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Theory perspectives on rare Kaon decays and CPV

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I review rare kaon decays in the LHC era: we discuss interplay with B-anomalies and possible New Physics in direct CP violation in $K \to 2\pi$: very rare kaon decays like $K \to \pi\nu\bar{\nu}$ are very important to this purpose. We discuss also the decays $K^0 \to \mu^+\mu^-$ due to the LHCB measurement

I. INTRODUCTION AND $K \to \pi\nu\bar{\nu}$

Rare kaon decays furnish challenging MFV probes and will severely constrain additional flavor physics motivated by NP [1]. SM predicts the $V-A \otimes V-A$ effective hamiltonian (Fig. 1)

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L \gamma^\mu \nu_L \times$$

$$\Big(\underbrace{V_{cs}^* V_{cd} \, X_{NL}}_{\lambda x_c} + \underbrace{V_{ts}^* V_{td} X(x_t)}_{A^2 \lambda^5 \, (1-\rho-i\eta) x_t} \Big) \qquad (1)$$

$x_q = m_q^2/M_W^2$, θ_W the Weak angle and X's are the Inami-Lin functions with Wilson coefficients known at two-loop electroweak corrections and the main uncertainties is due to the strong corrections in the charm loop contribution. The structure in (1) leads to a pure CP violating contribution to $K_L \to \pi^0\nu\bar{\nu}$, induced only from the top loop contribution and thus proportional to $\Im m(\lambda_t)$ ($\lambda_t = V_{ts}^* V_{td}$) and free of hadronic uncertainties. This leads to the SM prediction

$$K_L = (2.9 \pm 0.2) \times 10^{-11} \quad K^+ = (8.3 \pm 0.9) \times 10^{-11}.$$

where the parametric uncertainty due to the error on $|V_{cb}|$, ρ and η is shown.

Typical BSM predict new flavor structure that might affect $K \to \pi\nu\bar{\nu}$ that now can be tested at NA62 and KOTO [2]; we describe two different BSM

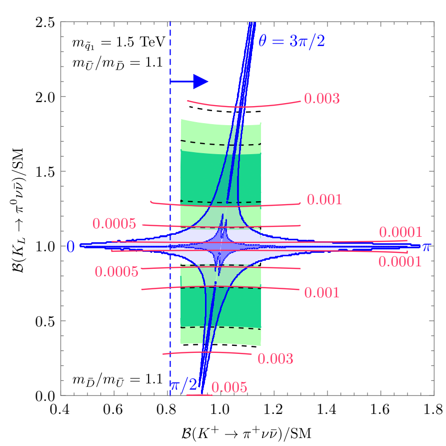

FIG. 1: $K \to \pi\nu\bar{\nu}$: NP from $K \to 2\pi$ susy isospin breaking terms ($\Im(A_2)$) [1, 3]

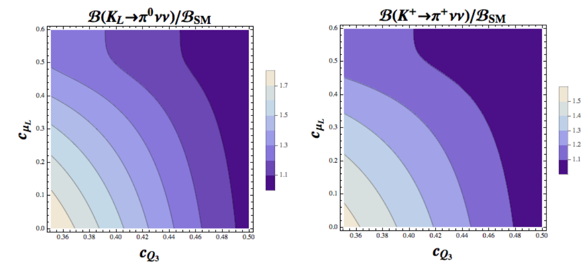

FIG. 2: RS scenario to explain B-anomalies: $B(K \to \pi\nu\bar{\nu})$ ranges as a function of fermion profiles (c_i's)

effects i) new flavor structures for ϵ' avoiding $\Delta S = 2$ constraints (Fig. 1) [1, 3] and ii) attempts to describe B-anomalies [4], typically induce large flavor effects at O(1) TeV [5]. i) the recent lattice results for $K \to 2\pi$ leave open the possibility of BSM for ϵ'; to isospin breaking terms in $\Im(A_2)$ have been studied [3] in Fig.1. We expect effects at most 10% in $K^+ \to \pi^+\nu\bar{\nu}$ while are more sizable for $K_L \to \pi^0\nu\bar{\nu}$. Theoretically addressing flavor in Randall Sundrum models is more challenging: we have studied the so called flavor anarchy scenario with 5D MFV and custodial symmetry; the only sources of flavor breaking are two 5D anarchic Yukawa matrices. These matrices also generate also the bulk masses, which are responsible for the resulting flavor hierarchy. The theory flows to a next to minimal flavor violation model where flavor violation is dominantly coming from the 3rd generation. We show that it is possible to find a range of parameters for bulk masses satisfying experimental flavor constraints, but also we explain the neutral B-anomalies, requiring NP flavor scale at O(1) TeV. Then we address $K \to \pi\nu\bar{\nu}-$decays: we show the TH predictions as a function of the bulk fermion masses in Fig.2 [5]. A natural issue is to test O(1) TeV physics at LHC; we are trying to apply the technique of Ref. [6] to this purpose.

