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Theoretical Interpretations of the XYZ Mesons

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The XYZ mesons are unexpected mesons containing a heavy quark-antiquark pair that have been discovered during the last decade. The models for the XYZ mesons that have been proposed include conventional quarkonium, quarkonium hybrids, and quarkonium tetraquarks, whose four constituents can be clustered in several possible ways. None of the models have provided a compelling pattern for the XYZ mesons. The most promising theoretical approaches within QCD are lattice QCD for the $c\bar{c}$ mesons, lattice NRQCD for the $b\bar{b}$ mesons, and the Born-Oppenheimer approximation. The additional hints that will be provided by future experiments guarantee the eventual solution of the XYZ puzzle.

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1 The XYZ Mesons

The XYZ mesons are unexpected mesons discovered during the last decade that contain a heavy quark-antiquark pair and are above the open-heavy-flavor threshold. Some of the more surprising of these XYZ mesons were

- $X(3872)$, discovered by the Belle Collaboration in 2003 [1]. It has comparable branching fractions into $J/\psi \rho$ and $J/\psi \omega$, implying a severe violation of isospin symmetry.
- $Y(4260)$, discovered by the BaBar Collaboration in 2005 [2]. It has J^{PC} quantum numbers 1^{--} , but it is produced very weakly in e^+e^- annihilation.
- $Z_b^+(10610)$ and $Z_b^+(10650)$, discovered by the Belle Collaboration in 2011 [3]. They both decay into $\Upsilon \pi^+$, revealing that they are tetraquark mesons with constituents $b\bar{b}u\bar{d}$.
- $Z_c^+(3900)$, discovered by the BESIII Collaboration in 2013 [4]. It decays into $J/\psi \pi^+$, revealing that it is a tetraquark meson with constituents $c\bar{c}u\bar{d}$.

Updated lists of the XYZ mesons in both the $c\bar{c}$ and $b\bar{b}$ sectors are given in Ref. [5]. The list of new $c\bar{c}$ mesons above the $D\bar{D}$ threshold includes 15 neutral and 5 charged mesons. The list of new $b\bar{b}$ mesons above the $B\bar{B}$ threshold includes 1 neutral and 2 charged mesons. Many of these mesons are surprisingly narrow. They reveal a serious gap in our understanding of the QCD spectrum.

The XYZ puzzle is the problem of understanding the nature of these new mesons. Most of the theoretical work on the XYZ mesons during the last decade has been carried out using simple-minded models that make no direct contact with QCD. The models that have been proposed are summarized in Section 2. The most promising theoretical approaches to the XYZ mesons within QCD all involve lattice gauge theory and are discussed in Section 3. The extensive information on the XYZ mesons that is already available and the additional hints that will be provided by future experiments almost guarantee the eventual solution of the XYZ puzzle.

2 Constituent Models

Most of the theoretical work on the XYZ mesons has been carried out using models that may be compatible with QCD but make no direct contact with the fundamental field theory. The models can be classified according to their constituents. The basic categories are (1) quarkonium, whose constituents are a heavy quark and a heavy antiquark, (2) quarkonium hybrid, which has an additional gluonic constituent, and (3) quarkonium tetraquark, whose additional constituents are a light quark and anti-quark. The tetraquark category can be subdivided into further categories according

to the way the four constituents are clustered inside the meson. None of the models that has been proposed thus far has provided a compelling pattern for the XYZ mesons that have been observed.

2.1 Conventional quarkonium

The constituents of a conventional quarkonium consist of a heavy quark Q and a heavy antiquark \bar{Q} only. There is a well-developed phenomenology for conventional quarkonium based on potential models. It provides an accurate description of the quarkonium states below the open-heavy-flavor threshold. Its accuracy for states above the open-heavy-flavor has not been tested, but it gives fairly well-defined predictions for their masses and for radiative transition rates to other quarkonium states [6, 7]. The predicted states form orbital-angular-momentum multiplets of states related by heavy-quark spin symmetry: S -wave multiplets with J^{PC} quantum numbers $\{0^{-+}, 1^{-+}\}$, P -wave multiplets $\{1^{+-}, (0, 1, 2)^{++}\}$, D -wave multiplet $\{2^{-+}, (1, 2, 3)^{-+}\}$, etc.

