \lambda |\mathcal{H}|^{4} + m^{2} \left(|H_{A}|^{2} - |H_{B}|^{2} \right) + \delta \left(|H_{A}|^{4} + |H_{B}|^{4} \right) , \qquad (4)$$

² We have used the $SU(4) \rightarrow SU(3)$ breaking pattern for simplicity. However, it is preferable in stronglycoupled UV completions (see Sec. III C) to use $SO(8) \rightarrow SO(7)$ (which also produces 7 pNGBs) as this preserves the custodial SU(2) symmetry.

with the first line preserving the global SU(4), while the second line does not. For the Higgs to be a pNGB the parameters of the second line must be small compared to those of the first. Small tree-level couplings are stable at one-loop due to the discrete twin symmetry. One finds that contributions to the symmetry breaking potential terms are at most logarithmically sensitive to $\Lambda_{\rm UV}$. This scenario can be extended to include not just a single twin sector, but many copies of the SM [26].

When the scalar potential in Eq. (4) has $\delta > 0$ and m = 0 [25] both H_A and H_B obtain equal VEVs and, consequently, equal couplings to SM and twin fields. These circumstances produce couplings of the observed Higgs boson to SM fields that are only half their SM value, contrary to LHC measurements. However, when $m^2 \neq 0$ the twin symmetry is softly broken, allowing $\langle H_B \rangle > \langle H_A \rangle$. The resulting Higgs couplings are closer to the SM predictions while preserving the one-loop protection of the Higgs mass. Choosing m^2 to give the correct Higgs VEV does constitute a tuning of the model, but typically a mild one, less tuned than 10%. This tuning can also be reduced in various ways, such as by inducing electroweak symmetry breaking through the tadpole of an auxiliary sector [27].

The logarithmic sensitivity in the Higgs potential parameters is now augmented by the Z_2 breaking. This can affect the twin Higgs protection at the two-loop level if, for instance, the SM and twin top Yukawa couplings run differently. Consequently, constructions like the twin Higgs typically only address the "little" hierarchy problem. They provide a significant improvement in fine-tuning, but must be completed by some other structure above the few TeV scale.

These gains in naturalness withstand significant variations from the mirror twin set-up. The so-called fraternal model [28] makes Z_2 breaking a main feature, only twinning the SM's third generation and allowing for modest deviations from the twin equality in some couplings. In effect, the model keeps only the minimal ingredients required for a twin-Higgs-like protection of the Higgs mass. In a similar vein, if the twin top quark is taken to be vector-like then even the third generation leptons can be removed without generating anomalies [29]. Rather than removing entire generations from the mirror twin model, simply allowing significant Z_2 breaking between the fermion Yukawa couplings (other than the top quark) is enough to eliminate some cosmological concerns [30]. In weakly-coupled UV completions, a hard breaking of the Z_2 in the quartic terms of the scalar potential can further reduce the need for fine-tuning [31]. Other efforts seek to completely preserve the discrete symmetry or explain how it breaks. In [32] the twin symmetry is preserved by much of the model (additional top-partner multiplets do play a role) leading to electroweak symmetry breaking with the correct hierarchy of VEVs, but without Z_2 breaking in the gauge and top quark sectors of the model. Vector-like leptons can also be introduced in each sector to produce a radiative Higgs potential that spontaneously breaks the Z_2 [33]. Spontaneous breaking can also be induced by simply including a new scalar field in each sector whose potential is similar to Eq. (4) but with $\delta < 0$. This leads to a completely Z_2 breaking vacuum with the VEV only in one sector. Such scalars can have other nonzero quantum numbers like hypercharge [34, 35] or color [35, 36] which lead to a variety of twin sectors by breaking twin gauge symmetries or giving additional masses to twin fermions.

Two Higgs doublet extensions of the SM have also been combined with the twin Higgs idea [37, 38]. This larger scalar sector can be used to explain the origin of Z_2 breaking. This includes spontaneously breaking the twin Z_2 symmetry [39], radiative breaking [40], and tadpole or quartic induced breaking [41]. Other models address neutrinos masses in conjunction with a spontaneous breaking of the discrete symmetry [35, 42]. The additional Higgs doublet can be combined with a flavorful Yukawa structure to address the SM flavor puzzle [43]. The additional doublet can also make mirror neutrons an asymmetric dark matter candidate [44].

1. UV Completions of the Twin Higgs

As mentioned above, the twin Higgs mechanism only addresses the little hierarchy problem. It is therefore important to construct realistic UV completions of the twin Higgs that address the big hierarchy problem. Additionally, one may then consider higher energies properties such as may be related to the SM flavor structure or other puzzles. Weakly-coupled UV completions are typically supersymmetric. These completions can be constructed to eliminate the fine-tuning of supersymmetric quartic couplings [45, 46], but simpler constructions [47] can retain small fine-tuning and preserve other virtues of SUSY models such as gauge coupling unification.

Within these supersymmetric UV completions other aspects of twin Higgs models can be explored unambiguously. In particular, various ways to further reduce fine-tuning have been found, including a hard breaking of the twin symmetry [31] and new *D*-term contributions to the SU(4) symmetric quartic [48–50]. A "turtle" construction is a UV extension of the twin Higgs, which can also precede a SUSY completion [51], again reducing fine-tuning. Supersymmetric completions of the twin Higgs can also be extended to solve the strong CP problem [52].

There are also several strongly-coupled UV completions of the twin Higgs framework. These include composite Higgs [53, 54], holographic constructions [55], and universal extra dimensions [56]. The EFT form factors of composite twin Higgs models are determined by the discrete symmetry, differing from other scenarios. This framework can also accommodate anarchic flavor with significantly less fine-tuning [57]. Precision electroweak constraints primarily apply to the Higgs quartic and are satisfied with only moderate tuning [58].

Clearly, the twin Higgs scenario can be incorporated within various high scale designs. In some cases a particular completion need not be selected, while for others a specific highenergy construction is essential. Interestingly, a renormalization group improved analysis of the twin Higgs effective potential shows that the Higgs mass can be a largely UV independent prediction [59].

B. Neutral Naturalness and SUSY

In contrast to the supersymmetric UV completions of the twin Higgs discussed above, SUSY can also be the fundamental symmetry that protects the Higgs mass, although it manifests in less familiar ways. These models share many characteristics with folded SUSY [60], so we outline its broad aspects in some detail.

