Problems of determination of $\sigma_{\text{tot}}$ at the LHC

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The analysis of the procedure of the determining the parameters of the hadron elastic scattering amplitude at high energy is presented. The exponential and non-exponential form of the scattering amplitude are taken into account. Especially, the impact of the real parts of the hadron scattering amplitude and electromagnetic hadron interaction on the determination of the total cross section is examined.

The experimental data obtained by the TOTEM collaboration at 7 TeV $^1$ do not coincide with the predictions of all theoretical models $^2$. The procedure of the extracting the parameters of the scattering amplitude from the data of the differential cross sections assumes some theoretical assumption and approximations $^3$. In $^4$, as an example, it was shown that the saturation regime, which can occur at the LHC energies, changes the behavior of the slope of the differential cross sections at small momentum transfer. As a result, the differential cross section cannot be described by a simple exponential form with the constant slope. Another example, in the $p\bar{p}$ scattering at $SPS$ there are two different measures of the size of $\rho$: $\rho = 0.24$ and $\rho = 0.139$. More careful analysis gave $\rho = 0.19$ for the data of the UA4 Collaboration $^5$ and $\rho = 0.16$ for the data of the UA4/2 Collaboration $^6$.

In our talk, we present some analysis of the new experimental data obtained by the TOTEM Collaboration at $\sqrt{s} = 7$ TeV at small momentum transfer. In all cases, we take into account all 5 spiral electromagnetic amplitudes and take into account the Coulomb-hadron interference phase $^7$. The hadron spin non-flip amplitude was chosen in the form with the possible non-exponential form

$$F(s, t) = (i + \rho) \frac{\sigma_{\text{tot}}}{4k\pi} \left[ B/2 + t^2/C/2 \left( \sqrt{q^2 - t} - 2\mu \right) \right]$$

where $k = 0.38938$ mb/GeV$^2$, $t = -q^2$, and $C$ GeV$^{-1}$ is some coefficient which reflects some additional part of the slope. We will examine the set of the TOTEM data at small $t$ with $N = 47$ points and $-t_{\text{max}} = 0.112$ GeV$^2$. The whole set ($N = 86$ points and $-t_{\text{max}} = 0.3$ GeV$^2$) will be examined only as an example. This interval of momentum transfer is large and the imaginary part of the scattering amplitude has some complicated form. In all our calculations we used only statistical errors.

Let us make the fit of the differential cross sections with the hadronic amplitude in form $^8$ and not take into account the electromagnetic interactions. We obtain the large $\sum_{i=1}^{N} \chi_i^2$ and the size of $\sigma_{\text{tot}} = 98.7$ mb. That is practically the same, as was obtained by the TOTEM Collaboration. The result does not feel the size of $\rho$. Now let us make the same fit but include the electromagnetic part of the elastic scattering amplitude. The first 4 rows of Table 1 present the fit without the additional part of the slope. In this case, the influence of the size of $\rho$ is visible and we can make the fit taking $\rho$ as free parameters (the row 4). We obtained minimum $\chi^2$ with the negative size of $\rho$. It is essentially far away from the TOTEM analysis and the predictions of the COMPETE Collaboration.
However, the size of $\sigma_{tot}$ is the same as in the previous case in the region of errors. If we take the additional part of the slope (the rows 5-7 of Table 1), the error of $\rho$ increases and its size is badly determined. But the size of the coefficient $C$ is determined especially with the fixed size of $\rho$. In this case, the size of $\sigma_{tot}$ increases with middle value 99.7 mb but, of course, with a large error.

In Fig. 1a, the dependence of the size of $\sigma_{tot}$ over $\rho$ (left picture) in the case without and with the contributions of the Coulomb and Coulomb-hadron interference terms is shown. The inclusion of the Coulomb dependence terms leads to an increase of $\sigma_{tot}$ at large $\rho$ and decrease in the case of small and negative $\rho$.