2.2 Quarkonium hybrid

A quarkonium hybrid consists of Q , \bar{Q} , and a gluonic excitation g . There are two important model-independent properties of a quarkonium hybrid:

- The wave function for the $Q\bar{Q}$ pair near the origin is very small. This follows from the fact that when the Q and \bar{Q} pair are close together, they must be in a color-octet state in which the Q and \bar{Q} have a repulsive $1/R$ potential at short distance.
- Decays into two S -wave mesons are suppressed, so decays into an S -wave and a P -wave meson are preferred, provided they are kinematically accessible [8, 9].

There are two rather different physical pictures of a quarkonium hybrid. In the constituent-gluon picture, the excitation of the gluon field is interpreted as a particle with definite J^{PC} quantum numbers. The lowest energy levels of a quarkonium hybrid can be reproduced by taking the constituent gluon to have $J^{PC} = 1^{+-}$, which can be interpreted as a vector particle in a P -wave orbital. If the $Q\bar{Q}$ pair is in an S -wave state, the resulting spin-symmetry multiplet for the hybrid is $\{1^{-+}, (0, 1, 2)^{++}\}$. If the $Q\bar{Q}$ pair is in a P -wave state, there are three spin-symmetry multiplets: $\{0^{++}, 1^{+-}\}$, $\{1^{++}, (0, 1, 2)^{+-}\}$, and $\{2^{++}, (1, 2, 3)^{+-}\}$. The 1^{-+} , 0^{+-} , and 2^{+-} states have exotic quantum numbers that are not possible if the only constituents are Q and \bar{Q} .

There is an alternative picture of a quarkonium hybrid that can be called a *Born-Oppenheimer hybrid*. The excitation of the gluon field is a configuration of chromoelectric and chromomagnetic fields in which the Q and \bar{Q} are embedded and for which

they provide the source. The gluon field energy provides a potential that determines the motion of the Q and \bar{Q} and the gluon field responds almost instantaneously to their motion. The possible gluon field configurations are traditionally labelled by upper-case Greek letters with subscripts and superscripts, such as Π_u , Σ_u^- , etc. The lowest energy level of the hybrid is a P -wave $Q\bar{Q}$ pair in the Π_u field, which consists of spin-symmetry multiplets $\{1^{--}, (0, 1, 2)^{-+}\}$ and $\{1^{++}, (0, 1, 2)^{+-}\}$. Another low-lying energy level is an S-wave $Q\bar{Q}$ pair in the Σ_u^- field, which gives a single spin-symmetry multiplet $\{0^{++}, 1^{+-}\}$. Note that the same spin-symmetry multiplets are obtained as in the constituent gluon picture, but they arise in different ways.

There is an XYZ state that is a compelling candidate for a charmonium hybrid: the $Y(4260)$. It has quantum numbers 1^{--} , which is one of the quantum numbers in the ground-state spin-symmetry multiplet. Despite these quantum numbers, it is produced very weakly in e^+e^- annihilation: the resonance appears as a small peak near a deep minimum in the cross section for e^+e^- annihilation into hadrons. This small production rate in e^+e^- annihilation indicates that the wave function for the $c\bar{c}$ pair at the origin is very small, which is a characteristic property of a quarkonium hybrid. The decay of the $Y(4260)$ into the charm mesons $D\bar{D}$ has not been observed in spite of the large available phase space. This is consistent with the suppression of decays of a hybrid into a pair of S -wave mesons. A convincing case for the $Y(4260)$ as a charmonium hybrid was made by Close and Page in 2005, shortly after its discovery [10]. The recent discovery of the unexpected $Z_c\pi$ decay mode of the $Y(4260)$ by the BESIII Collaboration [4] has not made the case any less compelling.

2.3 Compact tetraquark

A quarkonium tetraquark whose four constituents $Q\bar{Q}q\bar{q}$ all have overlapping spatial wavefunctions is called a *compact tetraquark*. A simple model for their interaction is the quark potential model in which each pair of constituents interacts through a potential. Careful solutions of the 4-body problem reveal that the tetraquark is unstable with respect to fall-apart decays into two mesons unless the mass of the tetraquark is below the thresholds for all pairs of mesons with the appropriate quantum numbers, both a pair of heavy-light mesons $Q\bar{q}$ and $\bar{Q}q$ and also a heavy quarkonium $Q\bar{Q}$ plus a light meson $q\bar{q}$ [11]. Since the XYZ mesons are above the threshold for $Q\bar{q}$ and $\bar{Q}q$ mesons, this implies that they cannot be compact tetraquarks unless the interactions between the constituents are more complicated than pairwise potentials, involving, for example, 3-body and 4-body potentials.