The essential idea behind folded SUSY is quite simple. Very schematically, SUSY connects bosonic and fermionic degrees of freedom. Roughly, we can consider that the top quark is extended to a SUSY multiplet

$$t \Rightarrow \begin{pmatrix} t \\ \tilde{t} \end{pmatrix} \bigg| \text{SUSY} ,$$
 (5)

where \tilde{t} denotes the scalar stop. This scalar partner to the top quark is a manifestation of the SUSY structure in a given model. This symmetry can also explain why the Higgs mass is insensitive to higher mass scales. Having two copies of these particles would also preserve Higgs naturalness; the cancellation between fermonic and bosonic contributions to the Higgs mass renormalization would simply happen twice in exactly the same way.

The authors of folded SUSY observed a generalization of this doubling idea, where each copy is charged under a different color group. If we label SM color as $SU(3)_A$ then we can label the copy (or folded color) as $SU(3)_B$ and write the top sector particles as

$$\begin{array}{c}
\operatorname{SU}(3)_A \times \operatorname{SU}(3)_B \\
\end{array} \\
\left(\begin{array}{c}
t_A & t_B \\
\widetilde{t}_A & \widetilde{t}_B
\end{array} \right) \\
\end{array} \\
\left| \operatorname{SUSY} .$$
(6)

This extension keeps the Higgs natural through the SUSY structure. In this construction the A and B sector fields are assumed to be related by a discrete Z_2 exchange symmetry to ensure the coupling structure is the same in each sector.

But what if the SM colored stop \tilde{t}_A and the folded top quark t_B were removed from the low-energy theory? Then the Higgs remains natural because the contributions from the SM top and the folded stop combine in precisely the same way as in standard SUSY, the fact that they are charged under different color groups is irrelevant. In this scenario the folded stops are much more difficult to produce at hadron colliders, as they do not carry SM color. Note that in this case no true SUSY is manifest in the low-energy theory as the fermions and bosons belong to different SUSY multiplets. The protection of the Higgs mass results from an accidental SUSY realized by the low-energy theory.

In folded SUSY the unwanted fields are removed using an additional spatial dimension. The extra dimension is taken to be a finite, flat interval between two 4D branes. It is also compactified on S_1/Z_2 , making it an orbifold, from which folded SUSY takes its name. The 5D superfields with $\mathcal{N} = 1$ SUSY can be usefully viewed as $\mathcal{N} = 2$ SUSY in 4D. The $\mathcal{N} = 2$ SUSY is broken to $\mathcal{N} = 1$ on each brane by the boundary conditions each bulk field has there. In the original folded-SUSY model the boundary conditions of the A sector colored scalars and B sector colored fermions are chosen such that their Kaluza-Klein tower has no zero-mode. This effectively eliminates them from the low-energy theory below the Kaluza-Klein scale. The gauginos are also projected out of the low-energy spectrum.

The $\mathcal{N} = 1$ SUSY preserved by the field boundary conditions on each brane can be *different*. This means that in the low-energy theory SUSY is completely broken by the Scherk-Schwarz [61, 62] mechanism. Crucially, the complete breaking of SUSY by the mismatch of the $\mathcal{N} = 1$ SUSYs on the two branes is nonlocal. Consequently, corrections to

the brane localized Higgs potential are finite and calculable. Thus, the simple concept of a SUSY-like protection of the Higgs mass through stops that are neutral to SM color is concretely realized.

Similar to the twin Higgs framework, extra dimensional constructions come with a UV cutoff that is typically not too far above the TeV scale. As shown in [63], however, the extra dimension can be deconstructed while preserving the natural virtues of folded SUSY. An estimate of the two-loop calculation of the folded-SUSY Higgs potential indicates that it may not produce the correct vacuum without modification. A simple change is to vary the Scherk-Schwarz twist away from its maximum value [64]. This can ensure electroweak symmetry breaking occurs at one-loop. The tuning in folded SUSY can also be improved by making the gauginos bifundementals of the two A and B color groups [65].

The squark fields of folded SUSY and its variations are color neutral, but carry SM electroweak charges. Two frameworks have been discovered that have scalar top partners that are complete SM gauge singlets. The tripled-top scenario [66, 67] demonstrates that the folded-SUSY-like cancellation can be accomplished in 4D using *two* copies of the top sector. In this construction the right-handed stops of each copy act as top partners, so they are not charged under $SU(2)_L$. The hypercharge of the top-sector copies may be freely specified, including the choice of scalar top partners with no hypercharge, which are SM gauge singlets. The Hyperbolic Higgs model [68] obtains SM singlet scalar top partners from a 5D set-up. In contrast to the approximate U(4) symmetry of the scalar potential of twin Higgs models, this scalar potential enjoys an approximate hyperbolic symmetry of U(2, 2). The kinetic terms of the scalar fields are not invariant under U(2, 2), but the form of the scalar potential is sufficient for SM gauge singlet scalars to cancel the divergent loop contributions of SM quarks.

C. Neutral Naturalness and pNGB Higgs

Neutral naturalness began as a pNGB construction. The twin Higgs scenario posits that the observed Higgs boson is the pNGB of an approximate SU(4) symmetry. However, the neutral natural pNGB possibilities include a rich variety of realizations. There is a class of theories that may be seen as the direct generalization of the twin Higgs, called orbifold Higgs models [69, 70]. From this perspective the twin Higgs gauge structure, $SU(3)_A \times$ $SU(2)_A \times SU(3)_B \times SU(2)_B \times Z_2$, results from the orbifold $[SU(6) \times SU(4)]/Z_2$. This provides an origin for the discrete symmetry that relates the gauge and Yukawa couplings in the two sectors which is essential to the one-loop protection of the Higgs. What is more, this orbifolding method can be used with other symmetry groups to produce different numbers and varieties of hidden sectors whose SM singlet top partners all cooperate to keep the Higgs natural.

There are other pNGB Higgs models that resemble folded SUSY in their set-up. The usual pNGB Higgs scenario assumes an approximate global symmetry is spontaneously broken at some scale f and the Higgs is one of the resulting pNGBs. The top quark is typically taken to belong to a representation of the larger global symmetry. The new fields required to fill out the multiplet are the top partners.

