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Lattice calculation
of
 D_s to $\eta^{(\prime)}$ semi-leptonic decay form factors

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We report lattice results of D_s meson semi-leptonic decay form factors to η and η' mesons. This decay process contains disconnected fermion loops, which are challenging in lattice calculations. Our result shows that the disconnected loops give significant contributions to the form factors.

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1 Introduction

Among semi-leptonic decays of charmed mesons, decays of D -mesons, such as $D \rightarrow l\bar{\nu}_l K$, are well studied both in experiment and in lattice calculations. The high precision measurements from experiment combined with high precision calculations of the relevant form factors from the lattice allows us to extract the value of the corresponding CKM matrix element. When it comes to charmed mesons with a strange quark, $D_s(\bar{D}_s)$, the situation is different, however. The major semi-leptonic mode is $D_s \rightarrow l\bar{\nu}_l\eta^{(\prime)}$ and only the branching fractions are available from the experiment. On the theory side, no lattice results have been published so far, while predictions from light cone QCD sum rules [1, 2] are available. These decays are interesting for flavor physics because of η - η' mixing and the investigation of these modes helps to understand the mixing angle and also possible gluonic contributions [3]. For decays involving the η' , the chiral anomaly should play some role and it is an interesting play ground for obtaining deeper understanding of the quantum field theory.

The relevant matrix element for these decay modes is

$$\langle \eta^{(\prime)}(k) | V^\mu(q^2) | D_s(p) \rangle = f_+(q^2) \left[(p+k)^\mu - \frac{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}{q^2} q^\mu, \quad (1)$$

where V^μ is a vector current and M_{D_s} and $M_{\eta^{(\prime)}}$ are the masses of the D_s and $\eta^{(\prime)}$, respectively. This matrix element is characterized by two form factors, $f_0(q^2)$ and $f_+(q^2)$. So far, we have focused on the scalar form factor $f_0(q^2)$, which we can also obtain from a scalar current $S = \bar{s}c$,

$$f_0(q^2) = \frac{m_c - m_s}{M_{D_s}^2 - M_{\eta^{(\prime)}}^2} \langle \eta^{(\prime)} | S | D_s \rangle. \quad (2)$$

We use this relation because the combination $(m_c - m_s)S$ is protected from renormalization due to the partially conserved vector current [4].

The three point function for the above matrix element contains fermion disconnected loops shown pictorially bellow (the second term is the disconnected part):

$$\langle \eta^{(\prime)}(\vec{k}) | S(\vec{q}) | D_s(\vec{p}) \rangle = \text{Diagram 1} - \sum_{l=u,d,s} \left(\text{Diagram 2} \right). \quad (3)$$

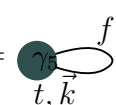
Here, only the valance quark lines are drawn and all possible gluon lines and sea quark loops are suppressed. The calculation of disconnected loops is expensive and the signal is noisy (but feasible [5]), so if the contributions were small it might be a good approximation to neglect them. This is not the case for the matrix element above, however. The disconnected loops should be summed over three light flavors which

enhances the magnitude roughly by a factor three. They contain the contributions from the chiral anomaly to the η' -meson. For these reasons, the disconnected loops may contribute significantly.

2 Lattice Setup

We use QCDSF $n_f = 2 + 1$ configurations [6, 7], which were generated with the tree level Symanzik improved gluon action and non-perturbative stout link improved clover fermion action. Although we neglect charm sea quarks but included relativistic valance charm quark. The charm quark mass was tuned to reproduce the spin averaged physical charmonium mass $M_{\overline{1S}}$. The u -, d - and s - quark masses are tuned so that their average $\frac{1}{3}(m_u + m_d + m_s)$ is fixed and that $2M_K^2 + M_\pi^2$ coincides with the physical values. Due to this strategy of choosing light quark masses, these configurations are particularly suitable for studying flavor physics using the SU(3) flavor basis. So far we have analyzed two sets of configurations, one is at the SU(3) flavor symmetric point ($m_{u,d} = m_s$) with $M_\pi \simeq 450$ MeV ($N_{\text{conf}} = 939$), and the other has lighter $m_{u,d}$ and heavier m_s , which gives $M_\pi \simeq 348$ MeV ($N_{\text{conf}} = 239$). Both sets have the lattice spacing $a \sim 0.08$ fm, and the lattice size $24^3 \times 48$ gives the physical extent $L \sim 1.9$ fm.