II. $K_{L,S} \to \mu^+\mu^-$

Recent $K_S \to \mu\overline{\mu}$ LHCB measurement is very interesting and unexpected

$$B(K_S \to \mu\overline{\mu})_{LHCB} < 9 \times 10^{-9} \text{ at } 90\ \%\ \text{CL} \quad (2)$$
$$B(K_S \to \mu\overline{\mu})_{SM} = (5.0 \pm 1.5) \times 10^{-12}. \quad (3)$$

It represents an important milestone since it has improved the previous limit, $< 3.2 \times 10^{-7}$ at 90 % CL, lasted 40 years. It is based on a production of 10^{13} K_S per fb^{-1} inside the LHCB acceptance and it is obtained using 1.0 fb^{-1} of pp collisions at $\sqrt{s} = 7$ TeV collected in 2011.

Two photon exchange generates the dominant contribution for both K_L and K_S decays to two muons [7]. The structure of weak and electromagnetic interactions entails a vanishing CP conserving short distance contribution to $K_S \to \mu^+\mu^-$. Indeed the SM short diagrams (similar to $K \to \pi\nu\overline{\nu}$ in Fig. 1) lead to the SM effective hamiltonian similar to eq. (1).

The LD contributions to $K_S \to \mu^+\mu^-$ Fig. (4) have been computed reliably in CHPT ($B = (5.0 \pm 1.5) \times 10^{-12}$). The relevant short distance contributions are

$$\begin{aligned} B(K_S \to \mu\overline{\mu})^{SD}_{SM} &= 1 \times 10^{-5}\ \ |\Im(V^*_{ts}V_{td})|^2 \sim 10^{-13} \\ B(K_S \to \mu\overline{\mu})_{NP} &\leq 10^{-11} \end{aligned} \quad (4)$$

We have shown that in some appealing susy scenario in Fig. (3) [8] large allowed new physics contributions (NP) can be substantially larger that SM SD contributions.

The short distance hamiltonian will contribute also to $K_L \to \mu\overline{\mu}$, through a CP conserving amplitude, $\Re(A_{\text{short}})$, that has to be disentangled from the large LD two-photon exchange contributions, $A_{\gamma\gamma}$: the absorptive LD contribution is much larger than SD, in the rate respectively 25 times larger than dispersive; total $B_{\text{expt}} = (6.84 \pm 0.11) \times 10^{-9}$. To extract SD info the situation would be better if we would know the sign of $A_{\gamma\gamma}$, theoretically and experimentally unknown. While K_L-decays outside the LHCB fiducial volume the interference $A(K_L \to \mu\overline{\mu}^* A(K_S \to \mu\overline{\mu}$ may affect the LHCB K_S-rates: we can study the time interference $K_{S,L} \to \mu\mu$; this can be done by flavor tagging $K^0\bar{K}^0$, specifically by detecting the associated $\pi^\pm$ and (or) $K^\mp$, determining the impurity parameter $D = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}$. Then interference term will affect the measured branching [7]:

$$\mathcal{B}(K^0_S \to \mu^+\mu^-)_{\text{eff}} = \tau_S \left(\int_{t_{\min}}^{t_{\max}} dt e^{-\Gamma_S t} \varepsilon(t) \right)^{-1}$$
$$\left[\int_{t_{\min}}^{t_{\max}} dt \left\{ \Gamma(K^0_S \to \mu^+\mu^-) e^{-\Gamma_S t} + \frac{D f_K^2 M_K^3 \beta_\mu}{8\pi} \cdot \right.\right.$$
$$\left.\left. \text{Re}\left[i \left(A_S A_L - \beta_\mu^2 B_S^* B_L \right) e^{-i\Delta M_K t} \right] e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \right\} \varepsilon(t) \right]$$

