

Top quark pair production at NNLO+NNLL' in QCD  
combined with electroweak correctionsMICHAŁ CZAKON<sup>a</sup>, ANDREA FERROGLIA<sup>b,c</sup>, ALEXANDER MITOV<sup>d</sup>, DAVIDE PAGANI<sup>e</sup>, ANDREW S. PAPANASTASIOU<sup>d</sup>, BENJAMIN D. PECJAK<sup>f</sup>, DARREN J. SCOTT<sup>g,h</sup>, IOANNIS TSINIKOS<sup>e</sup>, XING WANG<sup>i</sup>, LI LIN YANG<sup>i,j,1</sup>, MARCO ZARO<sup>h</sup><sup>a</sup>*Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany*<sup>b</sup>*New York City College of Technology, Brooklyn, NY 11201, USA*<sup>c</sup>*The Graduate School and University Center, The City University of New York, New York, NY 10016 USA*<sup>d</sup>*Cavendish Laboratory, University of Cambridge, CB3 0HE Cambridge, UK*<sup>e</sup>*Technische Universität München, James-Franck-Str. 1, D-85748 Garching, Germany*<sup>f</sup>*Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE Durham, UK*<sup>g</sup>*Institute for Theoretical Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands*<sup>h</sup>*Nikhef, Theory Group, Science Park 105, 1098 XG, Amsterdam, The Netherlands*<sup>i</sup>*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*<sup>j</sup>*Center for High Energy Physics, Peking University, Beijing 100871, China*

We describe predictions for top quark pair differential distributions at hadron colliders, which combine state-of-the-art NNLO QCD calculations with double resummation at NNLL' accuracy of threshold logarithms and small-mass logarithms. Numerical results are presented for the invariant mass distribution, the transverse momentum distribution as well as rapidity distributions. We further combine the NNLO+NNLL' result with electroweak corrections in order to achieve state-of-the-art precision in the Standard Model. This is the first time that such a combination has appeared in the literature.

PRESENTED AT

11<sup>th</sup> International Workshop on Top Quark Physics  
Bad Neuenahr, Germany, September 16–21, 2018

---

<sup>1</sup>Speaker

# 1 Introduction

Top quark pair production is one of the most important processes at the Large Hadron Collider (LHC). It allows us to precisely study the properties of the top quark which are related to many important questions in particle physics, such as the hierarchy problem, the stability of the electroweak vacuum, as well as the origin of fermion masses. Top quark pair production is also a major background in searches for many rare processes in the standard model (SM) and in new physics models beyond the SM.

Currently, the most precise fixed-order calculation in quantum chromodynamics (QCD) for top quark pair production is at the next-to-next-to-leading order (NNLO) [1, 2, 3, 4, 5, 6, 7]. Despite the high precision of the NNLO result, the complicated kinematics of  $t\bar{t}$  production makes it necessary to consider even higher order corrections. This is particularly important since the large collider energy of the LHC enables the study of “boosted” top quark pairs, where the energies of the top quarks are much larger than their rest mass  $m_t$ . In [7], it has been found that the NNLO QCD differential cross sections in the boosted regime are rather sensitive to the choice of factorization and renormalization scales. This scale dependence can be dramatically reduced by resumming certain towers of large logarithms to all orders in the strong coupling  $\alpha_s$  [8]. These include not only the threshold logarithms which arise when the partonic center-of-mass energy approaches the  $t\bar{t}$  invariant mass  $M_{t\bar{t}}$ , but also the small-mass logarithms of the form  $\ln^n(m_t^2/M_{t\bar{t}}^2)$  which develop in the boosted region  $M_{t\bar{t}} \gg m_t$ .

The resummation of the large logarithms starts from the factorization formula for the differential cross section with respect to the  $t\bar{t}$  invariant mass  $M_{t\bar{t}}$  and the scattering angle  $\theta$ . It can be conveniently written in moment space after a Mellin transform as

$$\frac{d^2\tilde{\sigma}(N)}{dM_{t\bar{t}}d\cos\theta} = \frac{8\pi\beta}{3sM} \sum_{ij} \tilde{\mathcal{L}}_{ij}(N, \mu_f) \tilde{c}_{ij}(N, M_{t\bar{t}}, \beta, \cos\theta, \mu_f), \quad (1)$$