The most computationally expensive quantity to calculate is the fermion one point loop,

$$C_{\text{1pt}}^f(t, \vec{k}) = \sum_{\vec{x}} \exp(i\vec{k} \cdot \vec{x}) \text{tr} \left[\sum_{\vec{x}', \vec{x}''} \gamma_5 \phi(\vec{x}, \vec{x}'') M_f^{-1}(t, \vec{x}''; t, \vec{x}') \phi(\vec{x}', \vec{x}) \right] = \text{loop diagram}, \quad (4)$$


where M_f is the Dirac operator for flavor f and $\phi(\vec{x}, \vec{x}')$ is a smearing function to make the wave function have finite spacial extent. Essentially, this is the trace of the inverse Dirac operator, which is a $O(10^7) \times O(10^7)$ matrix. A direct calculation is practically impossible. We use a stochastic method, which relies on an approximate completeness relation of random noise vectors $|n_i\rangle$: $\frac{1}{N} \sum_{i=1}^N |n_i\rangle \langle n_i| = 1 + O(\frac{1}{\sqrt{N}})$. To reduce the error from the $O(\frac{1}{\sqrt{N}})$ term, we combined several noise reduction techniques [8, 5].

3 Extracting η and η' states

To calculate the matrix element (3), we first need to build the interpolating operators to create the D_s meson and annihilate the $\eta^{(\prime)}$ meson. The operator for D_s is easy to obtain but ones for $\eta^{(\prime)}$ are non-trivial due to the mixing.

We start with SU(3) octet-singlet basis, $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and $\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$. We can build the following 2×2 two-point correlation function

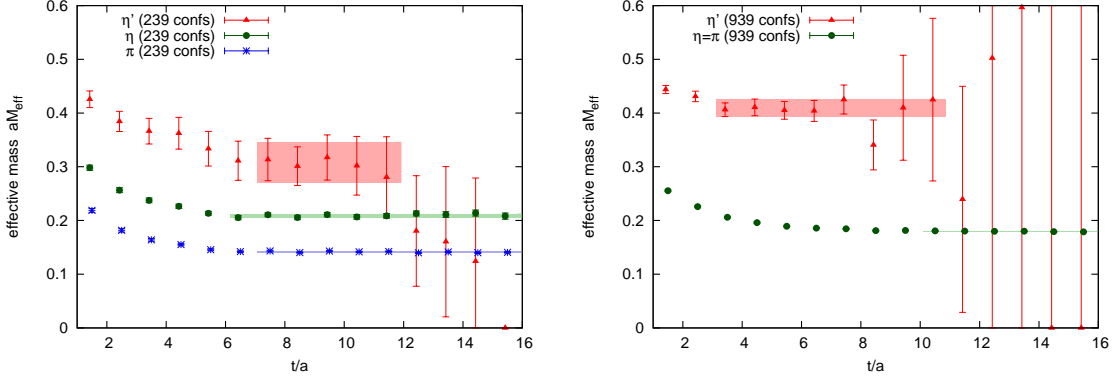


Figure 1: Effective mass plot of η' and η . For comparison, the effective mass for π is also plotted. Left panel: $M_\pi = 348$ MeV case. Right panel: $M_\pi = 450$ MeV case (SU(3) flavor symmetric), where $M_\pi = M_\eta = M_{\eta_8}$ and $M_{\eta'} = M_{\eta_1}$.

by using smeared interpolating operators, \mathcal{O}_8 for the octet and \mathcal{O}_1 for the singlet:

$$C_2(t; \vec{k}) = \begin{pmatrix} \langle \mathcal{O}_8(t; \vec{k}) \mathcal{O}_8^\dagger(0) \rangle & \langle \mathcal{O}_8(t; \vec{k}) \mathcal{O}_1^\dagger(0) \rangle \\ \langle \mathcal{O}_1(t; \vec{k}) \mathcal{O}_8^\dagger(0) \rangle & \langle \mathcal{O}_1(t; \vec{k}) \mathcal{O}_1^\dagger(0) \rangle \end{pmatrix}. \quad (5)$$

Here, t is the sink-source time separation and the operator at the sink is projected to momentum \vec{k} . Each element of eq. (5) contains both connected and disconnected fermion contributions.

Two point functions in the physical η and η' basis are obtained by diagonalizing eq. (5). The eigenvectors, which can be parameterized one mixing angle θ , give the interpolating operators for η and η' :

$$\mathcal{O}_\eta = \cos \theta \mathcal{O}_8 - \sin \theta \mathcal{O}_1, \quad \mathcal{O}_{\eta'} = \sin \theta \mathcal{O}_8 + \cos \theta \mathcal{O}_1. \quad (6)$$

The mixing angle above corresponds to a mixing between the interpolating operators and is not the mixing of the physical observables. The resulting two point correlators have the functional form:

$$C_{2\text{pt}}^{\eta^{(\prime)}}(t, \vec{k}) = \frac{|Z_{\eta^{(\prime)}}(\vec{k})|^2}{2E_{\eta^{(\prime)}}(\vec{k})} e^{-E_{\eta^{(\prime)}}(\vec{k})t} + (\text{contribution from excited states}). \quad (7)$$