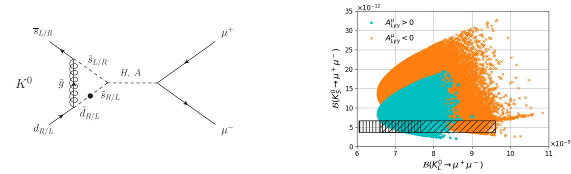

FIG. 3: Susy scenario: $K_S \to \mu\mu$ diagram (left), theory predictions: in dashed area no interference effects are considered (right)

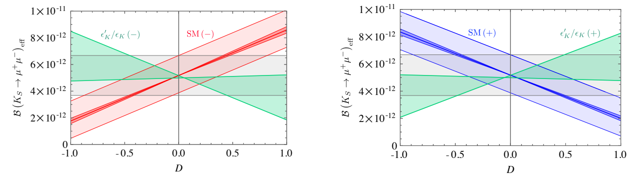

FIG. 4: K_S interference effect from eq. 5 on $\mathcal{B}(K_S \to \mu^+\mu^-)$ depending on the $A_{L\gamma\gamma}$ sign: negative (center and in red SM while in green NP contributions) and positive (right and in blue SM while in green NP contributions).

Then we are i) increasing the sensitivity to short distance and ii) possibly determining the sign $A_{L\gamma\gamma}$

$$\sum_{\text{spin}} \mathcal{A}(K_1 \to \mu^+\mu^-)^* \mathcal{A}(K_2 \to \mu^+\mu^-) \sim \underbrace{\text{Im}[\lambda_t] y'_{7A}}_{SD}$$
$$\left\{ \underbrace{A^\mu_{L\gamma\gamma}}_{LD} - 2\pi \sin^2\theta_W \left(\text{Re}[\lambda_t] y'_{7A} + \text{Re}[\lambda_c] y_c \right) \right\} \quad (5)$$

Experimentally, one can also access an *effective* branching ratio of $K^0_S \to \mu^+\mu^-$ [7] which includes an interference contribution with $K^0_L \to \mu^+\mu^-$ in the neutral kaon sample. LHCB has a beautiful kaon physics program [8, 9].

III. THE WEAK CHIRAL LAGRANGIAN

In Ref. [10] we have studied how to determine the weak O(p^4) chiral countertems in

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots$$
$$= G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \to \pi^+\gamma\gamma, K \to \pi l^+ l^-} \quad (6)$$

In fact as shown in Table I there is a subset of the 37 CT's, N^r_{14}, N^r_{15}, N^r_{16} and N_{17}, that can be determined from experiments. Due to the accurate NA48/2 study of the decays $K^\pm \to \pi^\pm\pi^0\gamma$ and $K^\pm \to \pi^\pm\pi^0 e^+e^-$ the subset of CT's in the table I can be determined

TABLE I: O(p^4) weak chiral countertems and their determination

$K^\pm \to \pi^\pm \gamma^*$	$N_{14}^r - N_{15}^r$	$a_+ = -0.578 \pm 0.016$	NA48/2
$K_S \to \pi^0 \gamma^*$	$2N_{14}^r + N_{15}^r$	$a_S = (1.06^{+0.26}_{-0.21} \pm 0.07)$	NA48/1
$K^\pm \to \pi^\pm \pi^0 \gamma$	$N_{14}^r - N_{15}^r - N_{16}^r - N_{17}$	$X_E = (-24 \pm 4 \pm 4)$ GeV^{-4}	NA48/2
$K^+ \to \pi^+ \gamma\gamma$	$N_{14}^r - N_{15}^r - 2N_{18}^r$	$\hat{c} = 1.56 \pm 0.23 \pm 0.11$	NA48/2

Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865 [12]
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2 [13]
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2 [14]

TABLE II: Fitted values of coefficients in the vector form factor.

IV. LEPTON FLAVOR UNIVERSALITY VIOLATION IN KAONS

The dominant contribution to $K^+ \to \pi^+ \ell^+ \ell^-$ is due to single virtual-photon exchange. The amplitude involves a vector form factor $V_+(z)$ which up to $O(p^6)$ in the chiral expansion, can be decomposed in the general form [11]

$$V_+(z) = a_+ + b_+ z + V_+^{\pi\pi}(z)\,, \qquad z = q^2/m_K^2\,. \quad (7)$$

Here the LECs a_+ and b_+ parametrise the polynomial part, while the rescattering contribution $V_+^{\pi\pi}$ can be determined from fits to $K \to \pi\pi$ and $K \to \pi\pi\pi$ data. Chiral symmetry alone does not constrain the values of the LECs,so instead, we consider the differential decay rate $d\Gamma/dz \propto |V_+(z)|^2$ as a means to extract a_+ and b_+ from experiment. The resulting fit to the decay spectra from all available high-statistics experiments is given in Table II.