For an SU(3) global symmetry we have the schematic form

$$\begin{pmatrix} t \\ b \end{pmatrix} = q_L \Rightarrow \begin{pmatrix} q_L \\ T \end{pmatrix} \bigg| SU(3) , \qquad (7)$$

where T is the top-partner field. The protection of the Higgs mass works just as well when it happens twice, including new fields charged under a new color group

$$\begin{pmatrix} \underbrace{q_{L_A} & q_{L_B}} \\ T_A & T_B \end{pmatrix} \bigg| SU(3) .$$
(8)

The one-loop protection persists even if the T_A and q_{L_B} fields are removed from the lowenergy theory. In this case the top partner T_B is not charged under SM color, though it would carry SM hypercharge. This is the essence of the Quirky Little Higgs model [71], where the unwanted fields are removed by the brane boundary conditions in an extra dimensional space, similar to folded SUSY. This symmetry breaking pattern also plays a role in some Dark Top constructions [72], though differences allow the top partner to be a stable dark matter candidate. The neutral top partners can also be right-handed neutrinos [73] for various symmetry breaking patterns.

While the SU(3)/SU(2) construction is in some sense minimal, it does not provide the Higgs with a custodial SU(2) symmetry that is highly motivated by precision electroweak measurements. Orthogonal groups can ensure the custodial form of Higgs structure. The minimal realization is SO(5)/SO(4) [74, 75] where boundary conditions in a warped extra dimension are employed to lift unwanted fields from the low-energy spectrum.

While the SO(5)/SO(4) symmetry breaking pattern preserves the custodial symmetry, it differs from the twin Higgs construction in how the top partners relate to the top quark. In the twin Higgs set-up, which can be thought of as SO(8)/SO(7), the top and its partner can be related by a discrete symmetry because they are embedded within the SO(8) in similar ways. This cannot be done when the symmetry breaking pattern is SO(5)/SO(4). The smallest orthogonal group structure that allows the top quark and its partner to be related by a twin parity is SO(6)/SO(5) [76, 77], which has also been extended into the UV holographically [78].

Though it allows for twin parity, the SO(6)/SO(5) symmetry breaking pattern also produces another pNGB which must be removed from the low-energy theory. Another construction which allows twin parity, leads to a custodial symmetry, and an embedding of both SM and twin $SU(2)_L$ is $SO(7)/G_2$ [79]. When SO(7) is spontaneously broken to G_2 seven pNGBs are produced, but six are eaten by the SM and twin $SU(2)_L$ gauge bosons, leaving behind a single physical Higgs, just as in the twin Higgs. The breaking pattern of SO(7)/SO(6) leads to custodial symmetry for the pNGB Higgs while the additional pNGBs become a viable complex scalar dark matter candidate [80].

Models employing a $SO(2N)/[SO(N) \times SO(N)]$ breaking pattern have also been investigated [81]. These models naturally produce a hierarchy between the Higgs VEV v and the global symmetry breaking scale f. In most pNGB models such a hierarchy is required to agree with Higgs data, but is a source of tuning. These so-called Twin Gegenbaur constructions agree with current Higgs measurements without additional tuning.

D. Reoccurring Themes

A few characteristics are common to most concrete manifestations of neutral naturalness. First, the existence of multiple sectors related by a discrete symmetry. In many cases, but not all, the number of sectors is two and the symmetry is Z_2 . The sectors may differ from each other in significant ways, but aspects of the third generation, more specifically the top quark sector, must have nearly identical couplings to the Higgs. Thus, the Higgs is an essential portal between the sectors; there may be others depending on the model. The top partners, and often other fields, in the hidden sector are charged under a QCDlike gauge group which is related to SM QCD by the discrete symmetry or a larger symmetry structure. However, the spectrum of particles can be quite different in the hidden sector, often with fewer light states. This can produce a hidden confinement scale that is significantly higher than the SM equivalent and a different spectrum of bound states.

Most of the models discussed are only effective theories up to the scale of a few TeV. Several UV completions have been discussed for twin Higgs models. Specific composite Higgs completions of pNGB Higgs constructions have also been developed. In other cases definite UV completions have not been specified. However, new colored states (and a variety of other signals) are generically expected at the few TeV scale.

Clearly, neutral naturalness models motivate rich hidden sectors and, as discussed in Sec. VI, provide many possible dark matter candidates. The discrete symmetry typically relates the dark matter to known SM fields and couplings. Consequently, the dark matter signals can be connected to the naturalness of the Higgs.

Beyond dark matter, these hidden sectors present a structure at least as rich and varied as the SM. While the hidden sectors are neutral under SM color, they may share other gauge interactions. Therefore, searches for new electroweak states are highly motivated by these constructions.

IV. COLLIDER PHENOMENOLOGY

The primary characteristic of neutral naturalness is that there are no new colored states up to the few TeV scale. Consequently, some of the most powerful hadron collider searches for new states do not apply. However, there are many other signatures of neutral naturalness that may appear at present day and future colliders. Indeed, it has been argued [2] that the combination of current machines along with future lepton and hadron colliders will be able to thoroughly probe symmetry-based solutions to the hierarchy problem.

A. Higgs Physics

The most robust prediction of neutral naturalness is the existence of new states that couple to the Higgs. Consequently, examining the Higgs for indications of these structures is highly motivated. In pNGB Higgs models the tree-level couplings of the Higgs to SM fields are modified, which already constrains such theories [82, 83]. The deviations from SM couplings can also have significant effects on, for instance, rare top quark decays [84]. In supersymmetric theories, or general bottom-up models with scalar top partners, the tree-level Higgs couplings may be unaffected. However, loop-level couplings, such as $h\gamma\gamma$ [85] and $hZ\gamma$ [86] (or even a loop-level contribution to hZZ [87]), can be modified by the contribution of the new particles to the loop calculations.

The Higgs often acquires new, or exotic, decay modes. For example, in the mirror twin Higgs model the Higgs couples to all twin quarks and leptons. The strengths of these couplings and the mass of the twin fermions are both controlled by the ratio of the SM Higgs VEV to the twin Higgs VEV, v/f. This quantity is also directly proportional to the tuning of the model, making both Higgs coupling deviations and the invisible Higgs width direct probes of naturalness. Even when these states are to heavy for the an on-shell Higgs to decay into them, they can be produced through an off-shell Higgs [88–91].

Most models require that some of the hidden-sector particles are charged under a hidden SU(3) gauge group. In many scenarios the fields charged under hidden color are much heavier than the hidden confinement scale, implying that the lowest lying hidden hadrons are glueballs [28]. The lightest glueball has the right quantum numbers to mix with the Higgs, 0^{++} , and does so through multi-loop processes [92, 93]. This can give the Higgs a small coupling to this glueball state, which can also decay, after traversing a macroscopic distance, back into SM fields. Despite the uncertainties of glueball hadronization [94], these exotic displaced decays of the Higgs [95–98] can be powerful probes of neutral natural frameworks.

