# Axion-Like Particles at High Energy Muon Colliders – A White paper for Snowmass 2021

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ABSTRACT: We study the discovery potential for heavy axion-like particles (ALPs) and the perspectives for determining their coupling properties at a muon collider. Focusing on their couplings to the Standard Model (SM) gauge bosons  $\gamma, Z, W^{\pm}$ , we show that a high-energy muon collider can substantially extend the mass coverage, essentially reaching the kinematic limit of the collider energy. The unique kinematics allow for non-ambiguous determination of the individual coupling strengths. The associated production via  $\mu^+\mu^-$  annihilation and the VBF processes with the tagged outgoing muons can be utilized to verify the CP property of the ALPs. We illustrate our results for a muon collider running at 3 TeV and 10 TeV.

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# 1 Introduction

Axion-like particles (ALPs) exist in a variety of theories beyond the Standard Model (SM) [1, 2]. They are CP-odd scalars, pseudo-Nambu-Goldstone bosons associated with a spontaneous global U(1) symmetry breaking, and singlets under the SM charges. The best known example is the QCD axion [3–6] (for a recent review see Ref. [7]), originally proposed to solve the strong CP problem [8–15]. It is soon realized that an ALP may appear in many theoretical constructions, such as composite models [16–32], extra dimension models [33–39], Grand Unification models [40–59] and superstring theories [60–62]. Due to the vastly different theoretical motivations and incarnations, the mass ( $m_a$ ) and the scale ( $\Lambda$ ) associated with the ALP physics can be drastically different [12, 18, 63–73], ranging from a light axion in sub-eV [10–13, 74] to a heavy one in TeV [28, 43, 75–79], and even beyond [80–82]. Therefore, the experimental search for the ALPs would require rather different techniques and facilities. Any experimental observation for such a state, on the other hand, would significantly extend our knowledge for physics beyond the SM [2, 7, 83].

High-energy colliders have been the primary tool for discoveries in the past decades. Once reaching a new energy threshold, the collider experiments will most effectively reveal the mass and interactions of the new states at the energy frontier. The same approach has been applied to the searches for ALPs at the CERN Large Hadron Collider (LHC) [84–93] and future lepton colliders such as ILC, FCC-ee and CLIC [88, 94–97] (see Ref. [98] and references therein for the current status of ALPs search at LEP, Belle-II, Tevatron and LHC). High-energy colliders have been the primary tool for discoveries at the energy frontier. After the discovery of the Higgs boson at the LHC, the luminosity upgrade of the LHC will take the lead in searching for new physics beyond the Standard Model [99]. There also have been considerations to construct next generator of hadron colliders of the order of 100 TeV in c.m. energy [100, 101].

Recently, high energy muon colliders have gained much attention in the community after the endorsement of its R&D by the European strategy and the subsequent formation of the Muon Collider Collaboration [102]. The recent technological development [103–105] has encouraged the community to consider the high-energy option at multi-TeV, thus provides tremendous opportunities to produce and discover new heavy EW particles. An optimistic scheme is to target on a high integrated luminosity and to scale it with energy quadratically as

$$\mathcal{L} = \left(\frac{\sqrt{s}}{10 \text{ TeV}}\right)^2 10 \text{ ab}^{-1}.$$
(1.1)

In particular, we consider two benchmark choices of the collider energies and the corresponding integrated luminosities for illustration,

$$\sqrt{s} = 3 \text{ and } 10 \text{ TeV}, \quad \mathcal{L} = 1 \text{ and } 10 \text{ ab}^{-1}.$$
 (1.2)

The first choice corresponds to a comparative benchmark associated with the CLIC [106]. The second choice is the high energy option targeted for a future muon collider.

Motivated by this exciting perspective, we perform a first study on the reach of muon colliders for the ALPs. Based on the general parameterization of the ALP couplings to the SM gauge bosons, we calculate the ALP signal cross sections, their discovery potential, and spin-parity property determination. We demonstrate that a muon collider has the doubleadvantage for reaching a higher mass threshold via the  $\mu^+\mu^-$  direct annihilation, and for offering multiple production channels via vector boson fusion processes. The clean experimental environment and the well-defined kinematics in leptonic collisions provide the great opportunity for determining the ALPs properties.

