Top pair threshold production at a linear collider with WHIZARD

Fabian Bach, Maximilian Stahlhofen
DESY, Notkestraße 85, 22607 Hamburg, Germany
E-mail: fabian.bach@desy.de, maximilian.stahlhofen@desy.de

Abstract. We briefly describe how the Monte Carlo generator WHIZARD 2.2 can be employed to study large QCD effects enhancing the top-antitop production threshold at a next-generation lepton collider. While present state-of-the-art predictions at NNLL order are confined to inclusive total cross sections, our tool can be used to simulate differential distributions including NLL threshold resummation in the production, and with off-shell decaying tops. The new model will be shipped with WHIZARD from version 2.2.3 onwards, to be released along with this article.

1. Introduction

The scan of the top-antitop threshold at the planned ILC will allow for a precise determination of the top quark mass, decay width and couplings. In particular, the expected precision for the top mass will be an order of magnitude better than what is possible at the LHC [1]. The total cross section for top-antitop threshold production has been calculated in the framework of NRQCD and is currently known to NNLL order [2]. We implement the NLL threshold resummation for the differential cross section for this process in the Monte Carlo (MC) event generator WHIZARD [3, 4, 5], allowing for off-shell top quarks and their (tree-level) decay. Our tool can thus be used for MC studies of exclusive observables or event shapes in top-antitop production at and above threshold with arbitrary experimental cuts on the decay products of the tops. Interferences with the non-resonant tree-level background are automatically taken into account.

2. WHIZARD–A universal event generator for elementary processes at colliders

The WHIZARD package [3, 4, 5] consists of two components, namely O'MEGA [6] for the generation of the hard parton level matrix elements, and the WHIZARD core program which performs the phase space integration and delivers all the necessary infrastructure including output formatting and interfaces. WHIZARD comes with a dedicated scripting language, SINDARIN, to facilitate the user control of the entire package functionality in one script file, from the process definition in a given model down to event analysis and histograms/plots. There is already an extensive library of SM extensions and BSM models in the package, ranging from various anomalous couplings and Little Higgs models to SUSY in its more common flavors as well as exotics like UED etc. Moreover, an interface to FEYNRULES [7] exists to further extend this list. Finally, there are ample package tools and interfaces available to the user to further control the initial and final state modeling for both hadron and lepton colliders (LHAPDF, ISR, FSR, MLM matching, hadronization, beamstrahlung, etc.) as well as I/O event formats.

Higher-order QCD effects at $t\bar{t}$ threshold production in e^+e^- collisions, cf. section 3, are implemented into WHIZARD via a new model `SM_tt_threshold` described in more detail in section 4. Along with this proceedings article, a new WHIZARD v2.2.3 containing the first official β-version of the described model will shortly be released at

www.hepforge.org/downloads/whizard

3. Theory

3.1. Top pair production at threshold

Near threshold the effective velocity of the top quarks in the center-of-mass frame, defined as

$$v \equiv \sqrt{\frac{\sqrt{s} - 2m + i\Gamma_t}{m}}\,, \tag{1}$$

is of similar size as the coupling, $v \sim \alpha_s \sim 0.1$, reflecting the bound-state-like dynamics of the top-antitop pair. To correctly describe top-antitop threshold production it is therefore crucial to resum Coulomb singular terms $\sim (\alpha_s/v)^n$ in the perturbation series for the cross section to all orders. This Coulomb resummation can be carried out by means of a Schrödinger-type equation within the nonrelativistic effective field theory NRQCD. The extended vNRQCD framework in addition allows the resummation of large logarithms of the top quark velocity $(\ln v)$. Schematically the normalized cross section (R-Ratio) close to threshold then takes the form

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \left\{1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2,\ \alpha_s v,\ v^2\,(\mathrm{NNLL}); \ldots\right\}. \tag{2}$$

The vNRQCD prediction for the total cross section of top-antitop threshold production has recently reached the NNLL level [8, 2]. Since $\alpha_{ew} \sim \alpha_s^2$, the electroweak background from non-resonant production of the top decay products $(W^+ b\, W^- \bar{b})$ starts to contribute at the NNLL level. However, there are also effects related to the interference of double- and single-resonant production that are parametrically of NLL order [9]. For the physical four-particle final state, $W^+ b\, W^- \bar{b}$, both of these electroweak corrections are automatically included in cross section predictions produced by WHIZARD, once the (virtual) top-pair threshold production is consistently implemented.

