New insights on the hyperon puzzle from quantum Monte Carlo calculations

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1   Introduction

The presence of hyperons in the inner core of a neutron star (NS) is still the subject of an open debate. The mechanism promoting the formation of heavier, distinguishable particles within dense interacting matter is quite intuitive at the non-relativistic level, and relies essentially on the Pauli principle. However, the energy and pressure reduction due to the presence of hyperons in the medium seems to be incompatible with the most recent astronomical observations. In particular, most of non-relativistic models predict a softening of the equation of state (EOS) which is too large to sustain a neutron star of mass $\sim 2M_\odot$ as the ones that have been recently observed [1, 2]. This apparent inconsistency between NS mass observations and theoretical calculations is a long standing problem known as *hyperon puzzle*.

   At present it is still too difficult to derive a first-principle effective Hamiltonian directly from LQCD results in matter with strangeness. Therefore it is still necessary to rely on the scarce experimental data to work out a realistic interaction between hyperons and nucleons in matter, and provide a reliable prediction for the EOS. We have been pursuing for a few years a program aiming to build up a phenomenological, realistic potential along the tracks of the Argonne/Illinois model. This work was started originally by Bodmer et al. [3], and continued in a series of papers by using variational Monte Carlo algorithms [4, 5, 6, 7, 8, 9, 10]. Our additional ingredient is the consistent use of auxiliary field diffusion Monte Carlo (AFDMC) [13, 14] calculations in oder to first assess the parameters of the interaction from measured

hyperon separation energies, and then to extrapolate the results to the case of neutron matter with strangeness. Other strange degrees of freedom might be relevant to the end of a correct determination of the EOS. However, in our approach we rely only on available experimental data. Essentially no measurements are available for Σ and Ξ hypernuclei, and therefore we exclude at the moment these channels from our treatment. The same argument applies to two- and many-hyperon forces, which, at present, are substantially unknown from the experimental point of view.

Most of our results have been recently published [15, 16, 17], and we refer to the original papers for all the detailed aspects of the algorithm and the actual calculations. In this proceeding we will briefly comment on a particular aspect of the hyperon-nucleon-nucleon force that nicely illustrates the difficulty in extracting the information on the Hamiltonian from experimental Λ separation energies.

2 A phenomenological, realistic hyperon-nucleon interaction

Within our non-relativistic many-body approach, Λ hypernuclei and Λ-neutron matter are described in terms of pointlike nucleons and lambdas, with masses m_N and m_Λ, respectively, whose dynamics are dictated by the Hamiltonian:

$$H_{\text{nuc}} = T_N + V_{NN} = \sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} \, , \tag{1}$$

$$H_{\text{hyp}} = H_{\text{nuc}} + T_\Lambda + V_{\Lambda N} + V_{\Lambda NN}$$

$$= H_{\text{nuc}} + \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda i} v_{\lambda i} + \sum_{\lambda, i<j} v_{\lambda ij} \, , \tag{2}$$

where A is the total number of baryons $A = \mathcal{N}_N + \mathcal{N}_\Lambda$, latin indices $i, j = 1, \ldots, \mathcal{N}_N$ label nucleons, and the greek symbol $\lambda = 1, \ldots, \mathcal{N}_\Lambda$ is used for Λ particles. The nuclear potential includes two- and three-nucleon contributions while in the strange sector we adopt explicit ΛN and ΛNN interactions.

In the non-strange sector we employ a simplified interaction in order to make the calculations feasible also for heavier hypernuclei. In particular we use the sum of the Argonne AV4' interaction [11], plus the central repulsive term of the three-body Urbana IX potential [12]. This choice provides a realistic description of energies, densities and radii. In Tab. 1 we report some examples of the binding energies obtained from AFDMC calculations for a set of closed shell nuclei.

