

# Theoretical results for top quark production

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## Abstract

I discuss and compare several approaches to higher-order calculations of top quark production. I study the relative effectiveness of the approaches in approximating the exact NNLO results for the top-pair total cross section and highlight the theoretical and numerical differences between them. I show that the results from my soft-gluon resummation method are nearly identical to the exact NNLO cross section at all LHC and Tevatron energies. This agreement has important consequences for the validity of existing approximate NNLO differential distributions, for further refinements of the predictions, and for applications to other processes such as single-top production. I also compare approximate NNLO top quark transverse momentum and rapidity distributions with recent LHC measurements.

## 1 Introduction

Top quark production has been a topic of intense theoretical study for a long time, with NLO calculations [1] appearing over two decades ago. Fixed-order calculations plus soft-gluon resummations of various kinds have been employed to predict the theoretical top-pair production cross section as well as various differential distributions. The theoretical formalisms that have been used for resummed calculations and resultant approximate NNLO results have large differences in scope and give a wide range of numerical values for phenomenologically interesting total and differential cross sections.

The recent calculation of the exact numerical NNLO total cross section [2] affords the possibility of evaluating the relative success of the various resummation/approximate NNLO approaches [3, 4, 5, 6, 7] and of drawing implications for differential quantities as well as for other related processes. This is the topic of this contribution.

## 2 A comparison of NNLO results for the top-pair cross section

A lot of work has been done in the last twenty years on soft-gluon resummations which have culminated in NNLL accuracy for top-pair production in various distinct approaches [3, 4, 5, 6] (for more details and references on the development of resummation see the review in Ref. [8]). The resummed expressions have been used as generators of approximate NNLO corrections, and cross section calculations for top-pair production have appeared in [3, 4, 5, 6, 7]. The differences between these various resummation/NNLO approximate approaches include:

cross section		Soft limit
1PI	$d\sigma/dp_T dy$	$s_4 = s + t_1 + u_1 \rightarrow 0$
PIM	$d\sigma/dM_{t\bar{t}} d\theta$	$1 - z = 1 - M_{t\bar{t}}^2/s \rightarrow 0$
total	$\sigma$	$\beta = \sqrt{1 - 4m_t^2/s} \rightarrow 0$

Table 1: The double-differential cross sections, in single-particle-inclusive (1PI) and pair-invariant-mass (PIM) kinematics, and the total cross section for which soft-gluon resummation have been developed. The variables that vanish in the soft limit for each case are indicated.

- Double-differential cross sections [3, 5] versus total-only cross sections [4, 6, 7]. These involve different definitions of threshold, see Table 1.

- Moment-space perturbative QCD (pQCD) [3, 4, 7] versus Soft-Collinear Effective Theory (SCET) [5, 6].

The more general approach is double-differential which allows the calculation of transverse momentum and rapidity distributions, as well as the total cross section by integrating over  $p_T$  and rapidity. This approach uses partonic threshold, i.e. the top quark is not necessarily produced at rest but can have arbitrarily large velocity. On the other hand, the total-cross-section-only approaches use absolute/production threshold (top produced at rest) and are thus a limit/special case of the more general partonic threshold. A detailed discussion of these matters can be found in Ref. [8] (see also [9, 10]). Further differences between the formalisms arise from what subleading terms are included, whether damping factors are used, and, for differential calculations, how the partonic threshold relation  $s + t_1 + u_1 = 0$  is used in the plus-distribution coefficients (again see Refs. [8] and [9, 10] for details). It is very important to note that while many of these differences are formally “subleading” they can be numerically very significant.