3.2. 1S mass scheme

Due to the cancellation of the leading renormalon between the pole mass and the QCD potential, the threshold production cross section is free of a $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon ambiguity, when expressed in terms of a suitable threshold mass scheme. For the WHIZARD top-antitop threshold model we use the 1S mass $M^{1\mathrm{S}}$ [10] as an input parameter, which can be related to other short-distance masses, like the $\overline{\mathrm{MS}}$ mass, in a renormalon-safe way. The determination of $M^{1\mathrm{S}}$ from a fit to future experimental data will therefore be stable against higher-order QCD corrections and is not limited by an intrinsic $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ uncertainty. In fact, a systematic and statistical error well below 100 MeV is expected for the 1S mass measured by a threshold scan at the ILC [1].

3.3. Top-antitop form factor

For the implementation of the NLL threshold effects in the WHIZARD framework we combine the vector and axial vector production current vertices with nonrelativistic S-wave and P-wave form factors, respectively. These contain the contributions from the vNRQCD resummation beyond tree level. The S/P-wave form factors are related to the Green's functions $G_{1/3}^{(x=0)}(E, p, \nu)$, which

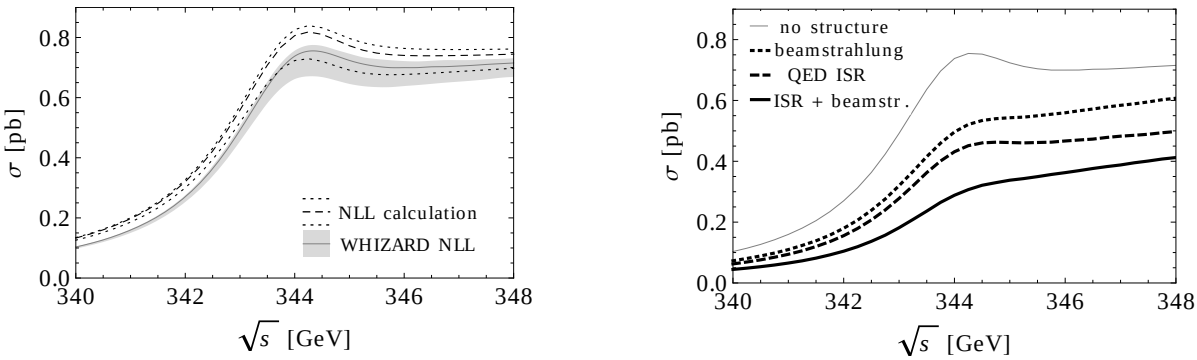

Figure 1. Comparison of the vNRQCD and WHIZARD inclusive cross sections at NLL, including error bands from combined scale variations as defined in [2] (left). Impact of various beam structure effects on the NLL threshold line-shape (right). For both plots a top invariant mass cut $\Delta M = 30\,\text{GeV}$ was applied.

depend on the total energy (E) of the top pair as well as on the top three-momentum in the c.m. frame (p), times the vNRQCD Wilson coefficient for the effective spin-triplet S/P-wave vector current $c_{1/3}(\nu)$ [11]. Both, G_i and c_i depend on the vNRQCD renormalization parameter ν and should be evaluated at $\nu \sim v$ in order to properly resum the logarithms $\sim (\alpha_s \ln v)^n$. The relevant NLL nonrelativistic momentum-dependent Green's functions can be computed numerically using the TOPPIK code [10]. In the nonrelativistic regime the virtuality of the tops is $\mathcal{O}(v^2)$. Hence the NLL nonrelativistic form factor is formally gauge and renormalization group invariant. This approximation however breaks down for relativistic top energies and/or momenta and the threshold resummation must therefore be smoothly turned off in a certain kinematic transition region such that the cross section matches the correct relativistic continuum obtained from a corresponding fixed-order QCD calculation. In a joint effort [12], we are currently working on a concrete matching procedure between the NLL threshold and the NLO relativistic continuum, which eventually will be implemented in WHIZARD. Until then the new WHIZARD model is only reliable in an energy range of a few GeV centered around the resonance peak at roughly $2M^{1S}$.

