Tensor networks for High Energy Physics: contribution to Snowmass 2021

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Abstract

Tensor network methods are becoming increasingly important for high-energy physics, condensed matter physics and quantum information science (QIS). We discuss the impact of tensor network methods on lattice field theory, quantum gravity and QIS in the context of High Energy Physics (HEP). These tools will target calculations for strongly interacting systems that are made difficult by sign problems when conventional Monte Carlo and other importance sampling methods are used. Further development of methods and software will be needed to make a significant impact in HEP. We discuss the roadmap to perform quantum chromodynamics (QCD) related calculations in the coming years. The research is labor intensive and requires state of the art computational science and computer science input for its development and validation. We briefly discuss the overlap with other science domains and industry.

CONTENTS

I. Introduction	6
II. Science goals for HEP	8
III. Relation with Quantum Computing and QIS	9
IV. Selected recent progress	11
V. Technologies and resources needed	13
VI. Overlap with other science domains and industry	16
VII. Conclusions	17
References	18
A. Brief background on tensor methods	24

Executive summary for "CompF6: Quantum computing"

Tensor network methods are playing an increasingly important role in several branches of physics and in quantum information science. Tensor networks provide a compact representation of entangled quantum systems and quantum states. They can be used to reformulate physical theories on a lattice and map them onto other quantum systems—such as quantum computers or quantum simulators. These physical theories can then either be studied classically using the original tensor network formulation, or in the mapped representation using quantum hardware.

In the original formulation, they present an increasingly attractive alternative to traditional Monte Carlo methods (used in lattice QCD) which are currently the only computational methods able to provide first-principle calculations for a large range of HEP problems, but which also consume large computational resources. In numerical methods based on coarse-graining, tensor network methods do not rely on statistical sampling, potentially providing solutions to problems plagued by *sign problems* that render Monte Carlo methods ineffective. In some fields, such as condensed matter physics, their use is fairly common. However, for problems of interest to HEP, tensor network methods are still very resource intensive, and are not currently feasible for complex theories in four space-time dimensions, such as QCD. Further development of tensor network methods and software is needed for them to make a significant impact in HEP like Monte Carlo.

In addition to providing an alternative to Monte Carlo methods in HEP, tensor networks are also useful in connecting HEP to QIS. They can provide a convenient representation for mapping lattice theories to quantum hardware (either in terms of digital quantum circuits or analog quantum simulators), and can also be used to perform classical simulations of quantum circuits. This is helpful for developing and testing quantum computing algorithms, and is an important area of research for determining the tipping point for quantum computers to provide a computational advantage over classical ones. They also assist in isolating "building blocks" that have limited complexity and can be optimally approximated for noisy intermediate-scale quantum (NISQ) hardware.

Here we summarize some of the key problems in HEP that could be addressed with tensor network methods, the current status of applications to HEP, and the developments in methods, software, and personnel needed to make progress in these areas.

Executive summary for "TheoryF10: Quantum Information Science"

Tensor network methods are playing an increasingly important role in several branches of physics and in quantum information science. In the context of HEP, tensor networks have appeared as ways to reformulate lattice gauge theory models to obtain a fully discrete formulation that is suitable for quantum computation and coarse graining methods. Tensor networks also provide tools to understand entanglement in conformal field theories and their connection to gravity.

In the context of lattice gauge theory, tensors can be seen as the translationally invariant, local *building blocks* of exact discretizations of the path integral. They encode both the local and global symmetries of the original model. It is easy to design approximations (truncations) that preserve these symmetries and to design simplified models that should have the same correct universal continuum limit as the original model. Developing these building blocks and optimizing the approximations for NISQ machines and classical computers are important tasks for the near-term future. Tensor networks can also be used to perform classical simulations of quantum circuits. This is useful for developing and testing quantum computing algorithms and quantum computational advantage.

There is an emerging international and interdisciplinary community developing new methods in this area, which includes an increasing number of researchers from the lattice gauge theory community. There is a clear road map to do QCD related calculations in the coming years. This effort is at the interface of quantum computing and classical high performance computing (HPC) and requires state of the art HPC for its development and validation.