There can be other scalars, besides the observed 125 GeV Higgs, in the Higgs sector, such as the twin Higgs of twin Higgs models or the heavy Higgses of SUSY models. These scalars often have couplings to both SM and hidden-sector fields, making them an interesting portal between the sectors. These motivate specific heavy scalar targets at current and future colliders [99–101] and can be used in conjunction with other collider signals to prove such scalars belong to a neutral naturalness scenario [102]. This may be accomplished through prompt heavy Higgs signals, and often with powerful complementarity through displaced heavy Higgs decays [103, 104].

B. Other Portals to SM Neutral Sectors

While the Higgs is a robust portal to the hidden sector, there are several other interesting ways that the sectors may be linked. The most obvious is by SM gauge bosons, if the hidden sectors include states with SM gauge charges. These scenarios are considered in the following section, IV C.

In the case of SM neutral hidden sectors, like in the twin Higgs, the visible and hidden sectors can still be joined through several possible portals. These analyses have, so far, focused on the twin Higgs set-up, so the term twin sector is appropriate. When twin hypercharge is gauged then there may be a kinetic mixing with the SM hypercharge gauge field. Within the low-energy theory such mixing requires at least four-loops [1, 105], which can be consistent with unbroken twin hypercharge [106, 107]. However, for kinetic mixing large enough to be relevant at colliders the twin hypercharge boson must have a mass. In this scenario both the twin photon and twin Z can be discovered at current or future hadron colliders [38].

The Higgs and hypercharge portals rely on fields with discrete symmetry partners, the twin Higgs or twin hypercharge bosons. However, if there are fields that the discrete symmetries take back to themselves, these can also serve as portals between the sectors. Such singletons—completely neutral fields with no Z_2 partners—may be scalar, fermions, or vectors and can produce distinct collider signals [35, 108]. For example, strongly-coupled UV completions of the twin Higgs may include a spontaneously broken scaling symmetry, producing a light dilaton which connects the visible and twin sectors [109].

C. New SM Charged States

Though the defining characteristic of neutral natural models is colorless top partners, specific realizations often include new states charged under the SM gauge groups. For instance, non-SUSY UV completions of the twin Higgs predict exotic quarks charged under SM color in the few to tens of TeV mass range, which can be thoroughly probed at the LHC and future colliders [110]. Other TeV scale states are SM color neutral but carry electroweak charge. If the discrete twin symmetry is preserved in the UV then these states may be out of reach of the LHC, but can be probed by future machines [111]. It may also be that the new electroweak charged states are significantly lighter the colored states. In this case the LHC can produce them at a meaningful rate. The twin color force draws the produced particles into bound states whose resonant decays can be discovered at the LHC [111, 112].

Supersymmetric models can also lead to novel electroweak states. Folded-SUSY sleptons [113] carry electroweak charges and can be discovered effectively at the LHC. In tripled-top constructions the Z portal can be the dominant connection to hidden-sector states [67, 114] with electroweak charges. These states are often bound by the hidden color force, which in some cases leads to "quirky" dynamics, which we focus on in the following section.

D. Quirks

Neutral natural models typically include a confining gauge group in the hidden sector. While the mirror twin Higgs model includes states at or below the hidden confining scale, this appears to be more the exception than the rule. In all frameworks with electroweak charged partner particles the bounds on new states are at the 100 GeV scale or above [115]. However, the hidden strong coupling is related by a discrete symmetry to the SM strong coupling in the TeV range. This typically leads to a hidden confinement scale in the neighborhood of a few GeV scale. Models like these, with no particles charged under the confining gauge group at or below the confinement scale, are said to exhibit quirky [116] dynamics.

Without light quarks, which can be pair produced to fragment tubes of confining color flux, states much heavier than the confinement scale evolve as though connected by a string of constant tension. These quirky bound states must shed energy until they reach low angular momentum configurations, after which they can decay efficiently. In the case of folded SUSY the folded squark bound states dominantly decay to a $W\gamma$ final state [117], also updated to LHC resonance searches at 8 TeV [82]. These and other SM vector decay modes can be used to detect pNGB Higgs models as well, such as SO(7)/SO(6) [80]. The SM exhibits a cancellation in $W\gamma$ production which can be used to strengthen these searches [118]. While the final decay product is the most conspicuous signal of these quirky states, the pattern of radiation they produce as they de-excite can also lead to signals in the underlying event [119].

In addition to decays into SM vectors, the quirks or squirks have a significant branching fraction into hidden gluons. Thus, heavy quirks can produce showers of glueballs [94, 120].

Some fraction of these glueballs, the 0⁺⁺ states, have displaced decays back into the SM, leading to striking signatures at current and upcoming colliders [121]. The quirky nature of excited state de-excitation can also play a significant role in determining the lifetime of these new states. When the top partners are SM singlets or only carry $SU(2)_L$ charge the slow de-excitation can lead to displaced signals, as demonstrated in [66] for some tripled-top models. While the quirk scenario is more general than neutral naturalness, these models point to a particularly motivated region of quirky models. As such, they serve as motivation for developing quirk collider searches [122–125].

V. CONNECTIONS TO LATTICE GAUGE THEORY

As discussed above, a recurring theme in neutral naturalness models is the appearance of a QCD-like SU(3) gauge group in the hidden sector. Lattice gauge theory calculations can provide direct estimates of masses and matrix elements for hidden-sector hadrons, which may otherwise be very difficult to estimate reliably.

A common situation for the hidden strong sector is that all of the hidden quarks are heavy compared to the confinement scale. In this case, the low-energy dynamics of the theory are well approximated by a "pure gauge" theory with no dynamical fermions at all. The mass spectrum of glueball states in the pure-gauge limit has been well-studied on the lattice, both for SU(3) [126, 127] and in the large- N_c limit [128, 129]. Lattice calculations can also provide matrix elements, such as the glueball-to-vacuum matrix element which determines glueball decay rates; these are generically more difficult to study than the glueball mass, but some results are available in [127, 130].