4. The WHIZARD model

The nonrelativistic threshold resummation effects are implemented in WHIZARD in terms of form factors for the ttZ/γ (vector, axial vector) vertices, as described in section 3.3. At each phase space point in the threshold region, the form factor is calculated as a function of the partonic $\sqrt{s}$ and the top 3-momentum magnitude p, either by evaluating an analytical expression at LL, or by running TOPPIK [10] for NLL precision. We employ M^{1S} as an independent user parameter for the top mass, cf. section 3.2. M^{1S} can be converted into the pole mass m_t^{pole}, which in the current WHIZARD version parametrizes the position of the off-shell top resonances. Therefore, we compute m_t^{pole} internally from the independent model parameters and provide it as a fixed parameter within SINDARIN, cf. table 1, in order to facilitate invariant mass cuts on the top resonances.

In the theoretical calculation, the top width is accounted for by a complex energy, cf. eq. (1). In WHIZARD, however, one can invoke the $2 \to 4$ process $e^+e^- \to bbW^+W^-$ including resonant $t\bar{t}$ diagrams with width Γ_t as well as the full non-resonant background. Therefore, the four-particle phase space is modeled correctly, and arbitrary experimental cuts can be applied to the final state. Although Γ_t is implemented as a user-definable parameter, note that the $2 \to 4$ process becomes inconsistent if Γ_t is chosen smaller than the partial decay width $t \to bW^+$ which is fixed by the other model parameters. WHIZARD will warn you whenever this happens.

Table 1. Model-specific WHIZARD parameters.

Parameter	Default value	Description
`alpha_em_i`	125.924	inverse QED coupling $1/\alpha_{em}$ at the $t\bar{t}$ threshold
`m1S`	172.0 GeV	top quark M^{1S} mass
`wtop`	1.54 GeV	top quark width Γ_t
`nloop`	1	NRQCD order: 0 (LL) or 1 (NLL)
`sh`	1.0	matching scale parameter h as defined in [2]
`sf`	1.0	renormalization scale parameter f as defined in [2]
`mtpole`	fixed	top quark pole mass m_t^{pole}

Finally, the sensitivity of the prediction on the choice for the matching and renormalization scales can be analyzed by varying the dimensionless parameters h and f, cf. [2]. In figure 1, we compare the threshold line-shape at NLL as produced by the WHIZARD model with the results of [2]. Note the visible difference between the two approaches coming from relativistic, phase space, and electroweak (interference) effects present in WHIZARD, but neglected in [2]. Moreover, as mentioned in section 2, the user can employ beam structure like QED ISR and beamstrahlung in WHIZARD. The impact on the threshold shape is illustrated in figure 1.

5. Conclusions and outlook

A precise assessment of the top-antitop threshold cross section will be one of the key physics cases of a future lepton collider, because it allows for a precise measurement of the top couplings including α_s and, in particular, the top mass (in a renormalon-free scheme). While NRQCD theory predictions have recently reached NNLL order accuracy for the total cross section, we present an implementation of the threshold enhancement at NLL order for off-shell $t\bar{t}$ production into the Monte Carlo generator WHIZARD (from version 2.2.3). We thus provide a fully differential tool facilitating experimental studies of threshold scans with arbitrary cuts on the final states of the (LO) decaying top quarks. In a future version of the model, the threshold process will be matched onto relativistic continuum production at NLO.

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