In the strange sector the ΛN interaction has been modeled with an Urbana-type

	AV4'	AV4'+UIX$_c$	exp.	diff. (%)
^{4}He	-32.83(5)	-26.63(3)	-28.296	6
^{16}O	-180.1(4)	-119.9(2)	-127.619	6
^{40}Ca	-597(3)	-382.9(6)	-342.051	12
^{48}Ca	-645(3)	-414.2(6)	-416.001	0.5

Table 1: Energies of a few nuclei computed by AFDMC using the AV4' nucleon-nucleon potential, and including a three-body repulsive contribution taken from the UIX potential.

potential [18], consistent with the available Λp scattering data

$$v_{\lambda i} = v_0(r_{\lambda i}) + \frac{1}{4} v_\sigma T_\pi^2(r_{\lambda i}) \, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i \, , \tag{3}$$

where $v_0(r) = v_c(r) - \overline{v} \, T_\pi^2(r)$ is a central term. The terms $\overline{v} = (v_s + 3v_t)/4$ and $v_\sigma = v_s - v_t$ are the spin-average and spin-dependent strengths, where v_s and v_t denote singlet- and triplet-state strengths, respectively. Note that both the spin-dependent and the central radial terms contain the usual regularized one pion exchange tensor operator $T_\pi(r)$

$$T_\pi(r) = \left[1 + \frac{3}{\mu_\pi r} + \frac{3}{(\mu_\pi r)^2} \right] \frac{e^{-\mu_\pi r}}{\mu_\pi r} \left(1 - e^{-cr^2} \right)^2 \, , \tag{4}$$

where μ_π is the reduced pion mass

$$\mu_\pi = \frac{m_{\pi^0} + 2 \, m_{\pi^\pm}}{3} \qquad \frac{1}{\mu_\pi} \simeq 1.4 \text{ fm} \, . \tag{5}$$

All the parameters defining the ΛN potential can be found, for example, in Ref. [9]. The three-body potential $v_{\lambda ij}$ can be conveniently decomposed in the 2π-exchange contributions $v_{\lambda ij}^{2\pi} = v_{\lambda ij}^{2\pi,P} + v_{\lambda ij}^{2\pi,S}$ and a spin-dependent dispersive term $v_{\lambda ij}^D$ as follows:

$$v_{\lambda ij}^{2\pi,P} = -\frac{C_P}{6} \left\{ X_{i\lambda} \, , X_{\lambda j} \right\} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \, , \tag{6}$$

$$v_{\lambda ij}^{2\pi,S} = C_S \, Z \left(r_{\lambda i} \right) Z \left(r_{\lambda j} \right) \boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{i\lambda} \, \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{j\lambda} \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \, , \tag{7}$$

$$v_{\lambda ij}^D = W_D \, T_\pi^2 \left(r_{\lambda i} \right) T_\pi^2 \left(r_{\lambda j} \right) \left[1 + \frac{1}{6} \boldsymbol{\sigma}_\lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right] \, . \tag{8}$$

The function $T_\pi(r)$ is the same as in Eq. (4), while the $X_{\lambda i}$ and $Z(r)$ are defined by

$$
\begin{aligned}
X_{\lambda i} &= Y_\pi(r_{\lambda i})\,\boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i + T_\pi(r_{\lambda i})\,S_{\lambda i}\;, \\
Z(r) &= \frac{\mu_\pi r}{3}\Big[Y_\pi(r) - T_\pi(r)\Big]\;,
\end{aligned}
\tag{9}
$$

where

$$
Y_\pi(r) = \frac{e^{-\mu_\pi r}}{\mu_\pi r}\Big(1 - e^{-cr^2}\Big)
\tag{10}
$$

is the regularized Yukawa potential and $S_{\lambda i}$ is the usual tensor operator. The range of parameters C_P, C_S and W_D that have been used in our calculations can be found in Ref. [16].