2.4 Meson molecule

A quarkonium tetraquark whose substructure consists of a pair of mesons, which are color-singlet clusters with constituents $Q\bar{q}$ and $\bar{Q}q$, is called a *meson molecule*. In

order to be a plausible constituent for a molecule, a meson must be rather narrow. The nonstrange charm mesons that are plausible constituents are D , D^* , D_0^* , and D_1 . The strange charm mesons that are plausible constituents are D_s , D_s^* , D_{s0}^* , D_{s1} , and D_{s2}^* .

One of the motivations for meson molecules is that many of the XYZ mesons have mass near a threshold for a pair of heavy-light mesons. This could be a coincidence or it could be a consequence of attractive interactions between the mesons that create the molecule. Since there are 25 thresholds between 3770 MeV and 5150 MeV for pairs of mesons with no net strangeness, a randomly chosen mass in this region is likely to be within 50 MeV of some threshold. It is therefore not implausible that the proximity of the masses to thresholds is coincidental.

The possibility that charm mesons could be bound into molecules by their interactions was first studied quantitatively by Tornqvist in 1995 [12]. He considered pion-exchange interactions between the charm mesons with an ultraviolet cutoff to regularize a divergence from short distances. This model predicts no molecules in isospin-1 channels, but it predicts bound states near threshold in several isospin-0 J^{PC} channels: $D^*\bar{D}$ with 0^{-+} and 1^{++} and $D^*\bar{D}^*$ with 0^{++} , 0^{-+} , 1^{+-} , and 2^{++} . Whether or not the molecule is actually bound, with mass below the threshold, depends on the ultraviolet cutoff. The binding energy is also sensitive to additional interactions between the charm mesons, such as the potential from the exchange of ρ , ω , and other mesons. In the case of bottom mesons, the binding energies are larger than for charm mesons by about 50 MeV. The meson pairs $B^*\bar{B}$ and $B^*\bar{B}^*$ should therefore be bound in all the J^{PC} channels listed above. However, the more strongly the mesons are bound, the more likely it is that QCD interactions rearrange their constituents in a way that is not well-approximated by a pair of mesons.

There is one XYZ meson whose identification as a meson molecule is completely unambiguous and this is the $X(3872)$. This follows from the universality of S-wave near-threshold resonances [13]. If a resonance is close enough to a threshold and if it has quantum numbers that allow an S-wave coupling to the threshold, it is transformed by its interactions into a weakly-bound molecule with universal properties that are determined only by its binding energy E_b . One such property is the mean-square separation of its constituents $\langle r^2 \rangle = 1/(4\mu E_b)$, where μ is the reduced mass of the constituents. The universal behavior requires the state to be very close to threshold, within 10 MeV for charm mesons and within 3 MeV for bottom mesons. This criterion is easily satisfied by the $X(3872)$. Precise measurements of its mass by the CDF and Belle Collaborations imply that it is below the $D^{*0}\bar{D}^0$ threshold by 0.3 ± 0.4 MeV [14, 15]. The quantum numbers of the $X(3872)$ have recently been determined definitively by the LHC Collaboration to be 1^{++} [16]. This implies that it has an S-wave coupling to $D^{*0}\bar{D}^0$. Thus, regardless of the source of the attraction between the charm mesons, the $X(3872)$ must be a loosely bound charm meson

molecule whose constituents are a superposition of $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$. The universal prediction for the rms separation of the charm mesons is about 5 fm if the binding energy is 0.3 MeV. Thus the $X(3872)$ has a truly astonishing spatial extent that is probably an order of magnitude larger than that of most hadrons.

Molecular interpretations have been proposed for most of the XYZ mesons. In most cases, the identification as a molecule has not proved to be very predictive. One exception is the $Y(4260)$, which has been proposed as a $D_1\bar{D}$ molecule [17, 18]. Another exception is the $Z_b(10610)$ and $Z_b(10650)$, which have been proposed as $B^*\bar{B}$ and $B^*\bar{B}^*$ molecules, respectively [19].