In the limit that their masses are very large, the properties of the heavy "quirk" states can also be studied in pure-gauge theory as static quarks. The string tension σ , which characterizes the potential between static quarks, has been studied in pure gauge theory and in the large- N_c context by [131, 132]. These studies, as well as investigations of stringbreaking effects such as [133] (for QCD at near-physical quark masses) or lattice calculations of parton distributions [134], may also be of interest in modeling hidden hadronization effects. References [131, 132] also study the finite-temperature confining phase transition in the context of large- N_c ; in a neutral naturalness model, such a transition would be realized as a cosmological phase transition in the early Universe. Away from the pure-gauge limit, there is a substantial amount of existing lattice QCD work done at heavier-than-physical quark masses. These results are well-suited to being adapted for use in neutral naturalness models where the hidden quarks are not so heavy compared to the confinement scale that they can be completely ignored. A detailed survey and compilation of some of these lattice results is given in [135]. See also Ref. [136] for studies of bottomonium which can be directly repurposed for use in neutral naturalness models with very heavy hidden-sector quark masses.

In principle, lattice calculations could also be used to study the ultraviolet completions of composite twin Higgs or other pNGB Higgs neutral naturalness models. There is already a substantial literature on lattice calculations and composite Higgs models, see [137–139] for recent reviews. This connection is most readily made for purely gauge-fermion UV completions, such as those discussed in [22, 76]; extensions of work on fermionic UV completions for general composite Higgs theories [140, 141] to the case of composite neutral naturalness could be particularly useful.

VI. CONNECTIONS TO DARK MATTER

The bulk of neutral naturalness dark matter studies stem from the twin Higgs framework. We divide these into those more closely aligned to the mirror set-up and those that assume a fraternal starting point. After discussing twin Higgs dark matter we describe some of the other possibilities that have begun to be explored.

A. Mirror Twin Higgs Dark Matter

As described in Sec. III there are a variety of possible particle spectra in the twin sector and consequently many possibilities for stable twin particles. With the correct interactions and abundance, a stable twin particle could comprise part or all of the dark matter. In the fully Z_2 symmetric mirror twin Higgs model the list of stable twin particles includes the twin photon, the twin electron, twin neutrinos, and twin atoms. The twin photon, twin electron, and twin neutrinos are not sufficiently cold, while self-interactions of twin atoms are too large to make up the dark matter. Thus, in addition to the model building reasons for breaking the Z_2 symmetry, a viable dark matter candidate also requires Z_2 breaking. In the mirror twin set-up, if the twin neutron were lighter than the twin proton, it would be stablilized by twin baryon number, yielding a viable dark matter prospect. Its abundance can be set by an asymmetry in the twin baryons making the twin neutron a natural asymmetric dark matter candidate [30, 142, 143]. The asymmetry in baryon number and twin baryon number could have a common origin [144, 145]. In the simplest case, the twin neutron n_B would have a mass $m_{n_B} \approx (\Omega_{\text{DM}}/\Omega_{\text{baryon}})m_n \approx 5 \text{ GeV}$ [142]. If one allows the masses of the twin up quark, twin down quark, and twin electron to freely vary then the twin neutron, twin Hydrogen atom, and twin Helium atom are all possible dark matter candidates, depending on the hierarchy among masses [30, 143]. When the twin quark masses are taken above the twin QCD scale the preferred mass range for the dark matter is 1 - 30 GeV [30]. Obtaining a twin neutron that is lighter than the twin proton can be accomplished by expanding the Higgs sector to a mirror twin two Higgs doublet model [44] or adding a pair of vector-like quarks with up-type quantum numbers [146].

Whether the dark matter is composed of twin neutrons, twin Hydrogen, or twin Helium, it would interact with visible nucleons via the Higgs portal. Above 10 GeV the cross section is detectable with $\sigma_{\rm DD} \gtrsim 10^{-47}$ cm² [30, 143], while below a mass of 10 GeV this cross section falls below the neutrino floor [142]. For these low masses, introducing kinetic mixing between twin hypercharge and visible hypercharge allows for additional interactions between the dark matter and visible nucleons. For a kinetic mixing of $\epsilon \sim 10^{-11}$ twin Hydrogen or twin Helium dark matter could be detected through nuclear recoils and twin electrons could be detected through electron recoils [147]. Kinetic mixing also allows for the possibility that the twin Hydrogen accumulates in the center of stars. Even in small amounts, the twin Hydrogen could modify the luminosity function of white dwarfs at observable levels [148].

Other than the twin neutron, there are a few more exotic possibilities that exist within the mirror twin Higgs model. If the masses of the twin leptons and twin photon are raised, the lightest twin particle becomes the twin pion. When the twin pions [149], or other twin isospin states [150], have a mass of a few hundred MeV the correct relic abundance can be achieved via the SIMP mechanism. Alternatively, the mirror electron, at a few MeV, could also be the dark matter with an abundance set by freeze-in if a kinetic mixing of $\epsilon \sim 10^{-12}$ between the visible and twin hypercharges is present [105].

As discussed in Sec. III A 1 at energies above the ~ 10 TeV scale the twin Higgs model needs to be UV completed. The most common UV completions employ supersymmetry or a composite Higgs. In a supersymmetric UV completion, in addition to the visible electroweakinos, the twin electroweakinos are also possible dark matter candidates. One example is the twin bino with its freeze-out abundance determined by annihilation to twin fermions [151]. The twin stau is another candidate with analogous annihilation channels to the visible stau [152].

B. Fraternal Twin Higgs Dark Matter

The fraternal twin Higgs model automatically includes several dark matter candidates. In this model there is only a single generation of twin leptons, the twin τ and twin ν , at least one of which is stabilized by twin lepton number. When twin hypercharge is not gauged, the twin τ freezes out via interactions mediated by the twin W and is a viable WIMP for masses between roughly between 60 GeV and 150 GeV [153, 154]. If one allows for the twin ν mass to be comparable to the twin τ , then the dark matter becomes multicomponent, consisting of the twin τ , twin ν , and possibly also the twin W [153]. Weakly gauging twin hypercharge opens up another annihilation channel for the twin τ which lowers the preferred mass range to about 1 - 20 GeV [154]. In the fraternal twin model the twin τ unavoidably interacts with the Higgs leading to tension with current direction detection bounds [155–157]. With an additional set of hypercharged scalars the relation between the twin τ mass and direct detection cross section can be broken allowing for rates below those of the neutrino floor [158].

Small variations on the fraternal twin Higgs set-up can lead to interesting experimental signals and model building. If the first and second generation of twin quarks are reintroduced, the lightest twin particles can be long-lived pion-like mesons. The annihilation of the twin τ s in indirection detection leads to a QCD-like shower within the twin QCD sector ending either with stable twin pions or visible photons [159]. Alternatively, if twin electromagnetism is broken, the twin photon obtains a mass and the twin τ and twin ν can mix. After this mixing, the dominantly twin ν state can be the dark matter, between 0.1 and 10 GeV, where the relic density can be set by either co-annihilation or co-scattering, depending on the mass of the twin τ and twin photon [160].

