Naturalness, Supersymmetry and Light Higgsinos: A Snowmass Whitepaper

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June 13, 2013

We show that the electroweak fine-tuning parameter $\Delta_{\rm EW}$ derived from the well-known electroweak symmetry breaking condition written in terms of weak scale parameters leads to *a bound on fine-tuning in the MSSM* and explain its utility for phenomenological analyses. We argue that a small magnitude of the μ parameter, and the concomitant presence of light higgsinos, is the most basic consequence of naturalness in SUSY models, and list the resulting implications of this for experiments at the LHC and at future e^+e^- colliders.

The ultra-violet behaviour of softly broken SUSY theories, with SUSY broken at the weak scale, ensures that the low energy theory is at most logarithmically sensitive to high scale (HS) physics. Thus weak scale SUSY solves the *big hierarchy problem* endemic to the Standard Model (SM) and discussions of naturalness concern at most the logarithmic sensitivity to HS physics.

The well-known electroweak symmetry breaking condition that yields the value of M_Z using the one-loop renormalization group improved effective potential can be written as,

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \ . \tag{1}$$

Here, Σ_u^u and Σ_d^d are the one-loop corrections arising from loops of particles and their superpartners that couple directly to the Higgs doublets. Naturalness requires that there be no large cancellations between terms on the right-hand-side of (1). We are thus led to require that [1]

$$\Delta_{\rm EW} = max\left(\left|\frac{-m_{H_u}^2\tan^2\beta}{\tan^2\beta-1}\right|, \left|\frac{-\Sigma_u^u(\tilde{t}_1)\tan^2\beta}{\tan^2\beta-1}\right|, \cdots, |-\mu^2|\right)/(M_Z^2/2) \tag{2}$$

remain smaller than some pre-chosen value decided by how much fine-tuning one "deems reasonable".

We note that (2) is obtained from the *weak scale* Lagrangian and so contains no information about either the underlying HS theory or the logarithms that we mentioned above. To see these logs, we must rewrite $m_{H_u}^2$, $m_{H_d}^2$ and μ^2 in (1) in terms of these parameters defined at the high scale using $m_{H_u}^2 = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2$, *etc.*: the large logarithms are then contained in $\delta m_{H_u,H_d}^2$ and in $\delta\mu^2$. We can then define a fine-tuning measure $\Delta_{\rm HS}$, that "knows about the high energy theory" in an analogous manner to $\Delta_{\rm EW}$ [2]. Typically, $\delta m_{H_u}^2$ has dominant contributions from top squark loops; this has led many authors to argue that top squarks must be lighter than 400-600 GeV in natural SUSY models with Λ as low as ~ 10 TeV[3, 4] (limits are even lower for higher Λ). Note that $\Delta_{\rm HS}$ is a very strict measure of naturalness in that any cancellations between $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ that lead to low $m_{H_u}^2$ at the weak scale will lead to a large value of $\Delta_{\rm HS}$, but not of $\Delta_{\rm EW}$.

The reader may correctly note that $\Delta_{\rm HS}$ as we have defined it does not incorporate correlations amongst weak scale parameters that are inevitably present in HS models such as mSUGRA/CMSSM that are completely specified by a small number of HS parameters. Because of these correlations, the weak scale value of $m_{H_u}^2$ can be approximated (using the one-loop solutions of the RGEs for $\tan\beta = 10$ and working within mSUGRA/CMSSM) by [5],

$$-2m_{H_u}^2(m_{\rm weak}) = 3.78m_{1/2}^2 - 0.82A_0m_{1/2} + 0.22A_0^2 + 0.013m_0^2 \ , \tag{3}$$

with analogous expressions for $m_{H_d}^2$ and μ^2 that can be substituted in (1). The small coefficient of m_0^2, which arises mainly because of the underlying equality of the HS soft SUSY breaking mass parameters for the H_u, Q_3 and U_3 fields (see Eq. (28) of Ref. [5]), then implies that the fine-tuning in mSUGRA/CMSSM may be smaller than expected for large values of m_0, as long as $m_{1/2}$ is not very large. We remark, however, that in this region – the focus point/hyperbolic branch region of mSUGRA/CMSSM – $\Delta_{\rm EW}$ becomes large for very large m_0 because the Σ_u^u in (2) begins to dominate. Note that incorporating the correlations between parameters as in (3) allows a (partial) cancellation between $m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ *without a concomitant contribution to the fine-tuning measure*, so that this criterion is intermediate between $\Delta_{\rm HS}$ and $\Delta_{\rm EW}$. Because of the simple quadratic form in the expression for M_Z^2 that results in this approximation, these considerations lead to a fine-tuning measure numerically similar to the widely used Barbieri-Guidice measure $\Delta_{\rm BG} \equiv \frac{\partial \ln M_Z^2}{\partial \ln a_i}$ [6, 7] where the a_i are fundamental theory parameters. We thus have,