In Fig. 1 we provide a comparison of various NNLO approximate calculations [3-7] at 7 and 8 TeV LHC energy together with exact NLO [1] and NNLO [2] results. All results are with the same choice of parameters and including theoretical uncertainties from scale variation and other sources as described in each paper [3-7]; these uncertainties however do not include uncertainties from parton distribution functions (pdf) or  $\alpha_s$  which are extraneous to the theoretical method used and should be the same for all approaches. The results use the same value of top quark mass and the same MSTW 2008 NNLO pdf [11] and  $\alpha_s$  as implemented in LHAPDF. We note that the values chosen for some of these parameters were slightly different among the published results of Refs. [2-7] so we here have chosen common values for a better comparison. Note, however, that there is a very small change (at the per mille level) if one uses the published results with the different parameters, and thus no noticeable change in the comparison and no change in the conclusions reached. Fig. 2 plots corresponding results at 14 TeV LHC energy (left) and 1.96 TeV Tevatron energy (right).

The result in [3] is from a double-differential formalism and uses pQCD. The result in [4] is from a method for the total cross section only and uses pQCD as does [7]. The results in [5] and [6] use SCET for the double-differential and total cross section, respectively. There is a fairly wide spread in the numbers and hence the degree of success of the various approaches.

The result of Ref. [3] is very close to the exact NNLO; both the central values and the scale

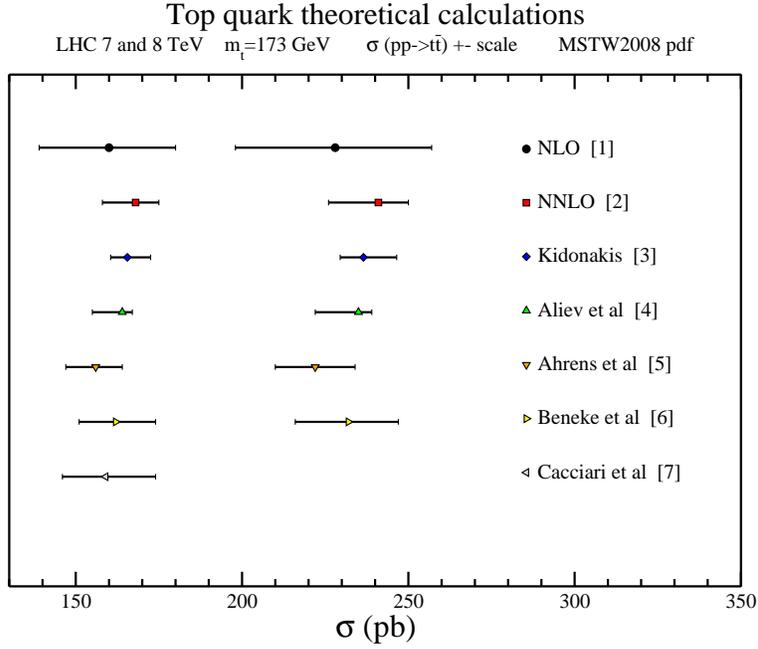


Figure 1: NNLO exact [2] and approximate [3-7] results for the  $t\bar{t}$  cross section at 7 TeV (bars on the left) and 8 TeV (right) LHC energy.