where  $N$  is the Mellin moment,  $\mu_f$  is the factorization scale,  $\beta = \sqrt{1 - 4m_t^2/M_{t\bar{t}}^2}$ ,  $\tilde{\mathcal{L}}_{ij}$  is the parton luminosity function, and  $\tilde{c}_{ij}$  is the hard-scattering kernel. The sum is over initial state partons  $i, j$ . The threshold limit corresponds to  $N \rightarrow \infty$ , where the hard-scattering kernel develops large logarithms of the form  $\ln^n N$ . In order to resum these threshold logarithms, one exploits the factorization formula [9, 10, 11]

$$\begin{aligned} \tilde{c}_{ij}(N, M_{t\bar{t}}, \beta, \cos\theta, \mu) = \text{Tr} \left[ \mathbf{H}_{ij}^m \left( \ln \frac{M_{t\bar{t}}^2}{\mu^2}, \beta, \cos\theta, \mu \right) \right. \\ \left. \times \tilde{\mathbf{s}}_{ij}^m \left( \ln \frac{M_{t\bar{t}}^2}{N^2 \mu^2}, \beta, \cos\theta, \mu \right) \right] + \mathcal{O}\left(\frac{1}{N}\right), \quad (2) \end{aligned}$$

where  $\bar{N} = Ne^{\gamma_E}$  with  $\gamma_E$  the Euler constant, while  $\mathbf{H}_{ij}^m$  and  $\tilde{\mathbf{s}}_{ij}^m$  are the massive hard and soft functions, which are both matrices in color space as indicated by the bold font. The resummation then proceeds by choosing an appropriate hard scale  $\mu_h$  for  $\mathbf{H}_{ij}^m$  and an appropriate soft scale  $\mu_s$  for  $\tilde{\mathbf{s}}_{ij}^m$ , and evolving the two functions to the factorization scale  $\mu_f$  via their renormalization group equations (RGEs).

The factorization formula (2) is valid whether or not the top quarks are boosted. However, in the boosted limit  $M_{t\bar{t}} \gg m_t$  or  $\beta \rightarrow 1$ , the massive hard and soft functions  $\mathbf{H}_{ij}^m$  and  $\tilde{\mathbf{s}}_{ij}^m$  themselves develop large logarithms of the form  $\ln^n(m_t^2/M_{t\bar{t}}^2)$  which also require resummation. In [12], it was shown that the massive hard and soft functions can be further factorized in the boosted limit as

$$\begin{aligned} \mathbf{H}_{ij}^m \left( \ln \frac{M_{t\bar{t}}^2}{\mu^2}, \beta, \cos \theta, \mu \right) &= \mathbf{H}_{ij} \left( \ln \frac{M_{t\bar{t}}^2}{\mu^2}, \cos \theta, \mu \right) C_D^2 \left( \ln \frac{m_t^2}{\mu^2}, \mu \right) + \mathcal{O} \left( \frac{m_t^2}{M_{t\bar{t}}^2} \right), \\ \tilde{\mathbf{s}}_{ij}^m \left( \ln \frac{M_{t\bar{t}}^2}{\bar{N}^2 \mu^2}, \beta, \cos \theta, \mu \right) &= \tilde{\mathbf{s}}_{ij} \left( \ln \frac{M_{t\bar{t}}^2}{\bar{N}^2 \mu^2}, \cos \theta, \mu \right) \tilde{s}_D^2 \left( \ln \frac{m_t^2}{\bar{N}^2 \mu^2}, \mu \right) + \mathcal{O} \left( \frac{m_t^2}{M_{t\bar{t}}^2} \right), \end{aligned} \quad (3)$$

where  $\mathbf{H}_{ij}$  and  $\tilde{\mathbf{s}}_{ij}$  are massless hard and soft functions describing the production of a highly boosted top quark pair, while  $C_D$  and  $\tilde{s}_D$  describe the fragmentation of a nearly massless top quark. Using this double factorization, one can simultaneously resum the threshold logarithms and the small-mass logarithms in the boosted region via the RGEs.