We can obtain the energy $E_{\eta^{(\prime)}}$ and overlapping factor $Z_{\eta^{(\prime)}} = \langle \eta^{(\prime)} | \mathcal{O}_{\eta^{(\prime)}} | 0 \rangle$ by fitting the two point function. We can even obtain the energy gap to the first excited state in some cases. Fig. 1 shows the effective mass $aM_{\text{eff}}(t + \frac{a}{2}) = -\ln \frac{C_{2\text{pt}}(t+a)}{C_{2\text{pt}}(t)}$ (or $\frac{C_{2\text{pt}}(t+a)}{C_{2\text{pt}}(t)} = \frac{\cosh[aM_{\text{eff}}(t+a/2)(t+a-T/2)]}{\cosh[aM_{\text{eff}}(t+a/2)(t-T/2)]}$ with the finite temporal size T) at $\vec{k} = \vec{0}$, together with the fitted value of the mass.

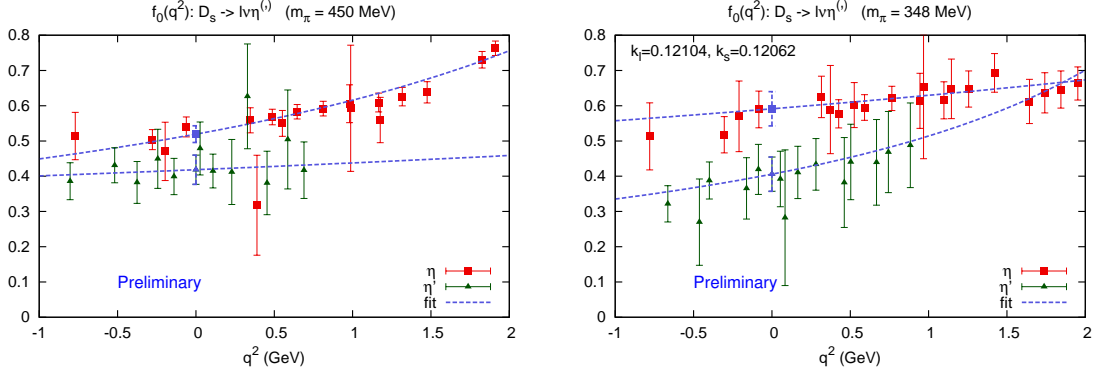


Figure 2: Preliminary results for the scalar form factor $f_0(q^2)$, at the SU(3) flavor symmetric point (left panel) and $M_\pi = 348$ MeV point (right panel).

4 Results

Having obtained the interpolating operators (6), we can construct the three point function needed for the form factor:

$$\begin{aligned}
 C_{3\text{pt}}(t, \vec{p}, \vec{q}, \vec{k}) &= \langle 0 | \mathcal{O}_{\eta^{(\prime)}}(\vec{k}, t_{\text{sep}}) S(\vec{q}, t) \mathcal{O}_{D_s}^\dagger(\vec{p}, 0) | 0 \rangle \\
 &= \frac{Z_{\eta^{(\prime)}}}{2E_{\eta^{(\prime)}}} \frac{Z_{D_s}}{2E_{D_s}} \exp[-E_{D_s}t - E_{\eta^{(\prime)}}(t_{\text{sep}} - t)] \times \left[\langle \eta^{(\prime)}(\vec{k}) | S(\vec{q}) | D_s(\vec{p}) \rangle + \dots \right],
 \end{aligned} \tag{8}$$

where t_{sep} is the sink-source separation. The factors in front of the matrix element $\langle \eta^{(\prime)}(\vec{k}) | S(\vec{q}) | D_s(\vec{p}) \rangle$ can be obtained by fitting the two point functions. Thus, we can obtain the scalar form factor $f_0(q^2)$ by using eq. (2). The results are plotted in Fig. 2, where the errors were calculated with jackknife analysis. We fit the results with one pole functions, $f_0(q^2) = f_0(0)/(1 - bq^2)$. The preliminary values at $q^2 = 0$ are $f_0^{D_s \rightarrow \eta}(0) = 0.52(2)$ and $f_0^{D_s \rightarrow \eta'}(0) = 0.42(4)$ for the SU(3) symmetric case, $f_0^{D_s \rightarrow \eta}(0) = 0.59(5)$ and $f_0^{D_s \rightarrow \eta'}(0) = 0.41(5)$ for the $M_\pi = 348$ MeV case.