Executive summary for "TheoryF5: Lattice Gauge Theory"

Tensor Lattice Field Theory (TLFT) is a new approach for understanding the nonperturbative structure of lattice theories including QCD. Starting from the conventional Lagrangian/path-integral formulation, it is possible to reformulate the models using tensors which are translationally invariant, local, and can be considered as *building blocks* of exact discretizations of the path integral. They encode the local and global symmetries of the original model. They provide a more general space of theories than the traditional Lagrangian and Hamiltonian densities. Most models studied by lattice gauge theorists can be reformulated in the language of TLFT. Truncations need to be used for practical purposes but can chosen to preserve symmetries in a generic way. This provides a broad class of models with the same universal continuum limit as the original model.

TLFT smoothly connects the Lagrangian and Hamiltonian approaches and is very useful in the context of quantum computing. Moreover, TLFT approaches to the path integral can also be combined with deterministic coarse-graining methods. This is very efficient for a broad range of couplings and also allows the study of complex actions. The truncations performed during coarse-graining are the only approximations used and, if they can be controlled, exponentially large volumes can be reached. This is potentially interesting in the study of models beyond the standard model with slowly running coupling constants.

Using TLFT for models directly relevant for HEP is a feasible goal in the next decade. Developing software and running codes in TLFT is labor intensive and requires access to high performance computing.

I. INTRODUCTION

The use of tensor network methods to tackle quantum field theory problems relevant for HEP is a recent and rapidly developing new area of research. This is a very interdisciplinary area and workshops involving several communities have taken place recently. For instance, a recent workshop at the Institute for Nuclear Theory [1] had more than 80 participants in High Energy, Nuclear and Condensed Matter Physics. Before discussing the science goals, the importance of the approach for quantum computation, recent progress, and resources needed, we first give a brief overview of the context in which tensor network methods appear, and their relations with other areas of research.

An important disclaimer: this white paper does not provide exhaustive lists of references but rather selected references that are most familiar to tensor practitioners in the US lattice community. Exhaustive lists of references can be found in recent reviews [2–6]. Additional information can also be found in some of the 2020 LOI's [7–9].

• Context

Tensor network methods are playing an increasingly important role in many branches of physics and data analysis. In the context of HEP and more specifically lattice gauge theory, tensor network methods appear at the interface of quantum and classical computing and target questions where sign problems are present or large volumes are needed. They can also play a role in understanding conformal and holographic theories. For recent reviews focused on the use of tensors in lattice gauge theory see [4, 5, 10].

• Terminology

The term "tensor" appears in many other contexts. For instance, energy-momentum tensors or Kalb-Ramond tensor fields are often used in HEP. In contrast, "tensor networks" refers to objects with multiple indices that can be assembled together to represent the states of a Hilbert space or transfer matrix, compute expectation values of operators, or express partition functions. Typically, the indices refer to quantum numbers or to a list of states in a localized Hilbert space.

• Relationship with quantum computing

The reformulations of lattice field theory models using finite sets of tensor indices provide Hilbert spaces suitable for quantum computing (see Section III). The spacetime tensor networks can be adapted to build quantum circuits. Additionally, certain quantum states that can be represented using a tensor network, e.g. matrix product states (MPS), can be prepared efficiently on a quantum computer. This allows one to address more general problems, e.g., quantum simulation, which are still inefficient for classical computers.

• Condensed matter origins

Many of the basic quantum information ideas behind the use of tensor networks in lattice field theory were originally developed in the context of condensed matter. White emphasized the need to keep track of the entanglement among coarse-grained blocks while performing real space renormalization group transformations and invented the density matrix renormalization group (DMRG) method to handle this [11]. DMRG is typically the most accurate method for computing ground states of one-dimensional lattice Hamiltonians and is related to tensor networks called matrix product states (MPS). Other tensors networks, for instance, called projected entangled pair states (PEPS) and the multi-scale entanglement renormalization ansatz (MERA), play an important role in dealing with other types of models. The quantum information theory community has developed a clear understanding of the convergence (or the lack thereof) of such algorithms in terms of area laws associated with entanglement entropy. For recent reviews see [12, 13].

• Tensors for HEP and nuclear physics

There has been a significant interdisciplinary effort starting around 2010, and highlighted in conferences and workshops at the Aspen Center for Physics, KITP, the INT and KITPC, to apply some of methods developed in the condensed matter community to problems in HEP and nuclear physics (NP). The annual lattice conferences have helped foster interactions among the communities involved [14–20]. The number of contributions has grown steadily with the years. Tensor reformulations of lattice gauge theory models starting from the standard Lagrangian path-integral formalism connect smoothly to Hamiltonian approaches developed in condensed matter physics.