In order to achieve the resummation at next-to-next-to-leading logarithmic (NNLL) accuracy, we need to know the various functions in the factorization formulas (2) and (3) to next-to-leading order (NLO), and the anomalous dimensions governing their evolution to order  $\alpha_s^2$ . These ingredients for the un-boosted case have been collected in [13, 14, 11] and the NNLL resummation of the threshold logarithms was performed in [11]. This result will be denoted as NNLL<sub>m</sub> in the following, where the subscript “m” means “massive”.\* In the boosted case, it is possible to improve the resummation accuracy to NNLL’ by including the NNLO contributions to the functions  $\mathbf{H}_{ij}$ ,  $\tilde{\mathbf{s}}_{ij}$ ,  $C_D$  and  $\tilde{s}_D$  [12, 16]. Among them the NNLO massless soft function was calculated in [17], while the corresponding hard function can be found in [18]. This NNLL’<sub>b</sub> (“b” meaning “boosted”) resummation can be combined with the NNLL<sub>m</sub> resummation to obtain an NNLL’<sub>b+m</sub> result valid both in the un-boosted and boosted regions. This was further matched to the NLO fixed-order calculation and finally arrived at the NLO+NNLL’ result in [19].

In this talk, we present some recent efforts towards improving the accuracy of the theoretical predictions. In Section 2, we discuss the construction of the NNLO+NNLL’ predictions combining the NNLO calculation with the double resummation at the

---

\*Note that the NNLO massive soft function has been calculated in [15], which provides an important ingredient for the NNLL’<sub>m</sub> resummation.

NNLL'<sub>b+m</sub> precision. In Section 3, we discuss the extension of the resummation framework to rapidity distributions. In Section 4, we further combine the QCD results with electroweak corrections, thus construction state-of-the-art Standard Model predictions. We summarize in Section 5.

## 2 Resummation at NNLO+NNLL' in QCD

In [19], the NNLL'<sub>b+m</sub> resummed result is only matched to the NLO fixed-order result. With the availability of the NNLO result with dynamic renormalization and factorization scales [7], it is desirable to combined these two state-of-the-art calculations, which was finally achieved in [8]. This is the first time a resummed calculation at full NNLO+NNLL' accuracy in QCD for a process with non-trivial color structure has been completed at the differential level.

Technically, the NNLO+NNLL' result involves three different contributions, two of which contain all-order information. Therefore one must be careful in combining them in order to ensure that there is no double-counting (or triple-counting) at any order in  $\alpha_s$ . We first match the resummation formulas in the soft and boosted-soft limit with each other. To do so, we need to remove the overlap between the NNLL'<sub>b</sub> and NNLL<sub>m</sub> results to all orders in  $\alpha_s$ . This can be done by exploiting the fact that the boosted-soft resummation formula is the small-mass limit of the soft-gluon resummation formula at any given order in  $\alpha_s$ . The combined NNLL'<sub>b+m</sub> result is thus given by

$$d\sigma^{\text{NNLL}'_{b+m}} = d\sigma^{\text{NNLL}'_b} + \left( d\sigma^{\text{NNLL}_m} - d\sigma^{\text{NNLL}_m} \Big|_{m_t \rightarrow 0} \right), \quad (4)$$

where the terms in the parenthesis account for contributions which are suppressed by  $m_t/M$  in the boosted-soft limit and thus not included in the NNLL'<sub>b</sub> result. Matching with the NNLO calculation then proceeds by subtracting the NNLO expansion of the resummed formula

$$d\sigma^{\text{NNLO+NNLL}'} = d\sigma^{\text{NNLL}'_{b+m}} + \left( d\sigma^{\text{NNLO}} - d\sigma^{\text{NNLL}'_{b+m}} \Big|_{\substack{\text{NNLO} \\ \text{expansion}}} \right). \quad (5)$$

With the above formulas, it is straightforward to perform the matching and obtain the NNLO+NNLL' predictions for the  $t\bar{t}$  invariant mass distribution as well as the top quark transverse momentum distribution. However, due to the complicated kinematics of  $t\bar{t}$  production, one should be careful about the choice of the factorization scale as well as the matching scales for each of the functions in the factorization formulas (2) and (3). In [7], it has been found that the  $t\bar{t}$  invariant mass distribution is quite sensitive to the choice of the factorization scale in the boosted region, even

at NNLO. By studying the convergence of the perturbative series, it was argued that the optimal choice should be

$$\mu_f = \frac{H_T}{4} \equiv \frac{1}{4} \left[ \sqrt{p_{T,t}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^2 + m_t^2} \right], \quad (6)$$

instead of one correlated with  $M_{t\bar{t}}$ . This fact also has implications for the choices of the other matching scales in the resummation formula, especially the hard and soft scales  $\mu_h$  and  $\mu_s$ . In [19], the hard scale was chosen to be correlated with  $M_{t\bar{t}}$ . However, a closer look at the hard function in the  $gg$ -channel reveals that in the boosted limit, the  $t$ - and  $u$ -channel propagators enhance the forward and backward regions:

$$m_t^2 - (p_1 - p_3)^2 \Big|_{m_t \rightarrow 0} \approx \frac{M^2}{2} (1 - \cos \theta) + m_t^2 \cos \theta \xrightarrow{\cos \theta \rightarrow 1} p_{T,t}^2 + m_t^2 \approx H_T^2/4, \quad (7)$$

$$m_t^2 - (p_2 - p_3)^2 \Big|_{m_t \rightarrow 0} \approx \frac{M^2}{2} (1 + \cos \theta) - m_t^2 \cos \theta \xrightarrow{\cos \theta \rightarrow -1} H_T^2/4. \quad (8)$$

As a result, the hard function is sensitive to the scale  $H_T/2$  instead of  $M_{t\bar{t}}$  when the top quarks are highly boosted. The analytic form and the numeric behavior of the hard function in the boosted region then lead to the default choice  $\mu_h = H_T/2$ , as concluded in [8]. The choice of the soft scale, on the other hand, is not as obvious. Study of the perturbative convergence of the massless soft function [8] has identified the default choice  $\mu_s = H_T/\bar{N}$ . This choice is also supported by the behavior of the massive soft function [15]. In the following, we will adopt these default choices, together with the choice of  $\mu_f$  as in Eq. (6).

We now show some phenomenological results at the LHC with  $\sqrt{s} = 13$  TeV. We use the NNPDF3.0 NNLO PDF sets with  $\alpha_s(M_Z) = 0.118$  [20], and take  $m_t = 173.3$  GeV. The perturbative uncertainties are estimated by varying the various scales by factors of two around the default values and adding the resulting uncertainties in quadrature. In Fig. 1, we show the results for the  $t\bar{t}$  invariant mass distribution (left plot) and the average top/anti-top quark transverse momentum distribution (right plot), where the  $p_{T,\text{avt}}$  distribution is defined by

$$\frac{d\sigma}{dp_{T,\text{avt}}} = \frac{1}{2} \left( \frac{d\sigma}{dp_{T,t}} + \frac{d\sigma}{dp_{T,\bar{t}}} \right). \quad (9)$$

A remarkable feature of this figure is that the NNLO+NNLL' and NNLO results are in close agreement in the whole range of  $M_{t\bar{t}}$  when  $\mu_f = H_T/4$  is chosen. To add context to this result, we display in Fig. 2 the results for the cross section in a sample bin  $M_{t\bar{t}} = [2500-3000]$  GeV in the boosted region. This figure delivers a couple of important messages. Firstly, the NNLO+NNLL' result is rather stable against switching the factorization scale between  $H_T$ -based and  $M_{t\bar{t}}$ -based schemes. This implies that the

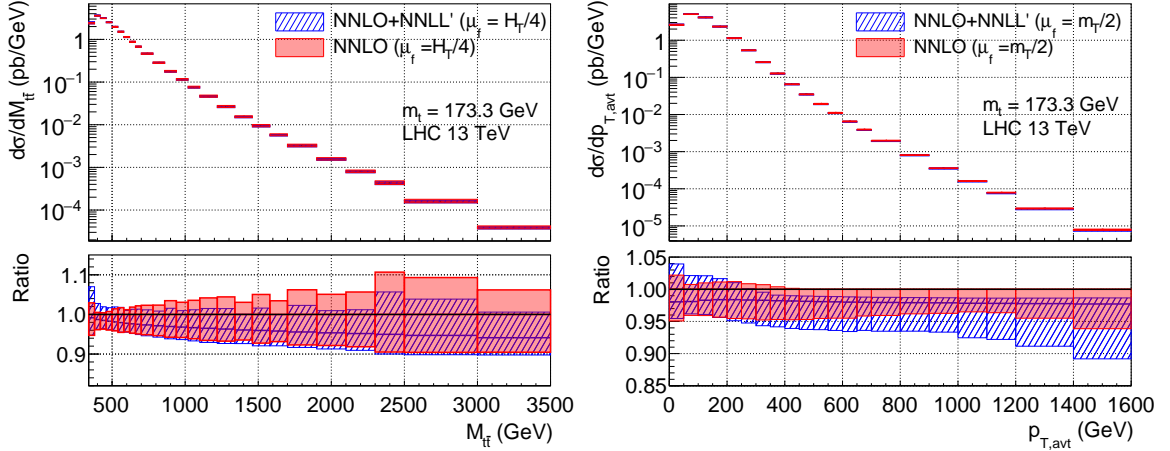


Figure 1: Results for the  $M_{t\bar{t}}$  distribution (left) and the  $p_{T,\text{avt}}$  distribution (right) at the LHC with  $\sqrt{s} = 13$  TeV. In all cases the ratio is to the NNLO result with  $\mu_f = H_T/4$ . The uncertainty bands reflect scale variations.