With only a single generation of twin quarks, the lightest twin baryon is the spin-3/2 baryon Δ_B with the quantum numbers of three twin b quarks $(b_B b_B b_B)$. When this state is

lighter than the twin τ , it is a viable asymmetric dark matter candidate both when m_{b_B} is parametrically above the twin QCD scale [161] or below the twin QCD scale [162]. When twin hypercharge is gauged, the lightest neutral state is a twin atom which consists of a Δ_B bound with a twin τ . This Δ_B asymmetry and the visible baryon asymmetry can be generated simultaneously, provided there is a sufficiently large kinetic mixing [42].

With a little additional structure, the fraternal twin Higgs model contains the correct moving pieces to implement either partially acoustic dark matter (PAcDM) or quasi-acoustic dark matter (QuAcDM) [163]. For PAcDM, the dark matter consists of a component that acts like traditional cold dark matter and a second sub-dominant component that interacts efficient with dark radiation. This interaction suppresses the matter power spectrum at small scales. The twin baryon Δ_B acts as the cold dark matter while the twin τ can comprise the sub-dominant dark matter component provided that the twin $U(1)_L$ is gauged to mediate efficient interactions between the twin τ and twin ν . QuAcDM, on the other hand, allows all species of DM to couple, slightly inefficiently, to dark radiation, also suppressing the matter power spectrum at small scales. Gauging $U(1)_{B-L}$ realizes this set-up where the Δ_B and twin τ couple to the twin ν via the weakly gauged $U(1)_{B-L}$.

C. Exotic Dark Matter

Additional particles introduced into twin models can also be the dark matter. Adding a singlet scalar S that couples to both Higgs doublets via $S^2(H_A^2 + H_B^2)$ adds a scalar portal on top of the twin Higgs model. The dark matter in this case is the scalar S and the twin mechanism, in addition to its usual advantages, now additionally lowers the direct detection cross section, re-opening viable masses for the S roughly above 100 GeV [164].

Twin models have also been proposed where the twin color SU(3) is broken to SU(2). This breaking causes the twin quarks to split into an SU(2) doublet and SU(2) singlet. The SU(2) singlet that results from the twin *b* quark is a viable asymmetric dark matter candidate [145].

Dark matter has been explored in neutral naturalness models other than the twin Higgs. The dark top model cancels the leading top quark divergences with 3 species of a fermionic top partner [72]. In the twin model there are 3 species of top partner due to the twin SU(3)group while in dark top models other group structures are possible. One example is the coset $SU(6)_C \times SU(3)_L/SU(6)_C \times SU(2)_L$ where the left-handed top quark can be embedded in a (6, $\overline{\mathbf{3}}$) of the global group which contains a (3, 1, 2), the SM top, and a (1, 3, 1), the dark top, where the first two labels are the $SU(3) \times SU(3)$ subgroup of $SU(6)_C$ and the third label is $SU(2)_L$. Without the additional structure of a twin Higgs model, the dark top is stable and thus a dark matter candidate. When the dark top is an electroweak singlet, its primary interactions with the Standard Model are through the Higgs which mediates both freeze-out and direct detection. For dark tops that are partly electroweak doublets, there is a possible Z interaction which greatly increases both the annihilation cross section and direct detection rate.

In expanded cosets, the additional pNGBs that accompany the Higgs multiplet could be the dark matter. The coset SO(7)/SO(6) is an example where the SO(6) contains the electroweak group and an additional unbroken U(1) [80]. This breaking leads to six pNGBs including the Higgs multiplet and a complex scalar χ that is stablized by the U(1). The scalar χ interacts with the Higgs both through higher-order Goldstone interactions and through interactions with heavy fermions. The momentum-dependence of the Goldstone interactions suppresses the direct detection rate relative to the typical Higgs-mediated rate.

VII. CONNECTIONS TO ASTROPHYSICS AND COSMOLOGY

In addition to a variety of potential dark matter candidates, the presence of a rich hidden sector in many neutral naturalness models has myriad implications for astrophysics and cosmology.

A. Cosmological Signals

One of the most notable challenges for the mirror twin Higgs model is the contribution of the light twin particles to the energy density, as measured by the parameter ΔN_{eff} . This is defined via

$$\rho_{\rm rad} = \rho_{\gamma} + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\rm eff} \,\rho_{\gamma},\tag{9}$$

where $\rho_{\rm rad}$ is the total energy density in radiation, ρ_{γ} is the energy density of visible photons, and $N_{\rm eff} = 3.044 + \Delta N_{\rm eff}$. For free streaming radiation $\Delta N_{\rm eff} \leq 0.30$ [165]. Without Z_2 breaking, three twin neutrinos would contribute ≈ 3 and the twin photon would contribute ≈ 4.4 for a total contribution of $\Delta N_{\text{eff}} = 7.4$ which is strongly excluded. Including Z_2 breaking decreases this to $\Delta N_{\text{eff}} = 5.6$ because when the visible and twin sectors decouple there are more degrees of freedom in the visible thermal bath which will slightly heat the visible sector radiation. The two sectors are kept in thermal equilibrium by the Higgs portal interaction which decouples around 3 GeV [25].

Within the mirror twin Higgs framework, there are a few approaches to reducing the contribution to ΔN_{eff} . The first is to asymmetrically reheat the visible and twin sectors such that the temperature of the twin sector is lower. Since $\rho_{\text{rad}} \propto T^4$, a slight reduction in the twin sector temperature is enough to satisfy observational constraints. Adding a right-handed neutrino N_A (and its twin N_B) can successfully heat the visible sector preferentially [166]. The heavy neutrinos dominate the energy density, then decay after the visible and twin sectors decouple, but before well-probed processes, like Big Bang nucleosynthesis, occur. The decays are dominantly to the visible sector because of the mass difference between the gauge bosons of each sector. A scalar singlet X could also be used instead of a right-handed neutrino [167]. With heavy right-handed neutrinos and a new colored scalar (and its twin), baryogenesis can be achieved along with asymmetric reheating [146].

Additional portals between the visible and twin sectors can allow entropy in the twin sector to return to the visible sector. For example, the neutrino portal allows the twin neutrino, when sufficiently heavy, to undergo a three-body decay into three visible neutrinos [35].