• Tensors as building blocks of computations

The tensors are local objects containing all the universal information about the model in question such as its dimension and symmetries. Most lattice models have a tensor reformulation and there is a clear plan to follow the road map that has been successful for QCD (the "Kogut sequence", see below). Tensors can be used to deal with sign problems, strongly interacting systems and conformal field theory (but efficient numerical methods remain to be fully developed in 2+1 and 3+1 dimensions). They provide new tools for the lattice field theory and AdS/CFT communities. Quantum circuits can also be viewed as tensor networks, providing alternative methods for constructing, transforming and evaluating quantum computations [21–23].

• The road map

The lattice gauge theory community worked its way over many years to being able to simulate full QCD with dynamical fermions by starting with simpler models in lower dimensions. This approximate sequence of models is sometimes called the "Kogut sequence" or the "Kogut ladder" after review articles [24, 25]. A similar road map is being followed with the tensor reformulations discussed here [5].

II. SCIENCE GOALS FOR HEP

• Quantitatively reliable coarse-graining for strong interactions

Quarks and gluons play an important role for problems with typical energies ranging from MeV to TeV. Hypothetical models beyond the standard model often involve effective coupling constants that run very slowly with energy scale. To tackle such problems it is crucial to develop reliable methods for performing coarse-graining and renormalization. Tensor renormalization group (TRG) methods have the potential to meet these expectations because they allow a clean partition of the degrees of freedom at different scales. Indeed such methods have been used in simple models to obtain critical exponents to very high precision. However, the truncation methods that must be employed still need optimization for use in higher dimensions with both fermions and gauge fields.

• Dealing with sign problems at finite density

A certain number of problems in astrophysics and nuclear physics require calculations involving quarks and gluons at finite density. These calculations are plagued by sign problems which prevent the use of conventional importance sampling. TRG methods rely on the singular value decomposition (SVD) and are typically insensitive to sign problems. In 1+1 dimensions, they are generically very efficient in dealing with problems involving finite density or complex couplings where the Euclidean Boltzmann weights are not real and positive. Developing efficient methods in higher dimensions is an important goal for the tensor community.

• Real-time evolution of strongly interacting particles

The interpretation of hadron collider data relies extensively on event generation algorithms such as Pythia [26]. These algorithms incorporate results from perturbative QCD that are reliable at short distances and use empirical models to describe the formation of hadrons at larger distances. Replacing these empirical models by abinitio calculations based on lattice QCD is one of the major motivations for quantum computing in HEP. This is discussed in a separate white paper [27]. Tensor methods can also in principle be used to handle real-time evolution and out-of equilibrium situations. This is a well-developed research area in condensed matter and we hope that this can also become the case in HEP.

III. RELATION WITH QUANTUM COMPUTING AND QIS

• Discretization of path-integrals

For qubit-based quantum computing a complete discretization of both field space and spacetime is necessary. Tensor network methods starting from a lattice path-integral formulation provide a general way to reach this goal.

• Character expansions

For compact groups, character expansions (e.g., Fourier expansion) provide a natural way to discretize the path integral of models with continuous variables (e.g., the gluon fields in QCD). They were extensively developed in the context of strong coupling expansions in the early days of lattice QCD but they also connect with modern worm algorithms and can be adapted to work well even at weak coupling.

• Symmetry compatible truncations

For practical purposes, truncations of the character expansions are necessary. Fortunately, this can be done in a way compatible with symmetries [5, 28, 29].

• Locality and effective theories

The (quasi) locality of microscopic interactions is a crucial ingredient of Lloyd's quantum supremacy argument [30]. It is present in tensor network reformulations and their effective theories obtained by coarse graining.

• Simulation of the building blocks of quantum algorithms

Quantum algorithms can be interpreted as tensor networks, where each tensor corresponds to a quantum gate. From this point of view, we can isolate building blocks of the quantum algorithm and use tensor network methods to perform the tensor contractions, providing a useful technique to simulate quantum algorithms on classical computers.

• State preparation

Tensor networks with some local structure allow one to produce state-preparation quantum circuits of small size or depth. This is beneficial for quantum simulation. It is also beneficial to study/verify new physical theories and for observing convergence by changing the bond dimension of the tensor.