The rest of this paper is organized as follows. In section 2, we set up our theoretical framework for the ALPs interactions with the SM gauge bosons, present the current search bounds on the theory parameters of the ALP mass and couplings, and comment on the prospects for searches from the other colliders. In section 3, we present our numerical analyses for the search and test of the ALP properties at a muon collider with the two sets of energy and luminosity benchmarks. We summarize our results and discuss directions for further exploration in section 4.

## 2 General Interactions for ALPs

Starting from the SM, we introduce a generic massive CP-odd scalar denoted by a, presumably a pseudo Nambu-Goldstone boson – an axion-like particle ALP associated with a global U(1) symmetry spontaneously broken above the electroweak scale. Besides the kinetic term for ALP, the most general effective Lagrangian for bosonic ALP interactions is composed of four dimension-five operators [85], respecting the SM gauge symmetry

$$\mathcal{L}_{eff} = C_{\tilde{G}}\mathcal{O}_{\tilde{G}} + C_{\tilde{B}}\mathcal{O}_{\tilde{B}} + C_{\tilde{W}}\mathcal{O}_{\tilde{W}} + C_{a\Phi}\mathcal{O}_{a\Phi} , \qquad (2.1)$$

with

$$\mathcal{O}_{\tilde{G}} \equiv -\frac{a}{f_a} G^i_{\mu\nu} \tilde{G}^{\mu\nu}_i , \quad \mathcal{O}_{\tilde{W}} \equiv -\frac{a}{f_a} W^j_{\mu\nu} \tilde{W}^{\mu\nu}_j , \qquad (2.2)$$

$$\mathcal{O}_{\tilde{B}} \equiv -\frac{a}{f_a} B_{\mu\nu} \tilde{B}^{\mu\nu} , \quad \mathcal{O}_{a\Phi} \equiv i \frac{\partial^{\mu} a}{f_a} (\Phi^{\dagger} \overleftrightarrow{D}_{\mu} \Phi) , \qquad (2.3)$$

where  $\Phi$  is the SM Higgs doublet,  $G^i_{\mu\nu}$   $(i = 1, \dots, 8)$ ,  $W^j_{\mu\nu}$  (j = 1, 2, 3) and  $B_{\mu\nu}$  denote the field strength tensors of  $SU(3)_c$ ,  $SU(2)_L$  and  $U(1)_Y$  gauge fields, respectively, and the dual field strengths are defined as  $\tilde{X}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} X_{\alpha\beta}$  with  $\epsilon^{0123} = 1$ . The new physics scale is denoted by the constant  $f_a$ . After the electroweak symmetry breaking, except the Yukawaaxion coupling induced by  $\mathcal{O}_{a\Phi}$ , the interactions between ALP and the physical SM gauge bosons are

$$\mathcal{L}_{eff} \supset -\frac{g_{agg}}{4} a G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a - \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{g_{a\gamma Z}}{4} a F_{\mu\nu} \tilde{Z}^{\mu\nu} -\frac{g_{aZZ}}{4} a Z_{\mu\nu} \tilde{Z}^{\mu\nu} - \frac{g_{aWW}}{4} a W_{\mu\nu} \tilde{W}^{\mu\nu} , \qquad (2.4)$$

where

$$g_{agg} = \frac{4}{f_a} C_{\tilde{G}} , \quad g_{a\gamma\gamma} = \frac{4}{f_a} (s_{\theta}^2 C_{\tilde{W}} + c_{\theta}^2 C_{\tilde{B}}) , \quad g_{aZZ} = \frac{4}{f_a} (c_{\theta}^2 C_{\tilde{W}} + s_{\theta}^2 C_{\tilde{B}}) ,$$
  

$$g_{a\gamma Z} = \frac{8}{f_a} s_{\theta} c_{\theta} (C_{\tilde{W}} - C_{\tilde{B}}) , \quad g_{aWW} = \frac{4}{f_a} C_{\tilde{W}} ,$$
(2.5)

