

## Void magnetic field and its primordial origin in inflation

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Since magnetic fields in galaxies, galactic clusters and even void regions are observed, theoretical attempts to explain their origin are strongly motivated. It is interesting to consider that inflation is responsible for the origin of the magnetic fields as well as the density perturbation. However, it is known that inflationary magnetogenesis suffers from several problems. We explore these problems by using a specific model, namely the kinetic coupling model, and show how the model is constrained. Model independent arguments are also discussed.

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# 1 Introduction

Various magnetic fields are known to exist in the universe. In the context of cosmology, magnetic fields in galaxies, galactic clusters and void regions are important. The magnetic fields in galaxies and clusters are  $B_{\text{gal}} \sim 10^{-6}\text{G}$  [1, 2]. The void magnetic field is reported to be stronger than  $B_{\text{void}} \gtrsim 10^{-15}\text{G}$  with an uncertainty of a few orders based on blazar observations [3, 4, 5, 6]. However, the origin of these magnetic fields is a long-term and outstanding problem in astrophysics and cosmology.

The candidates of the mechanism that generates the magnetic fields are divided into two main classes [7, 8, 9]. One class includes astrophysical processes which exploit plasma motions to produce magnetic fields in comparatively local regions while it may be difficult for these mechanisms to work in void regions. The other consists of cosmological processes which generate magnetic fields spread over the universe in the very early universe. The magnetic field produced by the latter class of models can directly dilute into the void magnetic field and also seed the galactic and cluster magnetic fields if the strength is sufficient. The scenario of the primordial magnetic field naturally explains the hierarchy between  $B_{\text{gal}}$  and  $B_{\text{void}}$  because the adiabatic compression and the dynamo mechanism may amplify it in galaxies and clusters while the magnetic field is expected to dilute due to the cosmic expansion in void regions. However, the primordial magnetic field is constrained by CMB observations as  $B_p \lesssim 10^{-9}\text{G}$  (see, e.g. [10], and references therein). Here, we focus on the magnetic fields with a primordial origin, especially an inflationary origin.

Inflation is a widely accepted paradigm of the very early universe and it can produce cosmological perturbations from quantum fluctuations. Since the initial density perturbation which seeds the large scale structures observed in the present universe originates from inflation, it is an attractive idea that the observed magnetic fields are also attributed to inflation. Although many models in which magnetic fields are generated during inflation are proposed so far [11, 12, 13, 14, 15, 16], these “inflationary magnetogenesis” models suffer from several problems. It is known that the strong coupling problem, the back reaction problem and the curvature perturbation problem spoil inflationary magnetogenesis models [17, 18, 20, 19, 21]. In the following section, we will explain these problems.

Inflationary magnetogenesis targets the magnetic field that is stronger than the blazar lower bound  $B_{\text{void}} \gtrsim 10^{-15}\text{G}$  because the void magnetic field is not amplified after reheating and reflects the primordial amplitude. \* Although  $10^{-15}\text{G}$  is very weak in comparison with, for example, earth’s magnetism ( $\sim 0.2 - 0.7\text{G}$ ), an remarkably strong magnetic field at the end of inflation is required. It is because the magnetic field dilutes in proportion to  $a^{-2}$  in the expanding universe. Furthermore, on super horizon scales during inflation, the electric field is stronger than the magnetic field which has to grow rapidly against the  $a^{-2}$  dilution. As we will see below, this extremely strong

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\*See, however, ref. [22, 23] in which the inverse cascade is discussed.

electric field makes magnetogenesis difficult.