2.5 Diquark-onium

A quarkonium tetraquark whose substructure consists of diquarks, which are colored clusters Qq and $\bar{Q}\bar{q}$, is called *diquark-onium* [20]. The diquark Qq is usually treated as a particle with anti-triplet color whose total spin can be 0 or 1. The spectrum of diquark-onium is then determined using purely algebraic methods, like in the naive quark model for mesons and baryons. If we only consider the light quarks u and d and if we take the diquarks to be in an S -wave state, the predicted diquark-onium states are degenerate isospin-0 and isospin-1 multiplets for each of six J^{PC} states: two with 0^{++} , two with 1^{++} , one with 1^{+-} , and one with 2^{++} . We can also allow the light quark to be s , and we can consider orbital angular momentum excitations and radial excitations of the diquark pair. Since there is no dynamics limiting the possible quantum numbers, the result is a proliferation of predicted diquark-onium states.

2.6 Hadro-quarkonium

A quarkonium tetraquark whose substructure consists of a compact color-singlet $Q\bar{Q}$ pair to which a light $q\bar{q}$ pair is bound is called *hadro-quarkonium* [21]. Alternatively, it can be regarded as a light $q\bar{q}$ meson bound to a quarkonium. The motivation for hadro-quarkonium is that many of the XYZ mesons have been observed only through a single hadronic transition to a quarkonium and a light meson, such as $J/\psi\omega$. Hadro-quarkonium provides a simple-minded explanation for this fact by essentially assuming that the quarkonium and the light meson are already preexisting in the XYZ meson.

2.7 Born-Oppenheimer tetraquark

A new proposal for the structure of a quarkonium tetraquark that was recently introduced is a *Born-Oppenheimer tetraquark* [22]. It is motivated by the Born-Oppenheimer picture for a quarkonium hybrid, in which the Q and \bar{Q} are embedded

in a gluon field configuration for which they provide the source. The field configuration in a quarkonium hybrid could also include light-quark fields, but it must be a flavor singlet. The proposal for Born-Oppenheimer tetraquarks is based on the observation that the configuration of gluon and light-quark fields in which the Q and \bar{Q} are embedded need not be a flavor singlet. It could instead have isospin 1, in which case the meson is a tetraquark. The color and spatial structure of a Born-Oppenheimer tetraquark is qualitatively different from each of the possibilities discussed in the previous four sections. It will be discussed in more detail in Section 3.3.

3 Theoretical approaches within QCD

A deep understanding of the XYZ mesons must eventually be based on the fundamental field theory of QCD. The most promising approaches to this problem all involve lattice gauge theory in an essential way. They are (1) lattice QCD, which can be applied directly to $c\bar{c}$ mesons, (2) lattice NRQCD, which can be applied to $b\bar{b}$ meson, and (3) the Born-Oppenheimer approximation, which can be applied to both $c\bar{c}$ and $b\bar{b}$ mesons.

3.1 Lattice QCD for $c\bar{c}$ mesons

The mass of the charm quark is small enough that lattice QCD can be applied directly to $c\bar{c}$ mesons using the computational resources that are currently available. Dudek, Edwards, Mathur, and Richards carried out pioneering calculations of the $c\bar{c}$ meson spectrum that included many states above the $D\bar{D}$ threshold [23]. These calculations have been extended by the Hadron Spectrum Collaboration [24]. They used a lattice with $24^3 \times 128$ sites and a spatial lattice spacing of 0.12 fm. They had dynamical light quarks, but the masses of the u and d quarks were unphysically heavy, corresponding to a pion mass of 400 MeV. Their results were not definitive, because they did not carry out the extrapolations to 0 lattice spacing and to the physical u and d quark masses that are necessary to quantify all systematic errors. Nevertheless their results are very impressive. They used the cross correlators of many operators to determine the spectrum of flavor-singlet $c\bar{c}$ mesons. They were able to identify 46 statistically significant states in various J^{PC} channels, with spins as high as 4 and with masses as high as 4.6 GeV. They also demonstrated that they were able to discriminate between charmonium and charmonium hybrids based on the strengths with which they couple to various operators. Their charmonium states filled out complete $1S$, $2S$, $3S$, $1P$, $2P$, $1D$, and $1F$ multiplets. The lowest charmonium hybrids formed the spin-symmetry multiplet $\{1^{--}, (0, 1, 2)^{-+}\}$. The higher charmonium hybrids filled out three additional multiplets: $\{1^{++}, (0, 1, 2)^{+-}\}$, $\{0^{++}, 1^{+-}\}$, and $\{2^{++}, (1, 2, 3)^{+-}\}$. The quantum numbers 1^{-+} , 0^{+-} , and 2^{+-} are exotic.