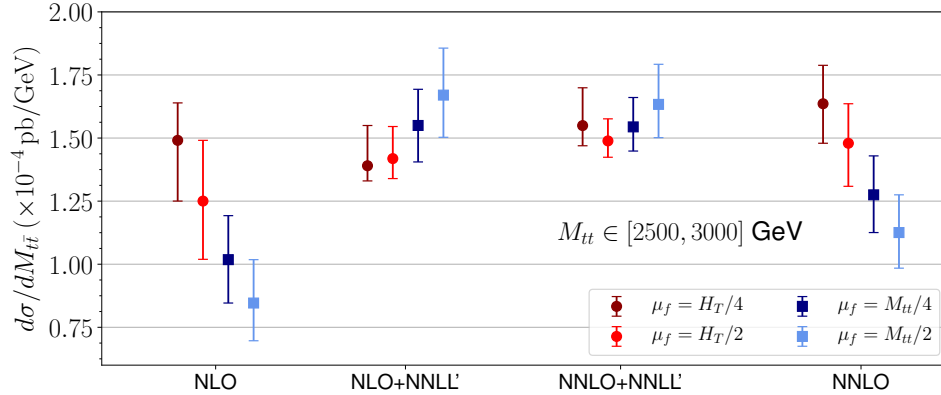


Figure 2: Cross sections obtained in a sample bin  $M_{t\bar{t}} = [2500-3000]$  GeV in the boosted region. The default value of  $\mu_f$  is indicated explicitly, and the error bars represent perturbative uncertainties estimated through scale variations.

even higher order corrections to the NNLO+NNLL' result are not so important. On the other hand, the NNLO result changes drastically when switching the schemes. In particular, higher order contributions beyond NNLO encoded in the resummation lead to large corrections for the choice  $\mu_f = M_{t\bar{t}}/2$ , as already foreseen in [19]. Given these observations, the close compatibility between the NNLO+NNLL' result (with either scale choice) and the NNLO result with  $\mu_f = H_T/4$  is a highly non-trivial fact. This provides an important confirmation of the result of [7], which favors the choice  $\mu_f = H_T/4$  for the fixed-order calculation of the  $t\bar{t}$  invariant mass distribution. The

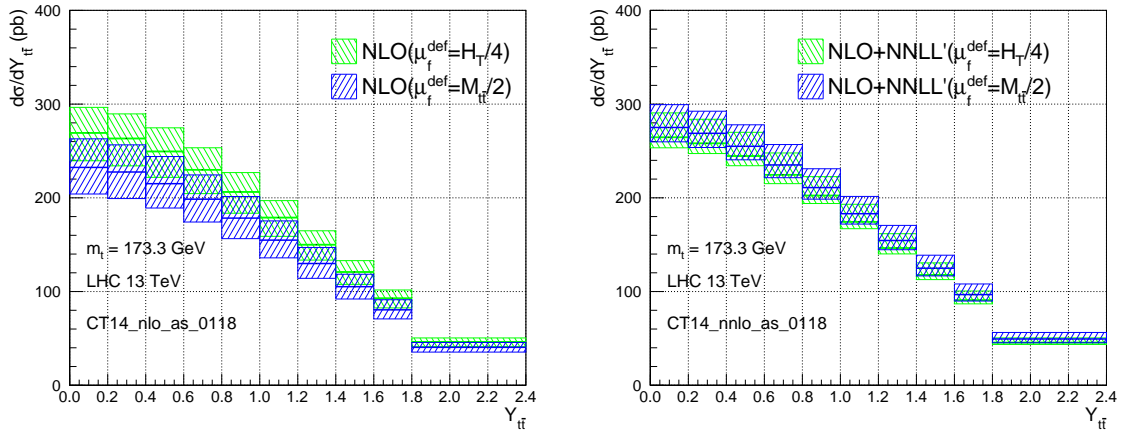


Figure 3: NLO and NLO+NNLL' predictions for the  $Y_{t\bar{t}}$  distribution with two choices of the default  $\mu_f$ .

overall picture emerging from the above analysis is that the perturbative description of the top-quark pair invariant mass distribution is under good control.