## 2 Sketch of inflationary magnetogenesis

The basics of inflationary magnetogenesis can be understood by reviewing a model. Let us sketch the kinetic coupling model (or  $IFF$  model) [12] as an example. The model action is

$$S = \int d\eta d^3x \sqrt{-g} \left[ -\frac{1}{4} I^2(\eta) F_{\mu\nu} F^{\mu\nu} \right], \quad (F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu), \quad (1)$$

where  $\eta$  is the conformal time,  $A_\mu$  is a gauge field and  $I(\eta)$  is originally considered as a function of a scalar field but is treated as a function of time. To solve the EoM of  $A_\mu$  analytically,  $I(\eta)$  is usually assumed as

$$I(\phi) = \begin{cases} (\eta/\eta_f)^n & (\eta < \eta_f) \\ 1 & (\eta \geq \eta_f) \end{cases}, \quad (2)$$

where “f” denotes the end of inflation and  $n$  is a constant. Without a time variation of  $I(\eta)$ , no fluctuation of  $A_\mu$  would not be generated because of the conformal invariance [11]. The EoM of  $A_i$  is given by

$$\left[ \partial_\eta^2 + k^2 - \frac{n(n-1)}{\eta^2} \right] (I\mathcal{A}_k) = 0, \quad (3)$$

where  $\mathcal{A}_k$  is the mode function of  $A_i$  expanded by the polarization vectors. If  $n < 0$ ,  $I(\eta) \ll 1$  during inflation and loop effects due to the coupling to charged fermions cannot be ignored. Then a reliable calculation is hardly done. It is known as the strong coupling problem [17]. Thus we choose  $n > 0$  and obtain the solution on super-horizon scale:

$$|I\mathcal{A}_k(\eta)| = \frac{\Gamma(n-1/2)}{\sqrt{2\pi k}} \left( \frac{-k\eta}{2} \right)^{1-n}, \quad \left( -k\eta \ll 1, n > \frac{1}{2} \right). \quad (4)$$

In the expanding universe, the power spectra of electromagnetic fields are given by

$$\mathcal{P}_E(\eta, k) \equiv \frac{k^3 |\partial_\eta \mathcal{A}_k|^2}{\pi^2 a^4}, \quad \mathcal{P}_B(\eta, k) \equiv \frac{k^5 |\mathcal{A}_k|^2}{\pi^2 a^4}. \quad (5)$$

It should be noted that the magnetic field is diluted in proportion to  $a^{-2}$ . Substituting eq. (4) into eq. (5), one finds that the resultant magnetic field at present is

$$\mathcal{P}_B^{1/2}(\eta_{\text{now}}, k) = \frac{\Gamma(n-\frac{1}{2})}{2^{\frac{3}{2}-n} \pi^{\frac{3}{2}}} (a_f H)^{n-1} k^{3-n} \sim 10^{23n-80} \text{G} \times \left( \frac{\rho_{\text{inf}}^{1/4}}{10^{16} \text{GeV}} \right)^{n-1} \left( \frac{k}{1 \text{Mpc}^{-1}} \right)^{3-n}, \quad (6)$$

in the case of the instant reheating. Here,  $\rho_{\text{inf}}$  is the inflation energy scale. Therefore  $n \gtrsim 3$  is necessary to produce the magnetic field with the sufficient strength,  $B(\eta_{\text{now}}) > 10^{-15} \text{G}$  at 1 Mpc scale.

### 3 Problems

#### 3.1 Back reaction problem

In the previous section, we assume that inflation continues regardless of the generation of the electromagnetic fields. However, if the energy density of the electromagnetic fields overtakes that of inflaton, the inflation dynamics and/or the generation of electromagnetic fields are altered [17]. This is the so-called back reaction problem.

Before evaluating the electromagnetic energy density, it is important to realize that, on super-horizon scales, the electric field is much stronger than the magnetic field in the kinetic coupling model:

$$\left| \frac{\mathcal{P}_E}{\mathcal{P}_B} \right| = \left| \frac{\partial_\eta \mathcal{A}_k}{k \mathcal{A}_k} \right|^2 \simeq \frac{1}{|k\eta|^2} = e^{2N_k} \gg 1, \quad (\text{super horizon}) \quad (7)$$

where  $N_k \equiv -\ln |k\eta|$  is the e-folding number of  $k$  mode. Thus we can focus on the electric field. Its energy density at the end of inflation is given by

$$\rho_{\text{em}}(\eta_f) \simeq \frac{1}{2} \int_{k_{\text{min}}}^{a_f H} \frac{dk}{k} \mathcal{P}_E(\eta_f, k) \simeq H^4 \left[ \frac{e^{2(n-2)N_{\text{tot}}} - 1}{2n - 4} \right], \quad (8)$$

where  $N_{\text{tot}}$  is *not* the total e-folds of inflation *but* the total e-folds of magnetogenesis (i.e. the time duration where  $I(\eta) \propto \eta^n$ ) and  $k_{\text{min}}$  is the mode which exits the horizon at the onset of magnetogenesis.  $H$  is the Hubble parameter during inflation. Note we drop a numerical factor for simplicity. One can see that for  $n > 2$ ,  $\rho_{\text{em}}$  becomes huge due to the IR contribution.