• Quantum state tomography

Tensor networks can also be used in the context of quantum state tomography [31]. By imposing a tensor network structure (e.g., a PEPS), there is a way perform efficient quantum state tomography, which requires exponential resources in general. Efficient quantum state tomography is important for many reasons, including the verification of quantum computing devices and experiments.

• Relation with other approaches

The study of the transfer matrix derived from the tensor formalism leads to finite algebras [29] that can be compared with quantum link constructions [32, 33]. Related methods [34, 35] have been used in the context of quantum computing. These related formulations should be compared in the continuum limit. For instance, it has been showed recently that truncations of the Abelian Higgs model in the charge representation leads to a phase transition associated with an enhanced symmetry.

IV. SELECTED RECENT PROGRESS

• Spin and gauge models with Abelian symmetries

It was found out early [36] that the character expansions for Abelian groups leads to simple factorization. The integration of the fields result in Kronecker deltas which represent the symmetry (modulo n for Z_n versions). The identities representing the symmetry depend only on these selection rules and not on the specific values taken by the tensor elements [28, 29]. Consequently, truncations preserve symmetries exactly and it is possible to attempt to reach the continuum limit with highly simplified microscopic formulations. In addition, for gauge theories, one can integrate completely the gauge fields without gauge fixing and the procedure is manifestly gauge invariant.

• Spin and gauge models with non-Abelian symmetries

The mass-gap of the O(3) nonlinear sigma model in 2D was studied in Ref. [37] by fitting the two-point correlation function. In Refs. [38] the CP(1) model in 2D with and without a θ -term was analyzed using the higher order tensor renormalization group, and the loop-tensor network renormalization algorithms. Reference [39] considered the non-Abelian Higgs model in 2D. This model is confining across the entire phase diagram, and the string tension was extracted from Polyakov loop correlators. Extracting spectra and binding energies from correlations functions is an important step on the path towards analyzing QCD. Understanding the systematics of the method in the presence of a θ -term – which has a sign problem – for a theory which is asymptotically free will help guide the method in higher dimensions with other non-Abelian groups.

• Scalar theories

The complex scalar ϕ^4 theory with finite chemical potential, a typical model with the sign problem, is analyzed in 2D and 4D [40, 41]. While the presence of a sign problem is a strong motivation to use tensor network methods, a notable accuracy of the critical coupling constant compared to other schemes including Monte Carlo simulations is reported in the 2D real ϕ^4 theory [42, 43] following the pioneering work by Shimizu [44]. To discretize the field space a Taylor expansion of the hopping factors combined with Gaussian quadratures is used.

• Fermions

Application of the Grassmann tensor renormalization group [45, 46] to relativistic fermion models started from a series of investigation on the 2D Schwinger model with Wilson fermions [47–49]. A key feature of these fermion models is the presence of Grassmann variables that decorate the tensor networks. In some special cases that include the 2D Schwinger model with staggered fermions, it is shown that one can construct a tensor network representation without Grassmann variables [50]. A coarse-graining algorithm for Grassmann variables in higher dimensions that goes well with the higher order TRG (HOTRG) [51] and its approximation schemes [52, 53] was proposed in [54].

• Supersymmetric models

The 2D $\mathcal{N} = 1$ Wess–Zumino model, the simplest supersymmetric model with a severe sign problem, is analyzed using the Grassmann tensor renormalization group [55]. In this work, a non-interacting case that is exactly solvable is employed as a numerical test. More complicated models including the interacting case of the same model and the $\mathcal{N} = (2, 2)$ Wess–Zumino model would be a possible and interesting direction.

• Quantum gravity

Tensor networks can be useful in the study of quantum gravity as well. One of the directions where tensor networks have appeared is in the "spin-foam" formulation of quantum gravity [56]. In Ref. [57], a tensor network formulation, as well as a coarsegraining scheme were developed for a spin-foam partition function. In Ref. [58] the authors form a tensor network for the partition function of two-dimensional gravity where the gauge symmetry has been extended to merge the tetrad and spin-connection variables into a single connection. In this work the Fisher zeros of the partition function are plotted in the complex-coupling plane. In a related work the authors in Ref. [59] model de Sitter space using the multi-scale entanglement renormalization ansatz tensor network.