AFDMC predictions for the Λ separation energy B_Λ, defined as the difference in the binding energy of the core nucleus and the corresponding hypernucleus, are in good agreement with experimental data for hypernuclei up to $^{49}_{\Lambda}$Ca and also for the Λ particle in different single particle states (see Fig. 1).
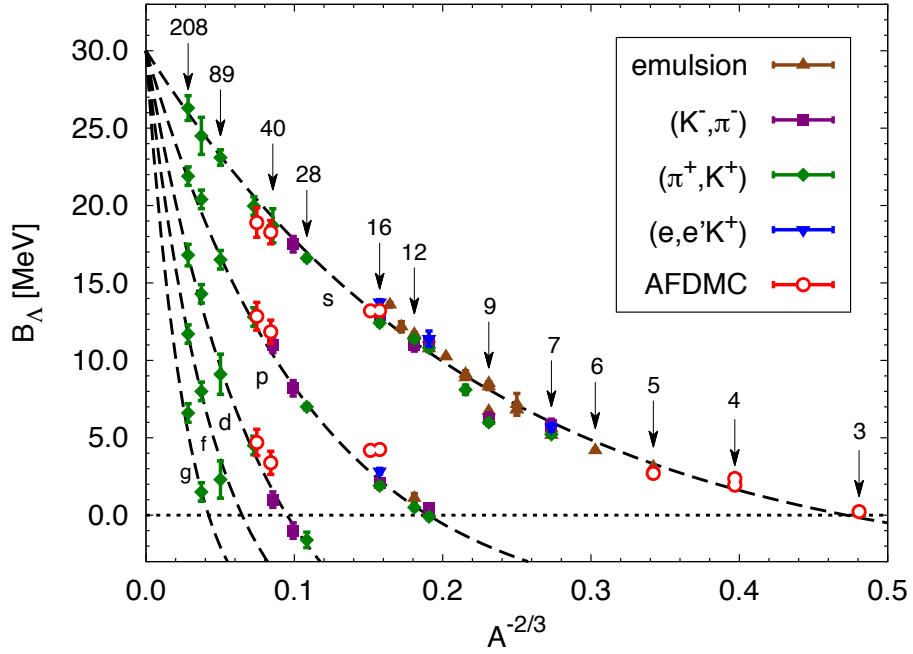

Figure 1: Measured [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] and computed Λ separation energies as a function of $A^{-2/3}$. Results for the Λ particle in different single particle states are also shown.

3 On the isospin dependence of the ΛNN interaction

As shown in Ref. [17], the parametrization of the hyperon-nucleon potential yielding the best prediction for B_Λ provides a repulsion that is large enough to prevent the appearance of strange degrees of freedom in the range of densities found in the inner core of a NS. While this result might look as a possible solution of the hyperon puzzle, a closer look to the interaction suggests that the constraints coming from an analysis of the hypernuclear data might not be sufficient to provide an accurate extrapolation to stellar conditions. In the following we present a case study to effectively illustrates this point.

The current version of the ΛNN potential does not depend on whether the two nucleons are in a singlet or a isospin triplet state. For symmetric hypernuclei the Pauli principle suppresses any strong contribution from the Λnn or Λpp channels. On the other hand, in neutron matter or in matter at β-equilibrium the contribution of the isospin triplet channel might become quite relevant. In order to test the sensitivity of our AFDMC predictions for B_Λ on the strength of the isospin triplet component, we consider the sum $v_{\lambda ij}^T$ of the S and P wave 2π exchange terms:

$$v_{\lambda ij}^T \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = \left[-\frac{C_P}{6} \left\{ X_{i\lambda}, X_{\lambda j} \right\} + C_S \, Z\left(r_{\lambda i}\right) Z\left(r_{\lambda j}\right) \boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{i\lambda} \, \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{j\lambda} \right] \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \; . \quad (11)$$

The operator $\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ can be written in terms of the projectors on the triplet ($T = 1$) and singlet ($T = 0$) nucleon isospin channels:

$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -3\, P_{ij}^{T=0} + P_{ij}^{T=1} \; . \quad (12)$$

The potential can be then rewritten as:

$$v_{\lambda ij}^T \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -3\, v_{\lambda ij}^T \, P_{ij}^{T=0} + C_T \, v_{\lambda ij}^T \, P_{ij}^{T=1} \; , \quad (13)$$

where the additional parameter C_T gauges the strength and the sign of the isospin triplet contribution. For $C_T = 1$ the original parametrization of the three-body force is recovered.