For the  $p_{T,\text{avt}}$  distribution shown in the right side in Fig. 1, the default factorization scale is chosen to be  $\mu_f = m_T/2$  (where  $m_T$  refers to the transverse mass of either the top or anti-top quark depending on the distribution under consideration), which is favored by the study of [7]. We see that the NNLO+NNLL' result is consistent with the NNLO one. On the other hand, it has been found that upgrading matching with fixed-order from NLO+NNLL' to NNLO+NNLL' is an important effect for the  $p_T$  distributions, especially in reducing the scale uncertainties in the high  $p_T$  region. This is an important fact to keep in mind when using NLO-based Monte Carlo event generators to model  $p_T$  distributions.

### 3 Rapidity distributions

The resummation framework employed in the last section can be straightforwardly generalized to the rapidity distributions [21]. We are concerned with the rapidity of the  $t\bar{t}$  pair  $Y_{t\bar{t}}$ , and the rapidity of the top quark or the anti-top quark  $y_{t/\bar{t}}$ .<sup>†</sup> In order to calculate the rapidity distributions, it is necessary to introduce a rapidity-dependent parton luminosity function. We refer to [21] for the technical details. In this talk, we present some phenomenological results.

In Fig. 3 we show the  $Y_{t\bar{t}}$  distribution at NLO and NLO+NNLL', with two choices of the default  $\mu_f$ . We find similar behaviour as in the  $M_{t\bar{t}}$  case, namely that resum-

<sup>†</sup>We also define the average distribution  $d\sigma/dy_{\text{avt}}$ , as in the  $p_T$  case.

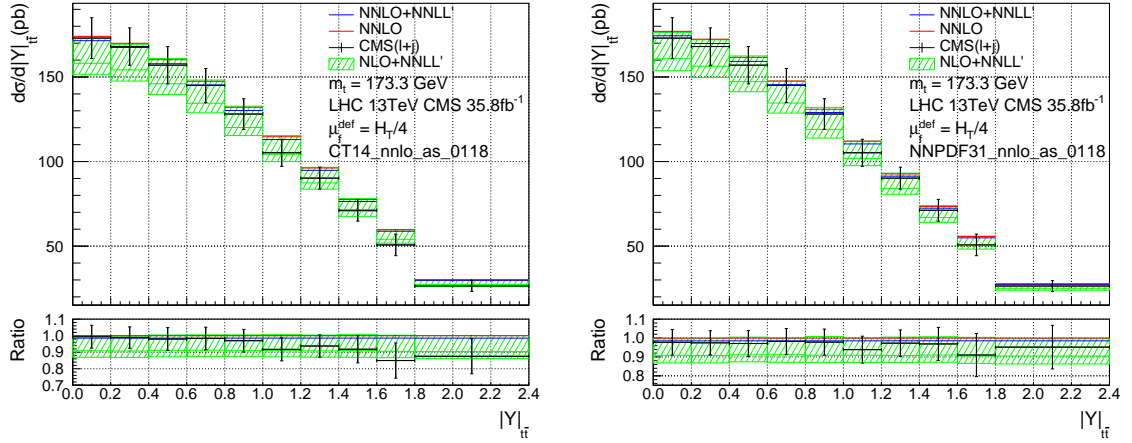


Figure 4: Comparison between the CMS data and various theoretical predictions for the  $Y_{t\bar{t}}$  distribution.

mation effects help to reduce the sensitivity of theoretical predictions to the choice of scales. The same conclusion can be drawn for the  $y_{avt}$  distribution.

In Fig. 4, we show predictions for the  $Y_{t\bar{t}}$  distribution using the CT14 [22] and the NNPDF3.1 [23] PDF sets, in comparison with the measurements from the CMS experiment [24]. We find excellent overall agreement of our results with the experimental data. However, we find that predictions from the two PDF sets have slightly different shapes. Especially in the tail region (large  $|Y_{t\bar{t}}|$ ), the CT14 PDFs tend to predict a higher production rate than the NNPDF3.1 PDFs. Such results can be used in the future to constrain the gluon PDF at large  $x$ , combining precision LHC data and state-of-the-art theoretical predictions.