Now for the crucial point: if lepton flavour universality applies, the coefficients a_+ and b_+ have to be equal for the ee and $\mu\mu$ channels, which within errors is indeed the case. Since the SM interactions are lepton flavour universal, deviations from zero in differences like $a_+^{\mu\mu} - a_+^{ee}$ would then be a sign of NP, and the corresponding effect would be necessarily short-distance [15].

To convert the allowed range on $a_+^{\rm NP}$ into a corresponding range in the Wilson coefficients $C_{7V}^{\ell\ell}$, we make use of the $O(p^2)$ chiral realization of the $SU(3)_L$ current

$$\bar{s}\gamma^\mu(1-\gamma_5)d \leftrightarrow iF_\pi^2(U\partial^\mu U^\dagger)_{23}\,, \qquad U = U(\pi, K, \eta)\,,$$

to obtain

$$a_+^{\rm NP} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* C_{7V}^{\rm NP}\,. \quad (8)$$

Contributions due to NP in $K^+ \to \pi^+ \ell^+ \ell^-$ can then be probed by considering the *difference* between the two channels

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*}\,. \quad (9)$$

If the framework of MFV, this can be converted into a constraint on the NP contribution to C_9^B:

$$C_9^{B,\mu\mu} - C_9^{B,ee} = -\frac{a_+^{\mu\mu} - a_+^{ee}}{\sqrt{2} V_{td} V_{ts}^*} \approx -19 \pm 79\,, \quad (10)$$

where we have averaged over the two electron experiments listed in Table II.

Evidently, the determination of $a_+^{\mu\mu} - a_+^{ee}$ needs to be improved by an $O(10)$ factor in order to probe the parameter space relevant for the B-anomalies, whose explanation involves Wilson coefficients $C_{9,10}^B = O(1)$ [16]. Improvements of this size may be possible at NA62, especially for the experimentally cleaner dimuon mode which currently has the larger uncertainty.

V. DETERMINING a_+ AND b_+

Recently we have addressed determining a_+ and b_+ from low energy data and short distance constraints[17, 18]. This is done by first part describes the contribution from the two-pion intermediate state to $V_+(z)$ in eq. 7. It is constructed upon assuming, in analogy with the electromagnetic form factor of the pion $F_V^\pi(s)$ that it is given by an unsubtracted dispersion integral,

$$V_+^{\pi\pi}(z) = \int_{4M_\pi^2}^\infty dx \frac{\rho_+^{\pi\pi}(x)}{x - zM_K^2 - i0}. \quad (11)$$

The absorptive part consists of the two-pion spectral density $\rho_+^{\pi\pi}(s)$, and is obtained upon inserting a two-pion intermediate state in the representation of the form factor given in eq. 7,

$$\rho_+^{\pi\pi}(s) = 16\pi^2 M_K^2 \times \tfrac{s-4M_\pi^2}{s}\,\theta(s-4M_\pi^2) \times F_V^\pi(s) \times \frac{f_1^{K^\pm\pi^\mp\to\pi^+\pi^-}(s)}{\lambda_{K\pi}^{1/2}(s)}. \quad (12)$$

Data provide the scattering amplitude $f_1^{K^\pm\pi^\mp\to\pi^+\pi^-}$ (fixing also the correct position of the ρ-poles !!), while matching short distance and resonances furnish also the remaining contributions in eq. 7. Our predictions are in Fig. 5 [17] and they have been improved in [18].

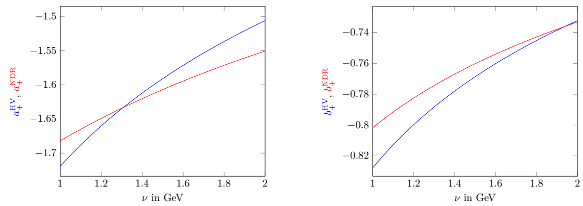

FIG. 5: The evolution of a_+ and b_+ with respect to ν in both NDR and HV schemes, for $M = 1$ GeV.

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