• Other approaches

As explained before, there is a large literature where MPS and PEPS are used to handle gauge theories [3, 4] that is not reviewed here. For recents progress, see for instance [60-63].

V. TECHNOLOGIES AND RESOURCES NEEDED

• Method development

A large variety of tensor network algorithms have been developed to approximately evaluate observables in Hamiltonian and Lagrangian systems (see section A in the appendix for a summary and references). The methods span a range of compromises between accuracy, compute time and memory use, with the most accurate methods generally requiring the most time and memory. For systems with one spatial dimension, the time and memory requirements of the most accurate methods can typically be accommodated on modern computers. However, for three spatial dimensions, the computing demands for the most accurate methods are well beyond what is presently possible, and a significant reduction in the size and connectivity of the tensors within the network is necessary. Several advances have been made to improve the scalability of methods, which has made initial simulations of some 3+1 dimensional models possible. However, high precision simulations of simple 3+1 dimensional systems and initial tests for more complex theories (such as QCD) still remain a significant challenge. Sustained research into understanding the tradeoffs among approaches, and developing new methods is necessary in order to reach the accuracy and scalability needed for many of the problems of interest to HEP.

• Tensor network software and library requirements

Tensor network calculations rely on efficient tensor (or matrix) math libraries. The most common operations needed are

- Reshaping the tensor and permuting the order of indices
- Element-wise operations (e.g. scaling elements, adding tensors)
- Contracting indices in a product of two (or more) tensors
- Tensor decompositions (typically SVD or eigen-decomposition)

A list of some tensor software can be found on the web [64] and a more comprehensive list of packages is available here [65]. Many of the tensor libraries were developed and are currently used by the condensed matter community, and may have MPS or other methods implemented. Popular programming languages are C++, Python and, more recently, Julia. For HEP research Numpy is still a popular choice for implementations of TRG and related algorithms. Since many of the tensor libraries use optimized GEMM routines for contractions, performance among them for large tensor sizes will be similar for the same set of operations. The main differences between them tend to be in the interface (e.g. whether contractions use Einstein notation, or some other helper functions). This can make a difference in the ease of use, however, the order of contractions can make a big difference in the memory and compute requirements, so interfaces that allow specifying a collection of contractions need to either determine the optimal order or let the user specify. Determining the optimal contraction order becomes particularly important for larger networks but is considered to be an NP-hard problem. Finding efficient methods to determine nearly-optimal orderings is an active area of research, especially for classical simulations of quantum circuits.

Tensor decompositions In many cases the exact contraction of tensor networks is not feasible, and approximations must be made. This is often done by using an SVD to reduce the size of intermediate tensors by keeping only the largest modes, and hence having an efficient SVD is important. Since typically only a small portion of the modes are being kept, a full SVD isn't necessary. A partial SVD, such as one using randomized linear algebra [66], can be more efficient as the tensor sizes grow.

Distributed memory As the tensor sizes grow, the memory available on a single computer will become a bottleneck. High accuracy calculations of theories such as QCD will likely require larger amounts of memory found on leadership computing resources. This will require having distributed memory tensor libraries that can efficiently perform the necessary contractions and SVD procedures on distributed tensors.

Symmetries Many problems of interest to HEP have symmetries that can also be captured in the TN formulation. This leads to sparse structure in the tensors which can be taken advantage of to make the calculations more efficient and even more accurate. Some tensor libraries developed for condensed matter applications can take symmetries into account. Adopting and modifying these for problems in HEP could be beneficial.

Software ecosystem As noted in [65], there are a large number of libraries available with overlapping capabilities, each developed for a specific research group or applica-

tion in mind. Due to the diverse set of target problems, methods and implementations, consolidating on a single or small number of libraries may be difficult. For HEP, having a coordinated research plan to develop tensor network software tailored for HEP problems, while leveraging libraries actively developed in the condensed matter and computer science communities may be a viable strategy to avoid further fragmentation.

• Access to large scale classical computers

Unlike traditional lattice field theory methods which have a cost in compute time and memory that grows with some power of the system volume, tensor network methods tend to have more modest resource growth for increasing volume. Many TN methods have a resource growth that is only logarithmic in the volume, and some can even work directly in the infinite volume limit. The major factors influencing the resource requirements for TN methods are

- The number of spatial dimensions in the theory
- The complexity of the theory (requiring a larger tensor on each site)
- The required accuracy of the final result

Increasing the accuracy of simulations requires growing the size of the intermediate tensors in the corresponding contraction and truncation procedure, to capture the necessary entanglement in the system.