Another approach to reducing ΔN_{eff} is to adjust the decoupling temperature between the visible and twin sectors. While the decoupling temperature cannot be increased without giving up the twin mechanism, it can be lowered if additional interactions are included to keep the sectors in thermal equilibrium longer. If the decoupling temperature is below the twin QCD phase transition (and above the visible QCD phase transition) then a larger number of degrees of freedom in the visible thermal bath heat the visible sector radiation substantially more [168]. A small mixing between the visible and twin neutrinos is sufficient to reduce ΔN_{eff} to ≈ 1 and additional model building can bring it into agreement with the bound $\Delta N_{\text{eff}} \leq 0.30$.

Yet a different approach to reducing ΔN_{eff} leverages the fact that the Higgs interacts weakly with the neutrinos. Therefore, if the twin fermions are taken to be very heavy then the twin neutrinos decouple from the thermal bath before the visible and twin sectors decouple [169]. Since the twin neutrinos leave the thermal bath earlier, their temperature is lower, reducing their contribution to ΔN_{eff} . The contribution from the twin photons can be reduced by including a mass and kinetic mixing with the visible photons so that their entropy is transferred to the visible photons.

In the mirror twin Higgs model, the presence of light twin electrons, twin protons, and twin photons results in twin baryon acoustic oscillations, analogous to the baryon acoustic oscillations in the visible sector [170]. These twin baryon acoustic oscillations occur prior to twin recombination and both suppress structure formulation and leave an oscillatory pattern in the matter power spectrum. The oscillations in the matter power spectrum only occur at small scales (corresponding to modes that are inside the horizon prior to twin recombination) and the inclusion of a twin sector, with a sufficiently low temperature and relative energy density, is fully consistent with existing Cosmic Microwave Background and large-scale structure data [171]. Future measurements of the matter power spectrum have the potential to observe twin baryon acoustic oscillations.

Without the twin electrons, twin protons, or twin photons, the fraternal twin Higgs model alone does not suppress the matter power spectrum. A suppression can be achieved in the fraternal twin Higgs model by gauging either twin $U(1)_{B-L}$ or twin $U(1)_L$ [163]. This additional force mediates interactions between some, or all, of the twin matter and the radiation-like twin neutrinos. This coupling of the dark matter to radiation suppresses the growth of density perturbations.

B. Cosmological Tensions

At present, there are two notable cosmological discrepancies. The first is a tension between the inferred value of the Hubble constant from Planck [165] and the measured late-time value [172]. The late-time value of Hubble is larger by $4 - 5\sigma$. The second is a tension between the inferred value of S_8 (the amplitude of matter fluctuations at scales of roughly $8h^{-1}$ Mpc) from Planck [165] and the value measured from weak lensing and galatic cluster surveys [173]. The value from Planck is larger by $2 - 3\sigma$.

One attractive feature of the mirror twin Higgs model is that it naturally addresses these tensions simultaneously. Reference [171] showed that the twin photon and twin neutrinos provide enough additional radiation to increase the value of the Hubble constant while the twin baryons comprising a subcomponent of the dark matter suppresses the matter power spectrum at small scales. It is estimated that the fraternal twin Higgs model (with either twin $U(1)_{B-L}$ or twin $U(1)_L$ gauged) might similarly address these tensions [163].

C. Phase Transitions, Gravitational Waves, and Baryogenesis

It is very interesting to consider the potential consequences of the various early Universe phase transitions expected in twin Higgs models. Reference [174] presents the basic conditions required for a strong first order phase transition in confining SU(N) hidden sectors and studies the associated gravitational wave signals and observational prospects. In particular, it is found that a strong first order mirror QCD phase transition and corresponding observable gravitational wave signal is expected in fraternal twin Higgs models since no light quark flavors are present in the spectrum. Complementing this work, a detailed study of the electroweak and global U(4) symmetry breaking phase transitions in mirror twin Higgs models was presented in Ref. [175]. They indicate that these phase transitions are generically not first order, which dampens the prospects for electroweak baryogenesis and gravitational wave signatures associated with these phase transitions.

Other works examine the possibility that electroweak symmetry may not be restored at high temperature in twin Higgs models. Reference [176] demonstrates that in mirror twin Higgs models, while the one-loop quadratic contributions to the Higgs potential at finite temperature from the SM degrees of freedom are cancelled by their same spin partners (a consequence of the twin Higgs mechanism), subleading corrections remain which drive the potential to a symmetry-restored phase at high temperatures. On the other hand, Ref. [177] points out that if the Yukawa couplings of twin fermions other than those of the twin top are larger than their SM counterparts, the electroweak symmetry may not be restored at scales below the global symmetry breaking scale $f \sim$ TeV. This could have important implications for electroweak baryogenesis.

Besides electroweak baryogenesis, several works propose mechanisms of baryogenesis, leptogenesis, and cogenesis of visible and dark matter asymmetries through the CP-violating out-of-equilibrium decays of heavy states. Reference [144] explores a simple model of twin baryogenesis, in which asymmetries in the twin and visible sectors are generated through the out-of-equilibrium decay of a TeV scale particle charged under both baryon and twin baryon number. The resulting twin baryons can serve as an asymmetric dark matter candidate and explain the relic abundance provided the twin baryon mass is around 5 GeV, which can be realized with the introduction of suitable sources of Z_2 breaking. As another example, Ref. [178] studies a model in which light Dirac neutrino masses are generated through a see-saw mechanism connecting the visible and mirror sectors. Leptogenesis occurs through the decays of the heavy singlet Dirac neutrinos to the SM leptons and mirror leptons. See also Refs. [42, 145, 146] for interesting related mechanisms of baryogenesis or leptogenesis.

D. Mirror Stars

In the simplest realizations of mirror twin Higgs models, the Z_2 symmetry implies the presence of both electromagnetism and nuclear physics in the mirror sector. It is therefore natural to speculate about astrophysical compact objects built out of mirror matter, i.e., mirror stars [148, 179–181]. References [179, 180] discuss the astrophysical signatures of such mirror stars. If the mirror and visible sector photons interact through a small kinetic mixing, visible matter may be captured in mirror star cores. This visible sector component of the mirror star—the "SM nugget"—is then heated through interactions with mirror matter. This may present discoverable electromagnetic signatures in both optical frequencies, due to bremsstrahlung cooling processes, and in X-rays, due to "Thompson conversion" of mirror photons to ordinary photons. A detailed search strategy for directly observing mirror stars using Gaia observations is presented in Ref. [181]. Furthermore, Ref. [182] explores the formation and structure of mirror neutron stars using realistic equations of state adapted from those describing ordinary neutron stars. The observational prospects for mirror neutron stars using gravitational waves and binary pulsars are also discussed.