We have performed calculations for several hypernuclei ($^4_\Lambda$H, $^4_\Lambda$He, $^5_\Lambda$He, $^{17}_\Lambda$O, $^{41}_\Lambda$Ca, $^{49}_\Lambda$Ca) varying the C_T parameter in the range $[-2; 3]$. The results are reported in Fig. 2 as the ratio of $B_\Lambda(C_T)$ and B_Λ at $C_T = 1$, i.e. with the original parametrization.

It can be observed that the sensitivity of the results on the value of C_T is not large over the whole interval $-1.0 \leq C_T \leq 1.5$, while strong deviations appear beyond this range. In general, outside of this range B_Λ tends to increase with respect to the original value. The only is exception is $^{49}_\Lambda$Ca, where the sensitivity appears to be larger, and B_Λ tends to decrease with respect to the original value. Given the substantial asymmetry of this hypernucleus, we can infer that the isospin triplet channel could be properly constrained only by looking at the binding energy of strongly asymmetric hypernuclei. Extrapolating results to neutron matter without taking into account this feature of the interaction might in principle lead to misleading results.
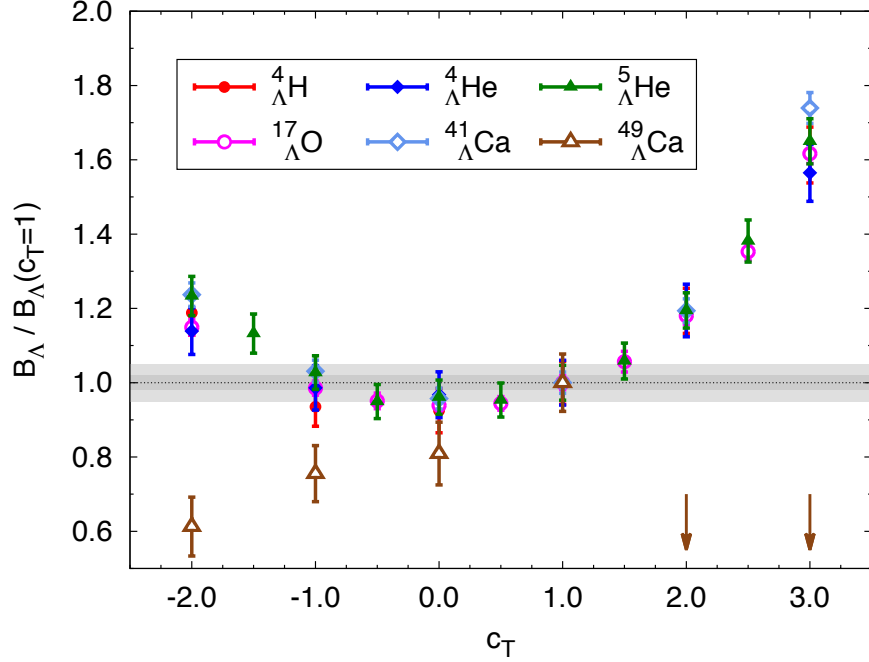
Figure 2: Λ separation energies normalized with respect to the $C_T = 1$ case as a function of C_T. Grey bands represent the 2% and 5% variations of the ratio $B_\Lambda/B_\Lambda(C_T = 1)$. Brown vertical arrows indicate the results for $^{49}_\Lambda$Ca in the case of $C_T = 2$ and $C_T = 3$, outside the scale of the plot.

4   Conclusions

In the last years auxiliary field diffusion Monte Carlo has been used to assess the properties of hypernuclear systems, from light- to medium-heavy hypernuclei and hyperneutron matter. One of the main findings is the key role played by the three-body hyperon-nucleon-nucleon interaction in the determination of the hyperon separation energy of hypernuclei and as a possible solution to the hyperon puzzle. However, there are still aspects of the employed hypernuclear potential that remain to be carefully investigated. For instance, we showed that the isospin dependence of the ΛNN force, which is crucial in determining the NS structure, is poorly constrained by the available experimental data. Future experiments on highly asymmetric hypernuclei such as $^{49}_\Lambda$Ca, $^{91}_\Lambda$Zr or even $^{209}_\Lambda$Pb would pin down fundamental properties of the hyperon-nucleon forces. This would thereby allow for a substantial step forward in understanding the deep connections between the physics at the km scale typical of NSs and the properties of matter at the fm scale that can be efficiently explored in terrestrial experiments.