The ground-state charmonium hybrid includes a 1^{--} state that can be identified with the $Y(4260)$. The results for the masses of charmonium hybrids in Ref. [24] are not definitive, because they were not extrapolated to zero lattice spacing or to the physical u and d quark masses. It is plausible that the mass splitting within spin-symmetry multiplets are less sensitive to these extrapolations than the masses themselves. If this is the case, we can get better predictions for the masses of charmonium hybrids in the ground-state multiplet by taking the mass of the $Y(4260)$ from experiment and the mass splittings from Ref. [24]. The resulting predictions for the masses of the 0^{-+} , 1^{-+} , and 2^{-+} charmonium hybrids are 4173 ± 21 MeV, 4195 ± 23 MeV, and 4312 ± 24 MeV, respectively [22]. It would of course be preferable to have definitive lattice QCD calculations of the hybrid charmonium masses which have been extrapolated to zero lattice spacing and to the physical masses of the light quarks.

3.2 Lattice NRQCD for $b\bar{b}$ mesons

The mass of the bottom quark is too large to apply lattice QCD directly to $b\bar{b}$ mesons with currently available computational resources. Instead it is necessary to use an effective field theory called *NonRelativistic QCD* (NRQCD) in which the b quark is treated nonrelativistically. Lattice NRQCD was used by Juge, Kuti, and Morningstar to calculate the masses of bottomonium hybrid [25]. They used a lattice with $15^3 \times 45$ sites and a spatial lattice spacing of 0.115 fm. They had no dynamical light quarks, so their results are only useful qualitatively. Their ground-state charmonium hybrid was the spin-symmetry multiplet $\{1^{--}, (0, 1, 2)^{-+}\}$. The next few multiplets were $\{1^{++}, (0, 1, 2)^{+-}\}$, $\{0^{++}, 1^{+-}\}$, and then a radially excited $\{1^{--}, (0, 1, 2)^{-+}\}$ multiplet.

3.3 Born-Oppenheimer approximation

Another approach to the XYZ mesons within QCD is based on the Born-Oppenheimer approximation. It provides a unified framework for describing conventional quarkonium, quarkonium hybrids, and quarkonium tetraquarks. The Born-Oppenheimer approximation is used in atomic physics to describe molecules. A Born-Oppenheimer potential is the energy of the electrons in the presence of static sources for the atomic nuclei separated by a distance R , and it can be interpreted as the potential energy of the nuclei. In the Born-Oppenheimer approximation, the motion of the nuclei is described by the Schroedinger equation in the Born-Oppenheimer potential, while the much lighter electrons are assumed to respond instantaneously to the motion of the nuclei.

The Born-Oppenheimer approximation for QCD applied to hadrons containing a heavy quark and antiquark was developed by Juge, Kuti, and Morningstar [25]. A Born-Oppenheimer (B-O) potential can be defined by the energy of gluon and light-

quark fields in the presence of static Q and \bar{Q} sources separated by a distance R , and it can be interpreted as the potential energy of the $Q\bar{Q}$ pair. In the Born-Oppenheimer approximation, the motion of the Q and \bar{Q} is described by the Schroedinger equation in the B-O potential, while the gluon and light-quark fields are assumed to respond instantaneously to the motion of the Q and \bar{Q} . The possible configurations of gluon and light-quark fields can be labelled by quantum numbers of symmetry operators in the presence of the Q and \bar{Q} sources. Juge, Kuti, and Morningstar followed the tradition of atomic physics of labeling them instead by upper-case Greek letters with a subscript and possibly a superscript: Σ_g^+ , Π_u , Σ_u^- , etc. They used lattice QCD without dynamical quarks to calculate the lowest few B-O potentials. Their lattice had only $10^3 \times 30$ sites and a spatial lattice spacing of 0.1 fm. The ground-state potential Σ_g^+ can be identified with the potential used in quark models, which is attractive and proportional to $1/R$ at short distances and repulsive and linear in R at long distances. The first excited potential Π_u and the second excited potential Σ_u^- have minima for R near 0.3 fm and 0.15 fm, respectively. They are both repulsive and linear in R at large R . At small R , they both approach the repulsive $1/R$ potential between Q and \bar{Q} in a color-octet state [26].