Demozzi et al. [17] show that by requiring  $\rho_{\text{em}} < \rho_{\text{inf}}$ , the magnetic field produced in the kinetic coupling model with the power-law kinetic function,  $I(\eta) \propto \eta^n$ , cannot exceed  $10^{-32}\text{G}$  today. <sup>†</sup> It is far smaller than the observational lower bound. Although their result is striking, it does not mean inflationary magnetogenesis is generally excluded because their analysis is based on the specific model. In ref. [18], nevertheless, the authors conduct a model independent argument in which the strong coupling problem and the back reaction problem are taken into account. They derive an universal upper bound on the inflation energy scale:

$$\rho_{\text{inf}}^{1/4} < 6 \times 10^{11} \text{GeV} \left( \frac{B(\eta_{\text{now}})}{10^{-15}\text{G}} \right)^{-2}. \quad (9)$$

Therefore the back reaction problem implies that low energy inflation is favored. In addition, this constraint can be translated into the bound on the tensor-to-scalar ratio:  $r < 10^{-19} (B/10^{-15}\text{G})^{-8}$ . Thus if the background gravitational waves are detected in the future, inflationary magnetogenesis is excluded.

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<sup>†</sup>They assume  $N_{\text{tot}} = 75$  and  $H = 10^{-6} M_{\text{Pl}}$ .

### 3.2 Curvature perturbation problem

The curvature perturbation problem refers that inflationary magnetogenesis can be constrained due to the curvature perturbation induced by the generated electromagnetic fields [19]. The electromagnetic fields produced during inflation behave as isocurvature perturbations and source the adiabatic perturbation [20, 21]:

$$\zeta^{\text{em}}(t, \mathbf{x}) = -\frac{2H}{\epsilon\rho_{\text{inf}}} \int_{t_0}^t dt' \rho_{\text{em}}(t', \mathbf{x}), \quad (10)$$

where  $t$  is the cosmic time and  $\epsilon$  is the slow-roll parameter. Regarding the curvature perturbation, not only the amplitude of the power spectrum  $\mathcal{P}_\zeta$  but also the non-linearity parameters,  $f_{\text{NL}}$  and  $\tau_{\text{NL}}$ , are observationally measured. Then those parameters induced by the electromagnetic fields should not exceed the observed values:

$$\mathcal{P}_\zeta^{\text{obs}} > \mathcal{P}_\zeta^{\text{em}}, \quad f_{\text{NL}}^{\text{obs}} > f_{\text{NL}}^{\text{em}}, \quad \tau_{\text{NL}}^{\text{obs}} > \tau_{\text{NL}}^{\text{em}}. \quad (11)$$

Considering  $\mathcal{P}_\zeta^{\text{obs}} > \mathcal{P}_\zeta^{\text{em}}$  in a model independent way, the authors in ref. [20] put the lower bound on the inflation energy scale:

$$\rho_{\text{inf}}^{1/4} > 3 \times 10^{13} \text{GeV} \left( \frac{B(\eta_{\text{now}})}{10^{-15} \text{G}} \right)^{1/2}. \quad (12)$$

Apparently, combined with eq. (9), this constraint eliminates inflationary magnetogenesis models in general. Nonetheless it should be noted in ref. [20] the authors assume that inflation is single slow-roll, the correlation length of the void magnetic field is 1 Mpc at present and the amplitudes of  $\mathcal{P}_\zeta$  of the minimal scale of inflation is same as that of the CMB scale,  $\mathcal{P}_\zeta(k_{\text{CMB}}) = \mathcal{P}_\zeta(k_{\text{max}})$ .