To extract the matrix element between the ground state of D_s and $\eta^{(\prime)}$, it is important to remove the excited contributions. After removing the leading exponential factors by taking a ratio, we have

$$\begin{aligned}
 R(t) &\equiv \frac{C_{3\text{pt}}(t, \vec{p}, \vec{q}, \vec{k})}{\frac{Z_{\eta^{(\prime)}}}{2E_{\eta^{(\prime)}}} \frac{Z_{D_s}}{2E_{D_s}} \exp(-E_{D_s}t - E_{\eta^{(\prime)}}(t_{\text{sep}} - t))} \\
 &= \langle \eta^{(\prime)}(\vec{k}) | S(\vec{q}) | D_s(\vec{p}) \rangle + A_1 \exp(-\Delta E_{D_s}t) + B_1 \exp(-\Delta E_{\eta^{(\prime)}}(t_{\text{sep}} - t)) + \dots,
 \end{aligned} \tag{9}$$

where ΔE_{D_s} and $\Delta E_{\eta^{(\prime)}}$ are the energy gaps to the first excited state. If t_{sep} and t were large enough, the residual pollution from the excited states would be exponentially

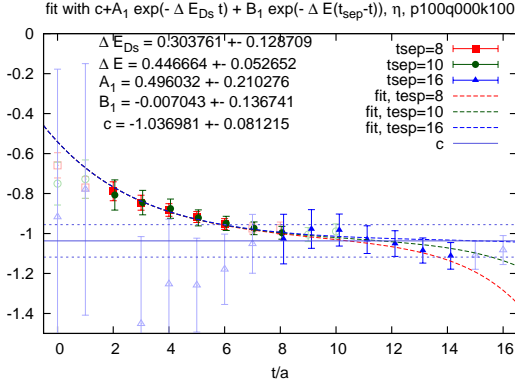


Figure 3: Fit of $R(t)$ with excited contributions from D_s and η . $D_s(\vec{p} = (1, 0, 0))$ is located at $t/a = 0$ and $\eta(\vec{k} = (1, 0, 0))$ is at $t/a = 8, 10$ and 16 . The data points with open symbols were not used in the fitting.

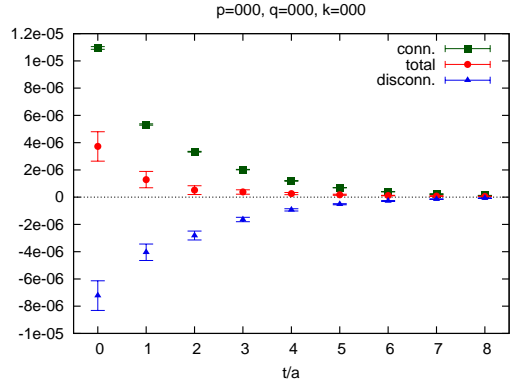


Figure 4: The connected, disconnected, and total contributions to $C_{3pt}(t)$ with $t_{sep} = 8$ for $D_s \rightarrow l\bar{\nu}\eta'$ at SU(3) symmetric point. D_s and η' are located at $t/a = 0$ and 8 , respectively.

small and negligible. We can not use such large t_{sep} and t , however, because the statistical error would increase and we would not obtain meaningful signals. We instead fit the ratios (9) with three terms in the r.h.s. simultaneously with three different $t_{sep}/a = (8, 10, 16)$ (for η in the SU(3) symmetric point, we also used $t_{sep}/a = 24$), using the energy gaps obtained from the two point functions. If we were not able to extract $\Delta E_{\eta^{(\nu)}}$ from the two point function, we only used the first two terms in eq. (9). A typical example of the fitting is shown in Fig.3. The plot shows that the ratio is well described with the fit function.

It is interesting to note that the disconnected contributions are really large in the $D_s \rightarrow \eta'$ three point function (Fig. 4). The disconnected part has almost the same magnitude as the connected part but the opposite sign, so the total three point function becomes much smaller in the magnitude than the connected contribution. Fig. 4 also implies that the error is mainly from the disconnected part.

5 Conclusions

We gave the preliminary results of a lattice calculation of semi-leptonic decay form factors for $D_s \rightarrow l\bar{\nu}\eta$ and $D_s \rightarrow l\bar{\nu}\eta'$, including the disconnected fermion loop contributions. This is the first lattice result for these form factors. The disconnected contributions to the decay $D \rightarrow l\bar{\nu}\eta'$ are significant. the decay into η' . We were able to obtain the scalar form factor $f_0(q^2)$ at $q^2 = 0$ in 10–15% statistical error.

We are planning to calculate the other form factor $f_+(q^2)$, which requires a renor-

malization factor. Another aim is to calculate form factors decaying into ϕ , for which a rigorous treatment requires disconnected contributions. The mixing of η and η' , and its quark mass dependence are also interesting.

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