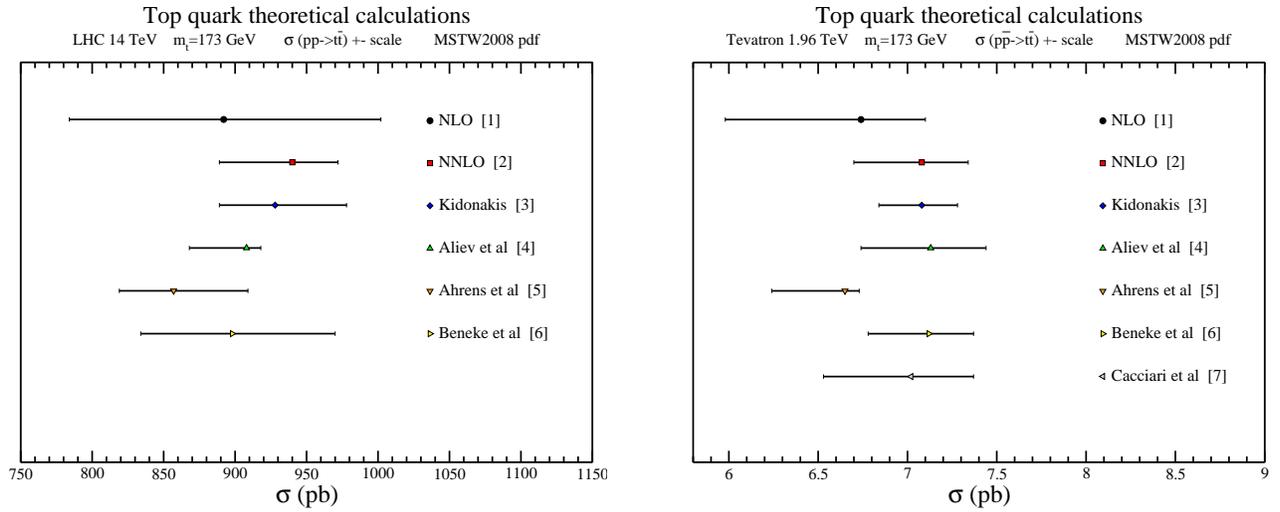


Figure 2: NNLO exact [2] and approximate [3-7] results for the  $t\bar{t}$  cross section at 14 TeV LHC energy (left) and at the Tevatron (right).

uncertainty are nearly the same. This is true for all collider energies, as can be seen from Figs. 1 and 2, and also for all top quark mass values from 130 GeV to 210 GeV as was checked with the results in Table III of [2]. This is in addition to the known excellent agreement between exact and approximate results at NLO. There is around 1% or less difference between approximate and exact cross sections at both NLO and NNLO.

The excellent agreement of [3] with preliminary exact NNLO results for the Tevatron energy was already discussed in detail in [9, 10]. The additional agreement with the exact NNLO results for all LHC energies proves the validity of the method of Ref. [3] in general and the theoretical arguments in its support (see [3, 9, 10, 12, 13]) and it shows that they are not restricted to a given energy of one collider. It is also well known that the results of [3] are in excellent agreement with cross section data from CDF and D0 at the Tevatron and from ATLAS and CMS at the LHC, see the figures in Refs. [9, 10] for several comparisons.

The fact that the results of Ref. [3] are very close to the exact NNLO [2] was expected from various theoretical reasons that were discussed in detail in [3, 9, 10]. This agreement was expected from the study of the NLO approximation, from the comparison of 1PI and PIM results in [12], and from other arguments regarding the analytical structure of the results and their implementation (see also discussion in [3, 9, 10, 13]). A double-differential calculation for partonic (as contrasted to absolute) threshold as used in [3] has a lot of theoretical/analytical information (also useful for deriving distributions), generality, and potential for numerical accuracy. This is an important point with clear consequences. Now that NNLO is fully known numerically [2] (though not analytically) for the total cross section, the next step is to add the approximate N<sup>3</sup>LO corrections (see [14] for previous results). For differential calculations approximate NNLO is still the state-of-the-art and is likely to be practically indistinguishable from any future exact NNLO, but one can add N<sup>3</sup>LO corrections to the differential distributions as well.

The stability of the theoretical NNLO approximate results in our formalism over the past decade [12, 13, 3, 15, 9] is notable; it is in contrast to the resummation formalism with the minimal prescription used in [2, 7] which has produced widely-ranging results for the threshold corrections in the past. The reliability and stability of the results from our formalism [3] and near-identical value to exact NNLO is very important for several reasons:

- (1) It provides confidence of application to other processes, in particular single-top [16].
- (2) The results have been used widely as backgrounds for many analyses (Higgs, etc).
- (3) It means that we presently have near-exact NNLO  $p_T$  and rapidity distributions.

Regarding point (1), the success of the formalism for single-top production in all three channels [16, 9, 10] in describing the Tevatron and LHC data complements the confidence gained from the above comparison that approximate NNLO [16] should also be a good approximation to exact NNLO for single top production. Regarding point (2), since the approximate NNLO results for both top-pair and single-top have been used as backgrounds in many Tevatron and LHC analyses, it is reassuring to know that any difference from exact NNLO is negligible and would not have materially affected these analyses. Finally, point (3) is also very important, and the remarkable success of the approximate NNLO distributions in describing Tevatron and LHC data further reinforces the theoretical arguments. In the next section, we discuss the top quark distributions and compare them with recent LHC data.