As these studies illustrate, the presence of rich hidden sectors in neutral naturalness theories may lead to highly novel astrophysical systems and signatures. Given the exciting program of current and planned astronomical surveys, further investigations in this direction would be timely.

VIII. CONNECTIONS TO NEUTRINOS

Neutral naturalness models can impact neutrino physics in a variety of ways, including through novel mechanisms for neutrino masses, new leptogenesis scenarios, connections with the dark radiation problem, and explanations of oscillation anomalies and other experimental discrepancies. First, in many neutral naturalness scenarios such as the twin Higgs, there are a host of new fermions that are neutral under the SM gauge symmetries. With suitable symmetry breaking mechanisms, such fermions can marry with SM neutrinos, effectively serving as right-handed neutrinos and generating neutrino masses through a see-saw mechanism. Along these lines, Ref. [73] proposed that the neutral top partners, responsible for cancelling the dominant corrections to the Higgs potential, may simultaneously function as right-handed neutrinos. In this framework, naturalness considerations robustly predict a TeV-scale see-saw, and large neutrino Yukawa couplings may be achieved through the inverse or linear see-saw mechanisms along with an associated rich phenomenology. Another interesting possibility for neutrino mass generation arises in models with fermionic singletons—completely neutral fields with no Z_2 partners—which can connect the visible and mirror lepton sectors [108]. There is clearly a wide scope for further studies of novel neutrino mass mechanisms within models of neutral naturalness.

The dynamics underlying neutrino masses may also have important consequences for cosmology in the mirror twin Higgs framework, as discussed in Sec. VII. In particular, the challenges associated with a large number of relativistic degrees of freedom in the mirror twin Higgs model can be circumvented in models with a low-scale (~GeV) see-saw model, as described in Ref. [166]. In this scenario, the right-handed neutrinos freeze-out with a large thermal abundance and later become non-relativistic due to the cosmological expansion, eventually dominating the energy density. After the SM and mirror sectors decouple, the right-handed neutrinos preferentially decay to SM states, increasing the energy density in the visible sector relative to that of the mirror sector, thereby alleviating the bounds from dark radiation. Reference [168] also investigated the impact of a low-scale see-saw on twin cosmology, demonstrating that mixing between the mirror and SM neutrinos can maintain thermal equilibrium between the two sectors until temperatures below the mirror QCD phase transition, thus lowering the prediction for $\Delta N_{\rm eff}$. Interesting scenarios of leptogenesis within the twin Higgs framework have also been put forth in Refs. [42, 178], and it would be of great interest to expand on these investigations; see Sec. VII C for further discussion.

In a separate direction, neutral naturalness models can potentially provide explanations for experimental anomalies associated with the neutrino sector. For example, Ref. [183] speculates that three twin neutrinos may potentially explain the LSND and MiniBooNE anomalies while evading the otherwise stringent constraints from disappearance experiments. As another illustration, Ref. [184] proposes an explanation of the EDGES 21-cm absorption signal that relies on a relic population of mirror sector neutrinos that produce mirror photons through their decays. The mirror photons are then resonantly converted to visible sector photons, enhancing the 21-cm absorption signal. Finally, in Refs. [185, 186] a potential resolution of the muon anomalous magnetic moment discrepancy in see-saw extensions of twin Higgs models is explored.

IX. CONNECTIONS TO FLAVOR

There are potentially far reaching implications for flavor physics in neutral naturalness models, although the existing literature has only scratched the surface of the subject. An interesting illustration comes from the consideration of flavor in composite twin Higgs models, as in the study of Ref. [57]. In ordinary composite Higgs models, the attractive hypothesis of quark flavor anarchy generically leads to stringent bounds on the scale of the composite resonances, which in turn requires a rather severe tuning of the composite Higgs potential, typically at the per-mille level. On the other hand, in composite twin Higgs models, the composite resonances can be freely raised to scales of order 10 TeV without incurring significant tuning, since the twin partners (and not the visible sector composite resonances) cancel the dominant quantum corrections to the Higgs potential. Raising the composite resonances then significantly weakens, though does not entirely eliminate, constraints from flavor, allowing a scenario with per-cent level tuning.

Also motivated by the flavor puzzle and the little hierarchy problem, Ref. [43] studied a twinned flavorful two-Higgs doublet model, in which one Higgs doublet generates the masses of the third generation while a second doublet is responsible for masses of the light generations. This scenario can provide a partial understanding of the hierarchical patterns observed in the fermion masses and also predicts novel signatures associated with the heavy scalar doublet in both flavor physics and at colliders.

While the examples above primarily concern flavor physics in the visible sector, there are also good motivations to consider the nature of fermion flavor in the mirror sector in twin Higgs models. The fraternal twin Higgs scenario [28] represents perhaps the most dramatic breaking of Z_2 symmetry in the the fermion flavor sector, with all but the third generation twin fermions removed from the spectrum. This scenario is well motivated from the a bottom-up naturalness perspective, and has important consequences for phenomenology and cosmology, as discussed above in Secs. III A,VI B,VII. Beyond the fraternal scenario, Ref. [30] considers a more modest breaking of the Z_2 exchange symmetry in the Yukawa couplings (except of course in the top Yukawa). This has several interesting implications, including the radiative generation of Z_2 breaking in the Higgs potential, which provides the correct vacuum alignment, the alleviation of bounds from too much dark radiation, and several potential dark matter candidates in the form of mirror baryons and leptons. Following this work, the authors proposed an effective theory that simultaneously explains the fermion mass hierarchies as in e.g., Froggatt-Nielsen models, and also realizes the Z_2 breaking in the Yukawa couplings through spontaneous Z_2 breaking between the visible and mirror flavor symmetries [143]. Novel spontaneous twin gauge symmetry breaking patterns can also lead to new flavor structures in the twin fermion sector [34, 35], with an array of associated phenomenological signatures.

It would be interesting to further explore solutions to the flavor puzzle within the neutral naturalness paradigm, particularly in the setting of concrete UV completions that permit extrapolation between the TeV scale and high scales where flavor is generated. Likewise, a worthy goal is to understand to what extent the structure of flavor in the hidden sectors of neutral naturalness theories can be tied through symmetry to visible sector observables at colliders and in precision measurements.

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