with  $s_{\theta}(c_{\theta})$  being the sine (cosine) of the weak mixing angle  $\theta_W$ . Below we assume the absence of the gluonic contribution and the Yukawa-axion coupling  $C_{\tilde{G}} = C_{a\Phi} = 0$  to focus on the EW sector only for simplicity. The Feynman rule between the ALP and two SM gauge bosons ( $V_1$ and  $V_2$ ) turns out to be<sup>1</sup>

$$-ig_{aV_1V_2} p_{V_1\alpha} p_{V_2\beta} \epsilon^{\mu\nu\alpha\beta} , \qquad (2.6)$$

with the momenta  $(p_{V_1}, p_{V_2})$  flowing inwards in the vertices. The independent model parameters to be studied are  $C_{\tilde{W}}/f_a$  and  $C_{\tilde{B}}/f_a$  in this formalism.

## 3 ALP phenomenology at Muon Colliders

The future high energy Muon colliders have great potential in probing ALP at multi-TeV scales. In the section, we study the ALP phenomenology at high energy muon colliders. In particular, we consider three benchmark scenarios where

(I) 
$$C_{\tilde{W}} = C_{\tilde{B}} \neq 0$$
; (3.1)

(II) 
$$C_{\tilde{W}} = 0, \quad C_{\tilde{B}} \neq 0;$$
 (3.2)

(III) 
$$C_{\tilde{W}} \neq 0, \quad C_{\tilde{B}} = 0.$$
 (3.3)

<sup>&</sup>lt;sup>1</sup>The complete Feynman rules from the bosonic ALP effective Lagrangian can be found in the Appendix B of Ref. [85]



Figure 1. The cross sections of different production modes from the direct annihilation  $\mu^+\mu^- \rightarrow \gamma a, Za$ , and the VBF processes, as a function of  $\sqrt{s}$  for  $m_a = 1$  TeV. The dashed brown curves indicate the  $\gamma Z$  destructive interference.

### 3.1 Production

The ALP can be produced at muon colliders through two different topologies, the associated production via  $\mu^+\mu^-$  annihilation and the electroweak vector-boson-fusion (VBF). In this section, we study the signature of different production modes and their projected sensitivities at muon colliders.

#### 3.1.1 Associated production

We first consider the ALP production associated with an electroweak vector boson through  $\mu^+\mu^-$  annihilation processes

$$\mu^+\mu^- \to Va, \quad V = \gamma, Z .$$
 (3.4)

Each production above is in general determined by the interference of two diagrams induced by two ALP couplings which are  $g_{a\gamma\gamma}, g_{a\gamma Z}$  (for  $\gamma a$  production) and  $g_{a\gamma Z}, g_{aZZ}$  (for Za production). Figures 1 and 2 show the production cross sections as a function of  $\sqrt{s}$  and  $m_a$ , respectively, where the associated productions are represented by red ( $\gamma a$ ) and green (Za) lines. The cross sections are normalized by  $C^2_{\tilde{W},\tilde{B}}/f_a^2$ . For the case with  $C_{\tilde{W}} = C_{\tilde{B}}$ , in particular, the coupling  $g_{a\gamma Z}$  vanishes and there appears only one relevant diagram. The momenta-dependence of the dimension-5 operators is cancelled by the *s*-channel propagator, and this cancellation leads to a constant behavior of cross section at high energies as shown by the red ( $\gamma a$ ) and green (Za) lines in Fig. 1

$$\sigma_{V_1^*V_2a} \propto g_{aV_1V_2}^2 \sim C_{\tilde{W},\tilde{B}}^2 / f_a^2 , \qquad (3.5)$$

where  $V_1^*$  denotes the gauge boson propagator in the *s*-channel and  $V_2$  is the one in final states. The falling behavior as a function of  $m_a$  in the panels of Fig. 2 only comes from the suppression of phase space.



Figure 2. The cross sections of different production modes from the direct annihilation  $\mu^+\mu^- \rightarrow \gamma a, Za$ , and the VBF processes, as a function of  $m_a$  at  $\sqrt{s} = 10$  TeV muon colliders. The dashed brown curves indicate the  $\gamma Z$  destructive interference.