Improved truncation schemes can help reduce the memory footprint in favor of more compute cycles (which can be advantageous as the compute power of computing technology tends to grow faster than memory capacity). However, when moving to 3+1 dimensions, the memory and compute requirements will both grow very fast as the accuracy is improved, and high accuracy simulations of complex systems such as QCD will likely require running across large scale leadership computing facilities. In this case having efficient distributed memory versions of the codes will be necessary. Having access to large machines (with large memory) will also be necessary to develop methods and to calculate results. Since there is no statistical sampling in TN methods, as opposed to standard lattice Monte Carlo methods, the total compute time needed could be less than for traditional lattice methods. The main bottleneck, however, is likely to be the memory size needed to store tensors required for the target accuracy. In some cases, storage of the large tensors for later use may also be desired, requiring sufficient storage space and I/O bandwidth for tensors of order of the size of the total available memory.

• Workforce

Progress on these problems will require a collaborative workforce across HEP and ASCR domains, along with collaborations with other science domains such as condensed matter physics. This will require skilled domain scientists familiar with the construction of lattice theories and relating numerical measurements to physical results. It will also require computational scientists developing improved tensor network methods that optimize the tradeoff between memory and compute time for a given science problem and computer hardware. They will also need to engage computer scientists and applied mathematicians who are developing libraries and methods for large scale parallelization of tensor network contractions and decompositions.

VI. OVERLAP WITH OTHER SCIENCE DOMAINS AND INDUSTRY

• Condensed matter physics

The development and application of tensor network methods is an active and vibrant field in condensed matter physics. More details on the background in condensed matter physics can be found in previous sections and related references. Many applications to HEP grew out of the related work by, and in collaboration with, condensed matter researchers. For example, the reformulation of lattice gauge theories was developed as part of a collaboration extending the methods of Tao Xiang's group [51].

• Classical simulation of quantum circuits

Tensor network methods can also be used to perform classical simulations of quantum circuits, used for testing and development of quantum algorithms. This can be done by using the tensor network to hold a compact approximation of the full state vector, for example using a PEPS network [67]. One can also use tensor network methods to simulate the output of quantum computers without storing the full state vector [68, 69]. The classical simulation of quantum circuits is an important means of developing and testing quantum algorithms for HEP applications. Additionally, improvements

in methods for simulating quantum circuits could have an impact in tensor network methods directly applied to computing HEP problems, and vice versa.

VII. CONCLUSIONS

In summary, tensor network methods are playing an increasingly important role in several branches of physics and are also becoming important in the context of HEP. Tensor networks provide a compact representation of entangled quantum systems and quantum states that can be used to reformulate physical theories on a lattice, map them onto other quantum systems, such as quantum computers or quantum simulators, and to classically evaluate simulation results either in the tensor network formulation, or in the mapped representation onto quantum hardware.

Tensor Lattice Field Theory (TLFT) is a new approach to models studied in the context of lattice QCD. It provides reformulations of these models where the microscopic tensors are the translationally invariant, local, *building blocks* of exact discretizations of the path integral. They encode the local and global symmetries of the original model. They also provide a more general space of theories than the traditional Lagrangian and Hamiltonian densities. Most model studied by lattice gauge theorists can be reformulated in TLFT. Truncations need to be used for practical purposes and preserve symmetries in a generic way. This provides a broader class of models with the same universal continuum limit as the original model. TLFT methods connect smoothly between the Lagrangian and Hamiltonian approaches and can be very useful in the context of quantum computing.

TLFT presents an increasingly attractive alternative to traditional Monte Carlo methods, which are currently the only computational methods able to provide first-principle calculations for a large range of HEP problems but have limitations. In numerical methods based on coarse-graining, tensor network methods do not rely on statistical sampling, so can also provide solutions to problems plagued by *sign problems* that render Monte Carlo methods ineffective. In some fields, such as condensed matter physics, their use is fairly common. However, for problems of interest to HEP, tensor network methods are still very resource intensive, and are not currently feasible for complex theories in four space-time dimensions, such as QCD. Further development in tensor network methods and software is needed for them to make a significant impact in HEP too. Tensor networks can also provide a convenient representation for mapping lattice theories to quantum hardware (either in terms of digital quantum circuits or analog quantum simulators). They can also be used to efficiently perform classical simulations of quantum circuits. This is useful for developing and testing quantum computing algorithms, and is an important area of research for determining the tipping point for quantum computers to provide a quantum advantage over classical ones. They help isolate "building blocks" that have a limited complexity and can be optimally approximated for NISQ hardware.