## 4 Combination with electroweak corrections

Besides higher order QCD effects, electroweak (EW) corrections can also have big impacts in some kinematic regions [25, 26]. In [26], the NLO EW corrections are combined with the NNLO QCD results using the multiplicative approach (denoted as QCD×EW in the following). It is straightforward to combine the electroweak effects with the NNLO+NNLL' QCD results in a similar way as Eq. (5). We first take the QCD×EW results in [26]. We then further add the resummed contributions and remove the overlaps as

$$d\sigma^{\text{QCD}\times\text{EW}+\text{Res}_{\text{QCD}}} = d\sigma^{\text{NNLL}'_{b+m}} + \left( d\sigma^{\text{QCD}\times\text{EW}} - d\sigma^{\text{NNLL}'_{b+m}} \Big|_{\substack{\text{NNLO} \\ \text{expansion}}} \right). \quad (10)$$



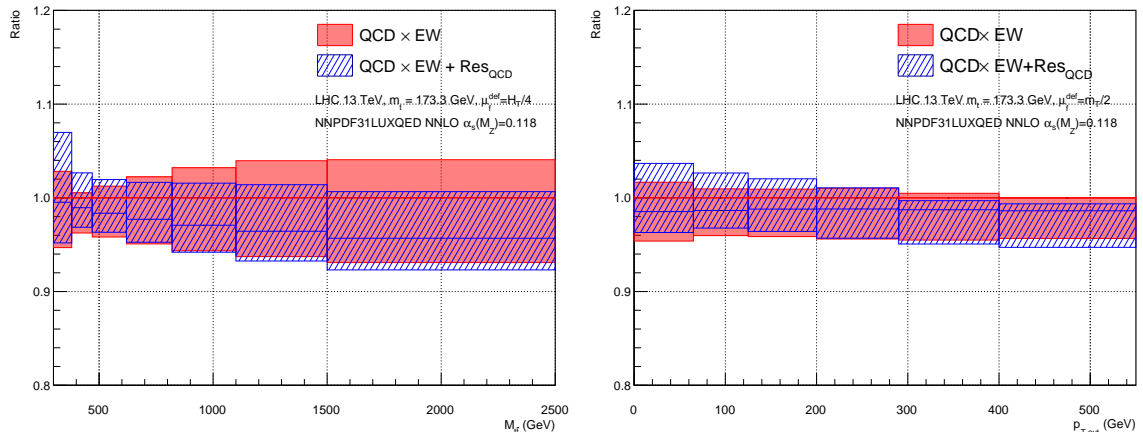


Figure 5: The state-of-the-art theoretical predictions for top pair  $M_{t\bar{t}}$  and  $p_{T,avt}$  distributions combine NNLO+NNLL' in QCD with NLO in EW.

In Fig. 5 we show predictions for the  $M_{t\bar{t}}$  and  $p_{T,avt}$  distributions including both QCD and electroweak corrections. Note that both the EW corrections and the resummation effects tend to soften the spectrums compared to the pure NNLO results. The combination of these two contributions thereby leads to a significant reduction of the differential cross sections in the boosted regime.

## 5 Summary

In this talk we have covered a number of recent developments related to the joint resummation of soft and small-mass logarithms in top quark pair production and the combination of these predictions with fixed order calculations at (N)NLO. The main effect of the resummation is to stabilize the dependence of the predictions on the choice of the factorization scale. It also shows that by carefully choosing the scales in the NNLO calculation, the higher order corrections are under good control. In section 3 we discussed the extension of our results to cover the rapidity distribution of the  $t\bar{t}$ -system,  $Y_{t\bar{t}}$  and the average rapidity distribution of the top/anti-top quark,  $y_{avt}$ . These observables can be used to constrain the gluon PDF in the future. Finally, we have presented for the first time the combination of our NNLO+NNLL' predictions in QCD with NLO EW corrections using the multiplicative approach. This is the first time that such a combination, which represents state-of-the-art precision in the Standard Model for top-quark pair differential cross sections, has appeared in the literature.

## ACKNOWLEDGEMENTS

L. L. Yang is supported in part by the National Natural Science Foundation of China under Grant No. 11575004 and 11635001. D. J. S. is supported under the ERC grant ERC-STG2015-677323. The work of D. P. and I. T. is supported by the Alexander von Humboldt Foundation, in the framework of the Sofja Kovalevskaja Award Project “Event Simulation for the Large Hadron Collider at High Precision”. The work of A. M. and A. P. is supported by the European Research Council Consolidator Grant NNLOforLHC2 and by the UK STFC grants ST/L002760/1 and ST/K004883/1. The work of M. C. was supported in part by a grant of the BMBF.