#### 3.1.2 Vector boson fusion production

At higher colliding energies, the productions of ALPs through VBF processes

$$\gamma\gamma, ZZ, \gamma Z, WW \to a$$
 (3.6)

become increasingly important. For c.m. energies  $\sqrt{s} \gg m_W$ , the initial muon beams substantially radiate EW gauge bosons under an approximately unbroken SM gauge symmetry. This becames particularly problematic for photon-photon fusions, where the collinear singularity is regulated by the tiny muon mass and and leads to large logarithm  $\ln(s/m_{\mu}^2)$ . We treat the vector bosons as initial state partons, and calculate the VBF cross sections, by utilizing the leading-order framework of electroweak parton distribution functions (EW PDFs) [107] with a dynamical scale  $Q = \sqrt{\hat{s}}/2$ , where  $\sqrt{\hat{s}}$  is the partonic c.m. energy. For the initial gauge boson partons  $V_i$  and  $V_j$ , the VBF production cross section can be factorized as the product of the parton luminosity  $d\mathcal{L}_{ij}/d\tau$  and the partonic cross section  $\hat{\sigma}$ 

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(V_i V_j \to F) ,$$
  
$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{d\xi}{\xi} \Big[ f_i(\xi, Q^2) f_j(\frac{\tau}{\xi}, Q^2) + (i \leftrightarrow j) \Big] , \qquad (3.7)$$

where F(X) denotes an exclusive final state (the underlying remnants),  $f_i(\xi, Q^2)$  is the EW PDF for vector  $V_i$  with Q being the factorization scale,  $\tau_0 = m_F^2/s$  and  $\tau = \hat{s}/s$ . By summing over all gauge boson initial states, one can obtain the total cross section of the "inclusive" production processes. The cross section is enhanced by collinear logarithm  $\ln(\hat{s}/m_{\mu}^2)$  for photon or  $\ln(\hat{s}/m_V^2)$  for massive gauge boson V at high beam energies. In contrast to the constant behavior in the associated production, such logarithmic enhancement can be seen in Fig. 1 where we choose the factorization scale as  $Q = \sqrt{\hat{s}}/2$ . Moreover, suppose the momenta of incoming two gauge bosons  $(p_{V_1\alpha}, p_{V_2\beta})$  are longitudinally back-to-back along the beam direction, the totally antisymmetric tensor  $\epsilon^{\mu\nu\alpha\beta}$  in the ALP vertex determines the polarization of them  $(\varepsilon_{V_1\mu}, \varepsilon_{V_2\nu})$  to be transverse. Thus, each VBF production is dominated by the transverse gauge boson  $(\gamma, W_T, Z_T)$  fusion.

In Figs. 1 and 2, we show the production cross sections of each individual fusion channels:

$$\gamma \gamma \rightarrow a$$
 (blue),  
 $ZZ \rightarrow a$  (cyan),  
 $\gamma Z \rightarrow a$  (orange),  
 $W^+W^- \rightarrow a$  (magenta).

Note that the  $\gamma$ - and Z-initiated fusion processes are physically indistinguishable, and in principle should be added coherently. For the sake of illustration, we compute such interference, also in the leading-log approximation, which is shown using brown lines in Figs. 1 and 2. The solid and dashed lines correspond to constructive and destructive interference, respectively. While the interference effects are usually orders of magnitude smaller, they do become important for heavy axion masses. As shown in Figs. 1 and 2, when the  $g_{a\gamma\gamma}$  coupling is not suppressed, the  $\gamma\gamma$ -fusion always dominates, due to its  $\ln(s/m_{\mu}^2)$  enhancement, as in scenarios (I) and (II). However, as in scenario (III), the  $g_{a\gamma\gamma}$  coupling is suppressed by  $s_{\theta}^2$  and the  $\gamma\gamma$ -fusion become significantly smaller.