$$\Delta_{\rm EW} < \Delta_{\rm BG} \lesssim \Delta_{\rm HS}.$$

While $\Delta_{\rm HS}$ is a popular (albeit strict) fine-tuning measure in a HS theory and Δ_{BG} is a traditional one, it is reasonable to wonder about the utility of $\Delta_{\rm EW}$, a quantity that is independent of the HS physics. We find that: $\Delta_{\rm EW}$ is clearly *a bound on the fine-tuning*[8]. [1] We can use it to infer [2] that models with large $\Delta_{\rm EW}$ (such as mSUGRA/CMSSM where Δ_{EW} always exceeds ~ 100) are necessarily fine-tuned.[2] Moreover, the properties of $\Delta_{\rm EW}$ allow us to make important phenomenological inferences.

- $\Delta_{\rm EW}$ is essentially determined by the physical spectrum. Thus, in principle, if a low energy spectrum leads to a large value of $\Delta_{\rm EW}$ we could infer the underlying theory is necessarily fine-tuned. *A low value of* $\Delta_{\rm EW}$ *does not imply the absence of fine-tuning,* but leaves open the possibility that there is an underlying theory with this spectrum that may not be fine-tuned. Since many observable cosequences are determined largely by the spectrum, we can study the phenomenology of these theories even without detailed knowledge of the underlying HS theory!

- Low $\Delta_{\rm EW} < 10$ (30) necessarily implies small $|\mu| < 200$ (300) GeV, and hence the existence of light higgsino states. An e^+e^- collider operating with $\sqrt{s} \gtrsim 2|\mu|$ may be able to see the low visible-energy-release events from $\widetilde{W}_1^+\widetilde{W}_1^-$, $\tilde{Z}_1\tilde{Z}_2$ and $\tilde{Z}_2\tilde{Z}_2$ higgsino pair production above SM two-photon backgrounds (as indicated by early analyses [9]). Thus, e^+e^- colliders will decisively probe $\Delta_{\rm EW} < \frac{1}{2}\left(\frac{E_{\rm CM}}{M_Z}\right)^2$.

- It is possible to find models with low values of $\Delta_{\rm EW} \sim 10-30$ but with top-squarks in the 1-5 TeV range [8]. For this reason, *we regard small* $|\mu|$ *as a more basic consequence of naturalness considerations than sub-TeV top squarks.* The importance of small $|\mu|$, though present, remains obscured in analyses that focus on the large top squark contributions to $\delta m_{H_u}^2$ mentioned above [4].

- If $|M_{1,2}|$ is coincidently comparable to $|\mu|$, charginos and neutralinos would be highly mixed gaugino-higgsino states and their electroweak production would lead to observable multilepton production at the LHC.

- If $|M_2| \leq 0.8-1$ TeV, but $|\mu|$ is small, hadronically quiet same-sign dilepton events from wino pair production $\widetilde{W}_2\tilde{Z}_4 \to W^\pm W^\pm + E_T^{\rm miss}$ may offer the greatest SUSY reach at a high luminosity LHC [10].

- If gaugino masses are large, the small mass gap between $\widetilde{W}_1 - \tilde{Z}_2$ and $\tilde{Z}_2 - \tilde{Z}_1$ ($\sim 10-15$ GeV) makes detection of higgsino pair production exceedingly difficult to see at the LHC because the visible decay products are very soft. In this case, it would be worth examining the initial state monojet and mono-photon radiation signal from higgsino pair production at LHC to see if they are possible avenues for discovery.

[1] In HS effective theories that are obtained from an overarching meta-theory that fixes the parameters of the HS theory so that cancellations between $m_{H_u}^2(\Lambda)$ and the log terms in $\delta m_{H_u}^2$ are *automatic*, we would not pay a fine-tuning price for the cancellation and $\Delta_{\rm EW}$ would be a valid fine-tuning measure.

[2] Implicit in this is an assumption that the SUSY conserving μ parameter and the soft SUSY breaking parameters have different origin, and so cancellations between these are unnatural.

Acknowledgments

This research was sponsored in part by grants from the US Department of Energy

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