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References

[1] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels, Nature 467, 1081 (2010).

[2] J. Antoniadis et al., Science 340, 1233232 (2013).

[3] A. R. Bodmer, Q. N. Usmani, and J. Carlson, Phys. Rev. C 29, 684 (1984).

[4] A. R. Bodmer and Q. N. Usmani, Phys. Rev. C 31, 1400 (1985).

[5] A. R. Bodmer and Q. N. Usmani, Nucl. Phys. A 477, 621 (1988).

[6] A. A. Usmani, Phys. Rev. C 52, 1773 (1995).

[7] A. A. Usmani, S. C. Pieper, and Q. N. Usmani, Phys. Rev. C 51, 2347 (1995).

[8] Q. N. Usmani and A. R. Bodmer, Phys. Rev. C 60, 055215 (1999).

[9] A. A. Usmani and F. C. Khanna, J. Phys. G 35, 025105 (2008).

[10] M. Imran, A. A. Usmani, M. Ikram, Z. Hasan, and F. C. Khanna, J. Phys. G 41, 065101 (2014).

[11] R. B. Wiringa, S. C. and Pieper, Phys. Rev. Lett. 89, 182501 (2002).

[12] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. C 56, 1720 (1997).

[13] K. E. Schmidt and S. Fantoni, Phys. Lett. B 446, 99 (1999).

[14] J. Carlson, S. Gandolfi, F. Pederiva, S. C. Pieper, R. Schiavilla, et al. arXiv:1412.3081.

[15] D. Lonardoni, S. Gandolfi, and F. Pederiva, Phys. Rev. C 87, 041303 (2013).

[16] D. Lonardoni, F. Pederiva, and S. Gandolfi, Phys. Rev. C 89, 014314 (2014).

[17] D. Lonardoni, A. Lovato, S. Gandolfi, and F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015).

[18] I. Lagaris and V. Pandharipande, Nucl. Phys. A 359, 331 (1981).

[19] M. Jurič et al., Nucl. Phys. B 52, 1-30 (1973).

[20] T. Cantwell et al., Nucl. Phys. A 236, 445 (1974).

[21] R. Bertini et al., Phys. Lett. B 83, 306 (1979).

[22] P. Pile et al., Phys. Rev. Lett. 66, 2585 (1991).

[23] T. Hasegawa et al., Phys. Rev. C 53, 1210 (1996).

[24] H. Takahashi et al., Phys. Rev. Lett. 87, 212502 (2001).

[25] H. Hotchi et al., Phys. Rev. C 64, 044302 (2001).

[26] T. Miyoshi et al., Phys. Rev, Lett. 90, 232502 (2003).

[27] L. Yuan et al., Phys. Rev. C 73, 044607 (2006).

[28] O. Hashimoto et al., Progr. Part. Nucl. Phys. 57, 564 (2006).

[29] F. Cusanno et al., Phys. Rev. Lett. 103, 202501 (2009).

[30] M. Agnello et al., Nucl. Phys. A 835, 414 (2010).

[31] K. Nakazawa, Nucl. Phys. A 835, 207 (2010).

[32] M. Agnello et al., Phys. Lett. B 698, 219 (2011).

[33] M. Agnello et al., Phys. Rev. Lett. 108, 042501 (2012).

[34] A. Feliciello, Mod. Phys. Lett. A 28, 1330029 (2013).

[35] J. K. Ahn et al., Phys. Rev. C 88, 014003 (2013).

[36] S. Nakamura et al., Phys. Rev. Lett. 110, 012502 (2013).

[37] L. Tang et al., Phys. Rev. C 90, 034320 (2014).