In comparison to the inclusive VBF production, for the  $\gamma$ - and Z-initiated fusion processes, one can also consider the exclusive VBF processes by requiring the outgoing  $\mu^+\mu^-$  to be observable in the detector coverage

$$10^{\circ} < \theta_{\mu^{\pm}} < 170^{\circ}.$$
 (3.8)

Such requirement would greatly suppress the production cross section, since the outgoing muons tend to be collinear to the beamline, especially for the  $\gamma\gamma$ -fusion. In addition, we also require

$$m_{\mu^+\mu^-} > 200 \text{ GeV},$$
 (3.9)

for the exclusive channel to enhance the its VBF topology. As shown by black lines in Figs. 1 and 2 labelled by "Dimuon", the cross section for exclusive processes are typically two orders of magnitude smaller than the ones for inclusive processes. Such difference becomes smaller in the case of  $C_{\tilde{B}} = 0$ , where the  $\gamma\gamma$ -fusion is suppressed by the relatively small coupling  $g_{a\gamma\gamma}$ . In spite of smaller cross sections, tagging the outgoing muons can still provide an extra handle for the signal event selection and help to reveal the CP property of the ALPs, as discussed in the next section.

#### 3.2 Projected bounds

To estimate the sensitivity, we consider the simplest decay channel as the signal

$$a \to \gamma \gamma$$
 . (3.10)

The leading background for the associated production is

$$\mu^+\mu^- \to V\gamma\gamma, \quad V = \gamma, Z .$$
 (3.11)

For VBF processes, we consider two different signal categories:

inclusive: 
$$\mu^+\mu^- \to a + X$$
, (3.12)

dimuon: 
$$\mu^+\mu^- \to a + \mu^+\mu^-$$
, (3.13)

where the exclusive dimuon channel is as described in the previous subsection, with the cuts in Eqs. (3.8) and (3.9) applied. The dominant background for the inclusive production is

$$\mu^+\mu^- \to \gamma\gamma \;, \tag{3.14}$$

where the invariant mass of the photon pair is smeared due to the initial-state-radiation (ISR) effect. We simulated such background using WHIZARD [108]. For the exclusive production, the dominant background becomes

$$\mu^+\mu^- \to \mu^+\mu^-\gamma\gamma \;. \tag{3.15}$$

We simulate the backgrounds in Eqs. (3.11), (3.14) and (3.15) using MadGraph [109] at the parton level.

For photon reconstruction, we impose the following basic cuts of transverse momentum, rapidity, and separation on the photons in final states

$$p_T(\gamma) > 10 \text{ GeV}, \quad |\eta(\gamma)| < 2.5, \quad \Delta R_{\gamma\gamma} > 0.4.$$
 (3.16)

To suppress the continuum background, we further impose the invariant mass on the diphoton resonance

$$\frac{|m_{\gamma\gamma} - m_a|}{m_a} < 0.05 . \tag{3.17}$$

We then estimate the significance and evaluate the projected sensitivities. The local significance is quantified as

$$N_{\rm SD} = \frac{S}{\sqrt{S+B}} , \qquad (3.18)$$

where S and B are the numbers of events for the signal and background, respectively.

In Fig. 3, we show the projected  $N_{\rm SD} = 5$  discovery limit on the couplings  $C_i/f_a$  as functions of the ALP mass  $m_a$ , for the two different muon collider benchmarks in Eq. (1.2). in dashed and solid lines, respectively.

In scenario (I)  $C_{\tilde{W}} = C_{\tilde{B}}$  and (II)  $C_{\tilde{W}} = 0$ , the inclusive VBF offers the strongest bounds, especially for relatively light ALP masses. This becomes even more true at  $\sqrt{s} = 10$  TeV, due to the large logarithmic enhancement from the photon PDF. The bounds from associated production remain mostly constant for a large range of ALP mass. They become slightly worse at lower masses and higher energy where diphoton from ALP decay turns out to be



Figure 3. Projected  $5\sigma$  sensitivity of  $C_{\tilde{B}}/f_a$  versus  $C_{\tilde{W}}/f_a$ . Selection cuts include Eq. (3.16),  $|m_{\gamma\gamma} - m_a|/m_a < 5\%$ , and  $m_{\mu^+\mu^-} > 200$  GeV. The leading backgrounds considered for  $\gamma a$ , Za, inclusive VBF, and Dimuon channels are  $\mu^+\mu^- \to \gamma\gamma\gamma$ ,  $\mu^+\mu^- \to Z\gamma\gamma$ ,  $\mu^+\mu^- \to \gamma\gamma$  with ISR, and  $\mu^+\mu^- \to \mu^+\mu^-\gamma\gamma$ , respectively.

collimated and boosted techniques are required to reconstruct the resonance. For ALP with mass  $m_a = 1$  TeV, the coefficients can be probed as low as  $|C_{\tilde{B}}|/f_a \sim 10^{-2} (10^{-3})$  TeV<sup>-1</sup> at  $\sqrt{s} = 3$  (10) TeV.