## References

- [1] P. Bärnreuther, M. Czakon and A. Mitov, Phys. Rev. Lett. **109**, 132001 (2012) [arXiv:1204.5201 [hep-ph]].
- [2] M. Czakon and A. Mitov, JHEP **1212**, 054 (2012) [arXiv:1207.0236 [hep-ph]].
- [3] M. Czakon and A. Mitov, JHEP **1301**, 080 (2013) [arXiv:1210.6832 [hep-ph]].
- [4] M. Czakon, P. Fiedler and A. Mitov, Phys. Rev. Lett. **110**, 252004 (2013) [arXiv:1303.6254 [hep-ph]].
- [5] M. Czakon, P. Fiedler and A. Mitov, Phys. Rev. Lett. **115**, no. 5, 052001 (2015) [arXiv:1411.3007 [hep-ph]].
- [6] M. Czakon, D. Heymes and A. Mitov, Phys. Rev. Lett. **116**, no. 8, 082003 (2016) [arXiv:1511.00549 [hep-ph]].
- [7] M. Czakon, D. Heymes and A. Mitov, JHEP **1704**, 071 (2017) [arXiv:1606.03350 [hep-ph]].
- [8] M. Czakon, A. Ferroglia, D. Heymes, A. Mitov, B. D. Pecjak, D. J. Scott, X. Wang and L. L. Yang, JHEP **1805**, 149 (2018) [arXiv:1803.07623 [hep-ph]].
- [9] N. Kidonakis and G. F. Sterman, Phys. Lett. B **387**, 867 (1996).
- [10] N. Kidonakis and G. F. Sterman, Nucl. Phys. B **505**, 321 (1997) [hep-ph/9705234].
- [11] V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, JHEP **1009**, 097 (2010) [arXiv:1003.5827 [hep-ph]].
- [12] A. Ferroglia, B. D. Pecjak and L. L. Yang, Phys. Rev. D **86**, 034010 (2012) [arXiv:1205.3662 [hep-ph]].

- [13] A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, Phys. Rev. Lett. **103**, 201601 (2009) [arXiv:0907.4791 [hep-ph]].
- [14] A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, JHEP **0911**, 062 (2009) [arXiv:0908.3676 [hep-ph]].
- [15] G. Wang, X. Xu, L. L. Yang and H. X. Zhu, JHEP **1806**, 013 (2018) [arXiv:1804.05218 [hep-ph]].
- [16] A. Ferroglia, B. D. Pecjak and L. L. Yang, JHEP **1309**, 032 (2013) [arXiv:1306.1537 [hep-ph]].
- [17] A. Ferroglia, B. D. Pecjak, L. L. Yang, B. D. Pecjak and L. L. Yang, JHEP **1210**, 180 (2012) [arXiv:1207.4798 [hep-ph]].
- [18] A. Broggio, A. Ferroglia, B. D. Pecjak and Z. Zhang, JHEP **1412**, 005 (2014) [arXiv:1409.5294 [hep-ph]].
- [19] B. D. Pecjak, D. J. Scott, X. Wang and L. L. Yang, Phys. Rev. Lett. **116**, no. 20, 202001 (2016) [arXiv:1601.07020 [hep-ph]].
- [20] R. D. Ball *et al.* [NNPDF Collaboration], JHEP **1504**, 040 (2015) [arXiv:1410.8849 [hep-ph]].
- [21] B. D. Pecjak, D. J. Scott, X. Wang and L. L. Yang, arXiv:1811.10527 [hep-ph].
- [22] S. Dulat *et al.*, Phys. Rev. D **93**, no. 3, 033006 (2016) [arXiv:1506.07443 [hep-ph]].
- [23] R. D. Ball *et al.* [NNPDF Collaboration], Eur. Phys. J. C **77**, no. 10, 663 (2017) [arXiv:1706.00428 [hep-ph]].
- [24] A. M. Sirunyan *et al.* [CMS Collaboration], Phys. Rev. D **97**, no. 11, 112003 (2018) [arXiv:1803.08856 [hep-ex]].
- [25] W. Bernreuther and Z. G. Si, Nucl. Phys. B **837**, 90 (2010) [arXiv:1003.3926 [hep-ph]].
- [26] M. Czakon, D. Heymes, A. Mitov, D. Pagani, I. Tsinikos and M. Zaro, JHEP **1710**, 186 (2017) [arXiv:1705.04105 [hep-ph]].