On the other hand, without these assumptions,  $\mathcal{P}_\zeta^{\text{em}}$ ,  $f_{\text{NL}}^{\text{em}}$  and  $\tau_{\text{NL}}^{\text{em}}$  are explicitly calculated and compared with the Planck result [24] under the framework of the kinetic coupling model in ref. [21]. Interestingly, it is found that the constraint from  $\tau_{\text{NL}}$  is the most stringent in the single slow-roll inflation case while the bound from the back reaction problem become the tightest when the single slow-roll assumption is relaxed (see fig.1). Unfortunately, in both cases, the allowed strength of the magnetic field is far smaller than the observational lower limit.

## 4 Summary and discussion

Since the magnetic fields in the universe are observed and their properties are constrained ( $B_{\text{gal}} \sim 10^{-6} \text{G}$ ,  $B_{\text{void}} \gtrsim 10^{-15} \text{G}$ ), theoretical attempts to explain their origin are strongly motivated. However, in spite of longstanding efforts and numerous papers, a successful quantitative model of magnetogenesis is not yet established.

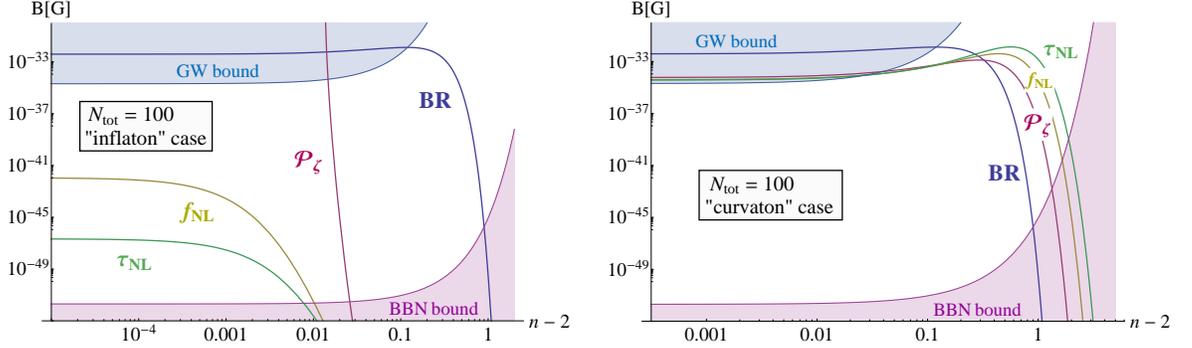


Figure 1: The upper limit of the current strength of the magnetic field for  $n \geq 2$  in the kinetic coupling model. It is assumed that inflaton generates all observed curvature perturbation in the left panel while that assumption is relaxed and instead  $\epsilon = 10^{-2}$  is adopted in the right panel. The total duration of the electromagnetic field generation is set as  $N_{\text{tot}} = 100$ . The shaded regions represent the restriction from gravitational wave (blue) and big bang nucleosynthesis (red), respectively.

In this paper, we explore inflationary magnetogenesis where the electromagnetic fields are generated during inflation. The idea that inflation produces the primordial magnetic field as well as the density perturbation looks natural. However, as we discussed above, inflationary magnetogenesis suffers from several problems and no promising model is known so far.

To determine whether inflationary magnetogenesis is possible or not, two ways can be considered. One is building a viable model and explicitly proving its existence. The other is conducting a model independent argument which generally constrains the possibility or gives a guidance for model building. As the general discussion of the strong coupling and back reaction problem [18] implies that low energy inflation is favored for magnetogenesis, a new general argument will provide a novel insight. For example, it seems that a model independent argument of the curvature perturbation problem without the assumptions can be made. <sup>‡</sup>

Note, we presume that the void magnetic field is generated purely during inflation and no additional amplification occurs. However, there is a chance that the magnetic field is amplified during reheating era or its dilution due to the cosmic expansion is partially compensated by the inverse cascade. Therefore even if *pure* inflationary magnetogenesis is excluded, the inflationary origin of the cosmic magnetic field combined with post-inflation dynamics may be viable.

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<sup>‡</sup>See ref. [25]

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