The solutions to the Schrodinger equation in the B-O potentials can be labelled nL , where $n = 1, 2, 3, \dots$ is a radial quantum number and $L = 0, 1, 2, \dots$ (or S, P, D, \dots) is an orbital-angular-momentum quantum number whose minimum value depends on the B-O potential. The energy levels in the ground-state B-O potential Σ_g^+ are conventional quarkonia. The energy levels in the excited B-O potentials Π_u , Σ_u^- , etc. are quarkonium hybrids. The lowest hybrid energy level is $1P$ in the Π_u potential, which consists of the two spin-symmetry multiplets $\{1^{--}, (0, 1, 2)^{+-}\}$ and $\{1^{++}, (0, 1, 2)^{+-}\}$. In the case of bottomonium, the next lowest energy levels are $1S$ in the Σ_u^- potential, which is a $\{0^{++}, 1^{+-}\}$ multiplet, and a $2P$ radial excitation in the Π_u potential. These results were obtained using lattice QCD without dynamical quarks. Calculations of the B-O potentials with dynamical quarks would be required to determine the correct ordering and spacing of the bottomonium hybrid energy levels.

Quarkonium tetraquarks can also be treated using the Born-Oppenheimer approximation. The appropriate B-O potentials are energies of gluon and light-quark fields that have isospin 1 instead of being flavor singlets. The isospin-1 B-O potentials for tetraquarks can presumably be calculated using lattice QCD, although the presence of the light valence quarks makes the calculations more demanding than those for flavor-singlet B-O potentials.

One can make plausible guesses for the qualitative behavior of the isospin-1 B-O potentials [22]. At small R , the isospin-1 Π_u and Σ_u^- potentials should both approach the repulsive $1/R$ potential of a color-octet $Q\bar{Q}$ pair, just like the flavor-singlet potentials except for an offset in the energy. At large R , they should both be repulsive and linear in R . Thus they should both have minima at intermediate R . It is therefore

plausible that the isospin-1 Π_u and Σ_u^- potentials both have similar shapes to the corresponding flavor-singlet potentials, except for an offset in the energy. The discovery of the $Z_c(3900)$ tetraquark with a smaller mass than the $Y(4260)$ hybrid implies that the isospin-1 B-O potentials are lower in energy than the flavor-singlet potentials. If they have the same shapes, the neutral members of the lowest tetraquark multiplet should have the same J^{PC} quantum numbers as the lowest hybrid multiplets. The lowest multiplets for charmonium tetraquarks in the Π_u and Σ_u^- potentials would then be $\{1^{--}, (0, 1, 2)^{-+}\}$, $\{1^{++}, (0, 1, 2)^{+-}\}$, and $\{0^{++}, 1^{+-}\}$.

4 Outlook

The discoveries of the neutral XYZ mesons, the bottomonium tetraquarks Z_b and Z'_b , and the charmonium tetraquark Z_c have revealed a serious gap in our understanding of the QCD spectrum. None of the proposed models for the XYZ mesons has revealed a compelling pattern. A promising recent proposal that has not yet been fully explored is that conventional quarkonia, quarkonium hybrids, and quarkonium tetraquarks can all be described in the same coherent framework based on the Born-Oppenheimer approximation.

The most promising theoretical approaches to the XYZ mesons within QCD all involve lattice gauge theory. There are several lattice calculations that would have a significant impact on our understanding of the XYZ mesons:

- definitive calculations of the spectrum of $c\bar{c}$ mesons using lattice QCD,
- definitive calculations of the spectrum of $b\bar{b}$ mesons using lattice NRQCD,
- calculations of the Born-Oppenheimer potentials using lattice QCD, both the flavor-singlet potentials for quarkonium hybrids and the isospin-1 potentials for quarkonium tetraquarks.

In addition, it is important to develop a phenomenological framework for estimating hadronic transition rates between conventional quarkonia, quarkonium hybrids, and quarkonium tetraquarks.

Many clues to the XYZ puzzle have already been provided by experiment. It is a little embarrassing for QCD theory that this problem has remained unsolved for so long. Fortunately, many additional hints can be expected from ongoing experiments at BESIII, current analyses by the LHC collaborations, and future experiments at Belle II and Panda. They make the solution to the XYZ puzzle almost inevitable.

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