In scenario (III)  $C_{\tilde{B}} = 0$ , the suppressed coupling  $g_{a\gamma\gamma}$  affects both the  $\gamma\gamma$ -fusion production and the diphoton branching fraction, resulting in looser bounds. It is worth noting that, at  $\sqrt{s} = 3$  TeV, the associated production of Za exceed the VBF processes for  $m_a \gtrsim 700$  GeV.

# 3.3 Characterizing the CP Property of ALPs at Muon Collider

To verify the CP property of the scalar particle produced at a muon collider, we first consider the associated production

$$\mu^+\mu^- \to Z\phi , \qquad (3.19)$$

followed by  $Z \to \ell^+ \ell^-$  and  $\phi = a$  as  $0^-$  pseudoscalar ALP or  $\phi = h$  as  $0^+$  SM Higgs scalar. The azimuthal angle  $\phi_{\ell\ell}$  is defined as the opening angle between the Z production and decay planes. The differential cross section of  $\phi_{\ell\ell}$  for ALP becomes

$$\frac{1}{\sigma}\frac{d\sigma}{d\phi_{\ell\ell}} = \frac{1}{2\pi}\left(1 - \frac{1}{4}\cos 2\phi_{\ell\ell}\right) \,. \tag{3.20}$$

We also examine the following exclusive VBF channels to probe the CP property

$$\mu^+\mu^- \to \mu^+\mu^-\phi \ . \tag{3.21}$$

We apply  $m_{\mu\mu} > 100$  GeV for final states to enhance the VBF topology and require the following basic cuts

$$p_T(\mu) > 10 \text{ GeV}, \quad 10^\circ < \theta_\mu < 170^\circ, \quad \Delta R_{\mu\mu} > 0.4.$$
 (3.22)



**Figure 4.** The normalized distributions of the observable  $\phi_{\ell\ell}$  in  $\mu^+\mu^- \to Z\phi \to \ell^+\ell^-\phi$  (left) and  $\mu^+\mu^- \to \mu^+\mu^-\phi$  (right) for  $\phi$  as ALP (red) or SM Higgs (blue). We assume  $m_a = 1$  TeV and  $C_{\tilde{W}} = C_{\tilde{B}} \neq 0$  for  $\phi$  as the ALP.

In this way we require to tag two forward muons, and define  $\phi_{\ell\ell}$  as the azimuthal angle between the two planes of the final state muons formed with respect to the beam direction. As seen in Fig. 4, the CP-even scalar yields a flat distribution as expected; and the CP-odd scalar exhibits explicitly different angular distribution as governed by the tensor form interaction in Eq. (2.6).

# 4 Executive Summary

In this document, we studied the search potential for a heavy ALPs at future high-energy and high-luminosity muon colliders. We considered the ALP production associated with a neutral electroweak gauge boson ( $\gamma$ , Z) and the various VBF processes together with the ALP decay into diphoton.

Our main results are summarized as follows.

- The  $\gamma\gamma$  fushion process is dominant in the ALP productions as long as the  $U(1)_Y$  gauge coupling is not suppressed.
- The ALPs can be probed as heavy as the colliding energy threshold above TeV in both associated production and VBF processes.
- The ALP gauge couplings can be reached as small as  $|C_{\tilde{W},\tilde{B}}|/f_a \sim 10^{-2} (10^{-3}) \text{ TeV}^{-1}$ at  $\sqrt{s} = 3$  (10) TeV for  $m_a = 1$  TeV.
- The associated productions and the VBF processes with the tagged outgoing muons can be utilized to reveal the CP property of the ALPs.

We conclude that a muon collider running at high energies with a high luminosity would have great potential in searching for the ALPs, essentially reaching the kinematic limit, and studying their interaction properties owing to the rather clean experimental environment at the lepton colliders.

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