Now let us fix additional normalization of the experimental data by the middle value $1/n$, with $n = 1.05$. In this case, we can take the size of $\sigma_{tot}$ as a free parameter. The results are shown in Table 2. If $C = 0$ (first 5 rows), the size of $\sigma_{tot}$ is less than the determined by the TOTEM Collaboration. Again, we can see that if we take $C = 0$ and free $\rho$ and $\sigma_{tot}$, we obtain the negative size of $\rho$. If we take as free parameters $\rho$, $C$ and $\sigma_{tot}$, we obtain $\rho$ near zero.

Let us check up our assumptions about the size and $t$ dependence of the real part of the scattering amplitude. We can use the method which was proposed and explored in [8, 9]. Let us introduce the value

$$\Delta_R(t) = (ReF_C(t) + ReF_H(s,t))^2 = \left[\frac{d\sigma}{dt}\right]_{exp}/n - k\pi \times (ImF_C + ImF_H)^2/(k\pi).$$

(2)

For the $pp$ high energy scattering the real part is positive at small $t$ and the Coulomb amplitude is negative. Hence, the $\Delta_R$ will have the minimum at some point of $t$ and then a wide maximum.

Let us take the parameters obtained by the TOTEM Collaboration $\sigma_{tot} = 98.6$ mb, $B = 19.9$ GeV$^{-2}$, $n = 1$, $\rho(0) = 0.14$ and calculate the value $\Delta_R$, using the left part of eq. (2). The result is shown in Fig.1b by the hard line. Now let us take these parameters for the imaginary part and calculate $\Delta_R$ using the experimental data in the right part of eq. (2). The triangles in Fig.1b present these calculations. The first and second calculations are very far from each other. If we take the real part with the parameters $\sigma_{tot} = 96.4$ mb, $B = 19.9$ GeV$^{-2}$, $\rho = 0.1$ and calculate $\Delta_R$ (short dashed line in Fig.1b), the position of the minimum moves to a higher $t$, but the difference remains large. If we take some other

<table>
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<th>$\sum \chi^2_i$</th>
<th>$\rho$</th>
<th>$C$</th>
<th>$\sigma_{tot}$, mb</th>
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<td>0.74 ± 0.8</td>
<td>99.7 ± 0.8</td>
</tr>
</tbody>
</table>

Table 2: The basic parameters of the model are determined by fitting experimental data with $n = 1.05$.
parameters for the imaginary part: \( \sigma_{\text{tot}} = 96.4 \text{ mb}, B = 20.3 \text{ GeV}^{-2}, n = 1.08, C = -0.05 \text{ GeV}^{-1} \), we obtained from eq. (2) the result which is shown in Fig.1b by circles. The contribution of only the Coulomb amplitude \((\rho = 0)\) in \(\Delta R\) is shown in Fig.1b by the long dashed line and the dotted line represents the calculation with \(\rho = -0.05\). We can see that only in the last case the difference between the calculations by eq. (2) (left part) and by eq. (2) (right part) is not large.

The analysis of the new experimental data obtained by the LHC TOTEM Collaboration \([1]\) shows that there are some additional specific moments which are to be taken into account to determine the size of \(\sigma_{\text{tot}}\). We cannot neglect the electromagnetic interactions. It is needed to check out the obtained, during the fitting procedure, real part of the scattering amplitude by using eq. (2). Maybe, there is some problem with the normalization of the separate parts of the experimental data, or there exists some additional (probably oscillation) term (see \([10]\)) which changes the form of the imaginary part. Finally, we should note that the best way to decrease the impact of the different assumptions, which are examined in the phenomenological model, consists in the determination of the sizes of \(\sigma_{\text{tot}}\) and \(\rho(s, t)\) simultaneously in one experiment.

![Figure 1: Size of \(\sigma_{\text{tot}}\) a)(left) over \(\rho\); b) (right) the calculations of \(\Delta R\).](image)

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**References**