We have summarized some of the key problems in HEP that could be addressed with tensor network methods, the current status of applications to HEP, and the developments in methods, software and personnel needed to make progress in these areas.

Using TLFT for models directly relevant for HEP is a feasible goal for the next decade. Developing software and running codes in TLFT is labor intensive and requires access to HPC.

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Appendix A: Brief background on tensor methods

This is a brief overview of some tensor network methods. More complete and detailed summaries can be found in several review articles [2–6].

The Density Matrix Renormalization Group (DMRG) [11] method (and the similar repre-

sentation known as Matrix Product States (MPS) [70]) was developed for 1D lattice Hamiltonians, and is now the most accurate method available for many of these types of problems. MPS can also be applied to 2D Hamiltonians, but this is typically limited to cases with low entanglement. The Projected Entanglement Pair States (PEPS) [71–73] is an extension of MPS that can capture 2D entanglement well, but at a greatly increased cost.

An alternative to MPS is provided by Tree Tensor Networks (TTN). These use a binary tree of tensors to represent the state, which can be more efficient in memory, especially for large systems, but generally does not provide as good a representation as MPS. This can be circumvented by the addition of *disentanglers* into the tree network, which provide extra connections within TTN that allow it to express much greater entanglement (by applying unitary rotations that remove entanglement without changing the physics). This is the basis of the Multiscale Entanglement Renormalization Ansatz (MERA) network [74]. The disentanglers require extra effort to optimize, but for Hamiltonian systems, they still allow for an efficient contraction of the network.

TTNs can be applied to 2D Hamiltonians as well, with and without disentanglers. An implementation with disentanglers has been shown to work expecially well [75], however it comes with a large cost.

For HEP applications, an important problem is the evaluation of partition functions, and related observables, that would come from Lagrangian systems (and also classical Hamiltonians). One of the original methods designed for the evaluation of 2D partition functions was the Tensor Renormalization Group (TRG) algorithm [76]. This method explicitly decomposed and contracted neighboring tensors together to produce a new renormalized tensor encoding the interactions of several sites. An improved method for performing the tensor block transformation, based on a higher order SVD for tensors, was introduced as the Higher Order TRG (HOTRG) method [51]. This method has a relatively simple form and can easily be extended to higher dimensions, though the entanglement is limited due to the tree-like structure of the projections.

Several methods have been introduced to improve on the accuracy of HOTRG. The Tensor Network Renormalization (TNR) method [77] includes extra disentangling steps, similar to those used in MERA. TNR has only been applied in 2D systems so far, and generalizing to higher dimensions is non-trivial. Other methods that attempt to remove unnecessary shortrange degrees of while preserving the long-range ones are Loop-TNR [78] and Gilt-TNR [79]. These are based the idea of removing degrees of freedom around a loop and are easier to apply in higher dimensions.

One method developed explicitly to scale well in higher dimensions is Anisotropic TRG [52]. This is based on an update step that is similar to the original TRG method, which is relatively cheap, but not as accurate as some of the above methods at a given bond dimension. Another variation developed to make the calculations even cheaper in memory and compute time is to decomposes the network so that it only contains triad tensors (ones with three indices) [53]. This representation can be combined with other methods such as HOTRG.

Choosing the best method for a given problem can be difficult as it depends on the details of the observables being measured, the required accuracy and even the details of the implementations and the computer hardware being used. For problems in 4D, memory used is a significant constraint. Methods that use less memory can be run with a larger bond dimension, than those that consume more memory. In some cases the larger bond dimension can make up for the accuracy lost (at fixed bond dimension) due to breaking up the tensor, or using a simpler update method. However this is not always the case, since many of the methods are based on local update steps which do not necessarily converge on the correct answer in a consistent manner. In some cases a more expensive, but more uniformly converging method might give better accuracy for a fixed hardware budget (memory and/or compute time). Searching for the best combination of efficiency and convergence for problems of interest to HEP is an open question.