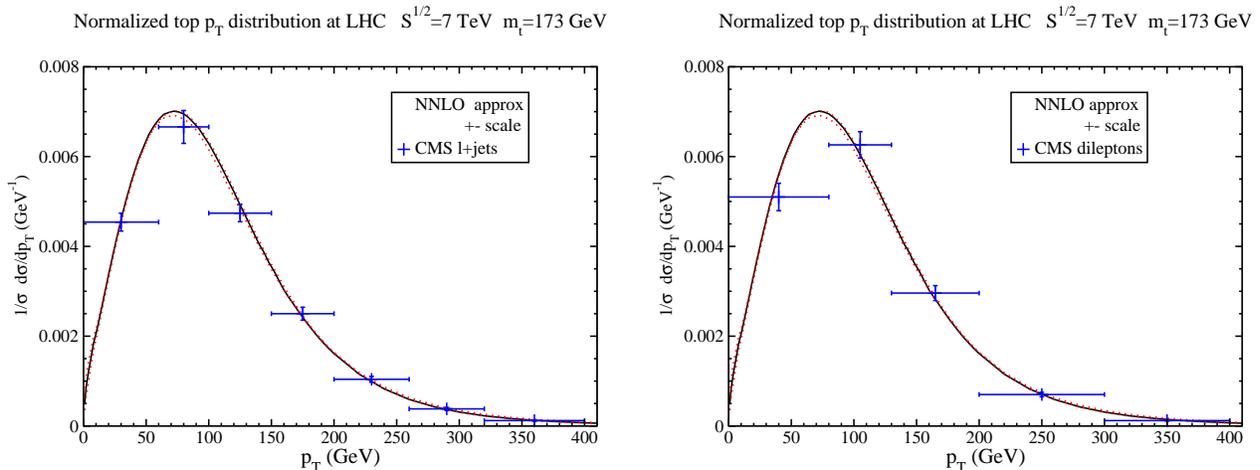


Figure 3: Normalized approximate NNLO top-quark  $p_T$  distributions [3] at the LHC and comparison with CMS data [17] in the  $\ell$ +jets (left) and dilepton (right) channels.

### 3 Top quark differential distributions

The soft-gluon approximation of [3, 15] works very well both for total cross sections and differential distributions. The approximation is known to be excellent at NLO, with  $\sim 1\%$  difference between NLO approximate and exact differential distributions, see Fig. 2 in [9]. Given the success of the approximation at NNLO for the total cross section, it is clear that the distributions should work very well at NNLO as well.

In Fig. 3 the theoretical top quark normalized transverse momentum distribution at approximate NNLO [3] for 7 TeV LHC energy is plotted and compared with recent data from the CMS collaboration [17] in the  $\ell$ +jets and dilepton channels. The central result is for  $\mu = m_t$  and the theoretical uncertainty from scale variation  $m_t/2 < \mu < 2m_t$  is also displayed. The agreement of the LHC data in both CMS channels with the theoretical prediction is very good. The theoretical uncertainties are much smaller than the experimental error bars. Similar results have also appeared for Tevatron energies and the agreement with D0 data [18] is excellent, see Fig. 4 in Ref. [10].

In Fig. 4 the theoretical top quark normalized rapidity distribution, again with uncertainty from scale variation, at approximate NNLO [15] for 7 TeV LHC energy is plotted and compared with recent data from the CMS collaboration in the  $\ell$ +jets and dilepton channels [17]. Again, the agreement between LHC data and theoretical results is very good and the theoretical uncertainty is very small.

### 4 Conclusions

Various methods for soft-gluon resummation and approximate NNLO calculations for top-quark pair production have been discussed and compared. We have shown in this paper that the soft-gluon approximation method of Ref. [3] works extremely well in approximating the exact

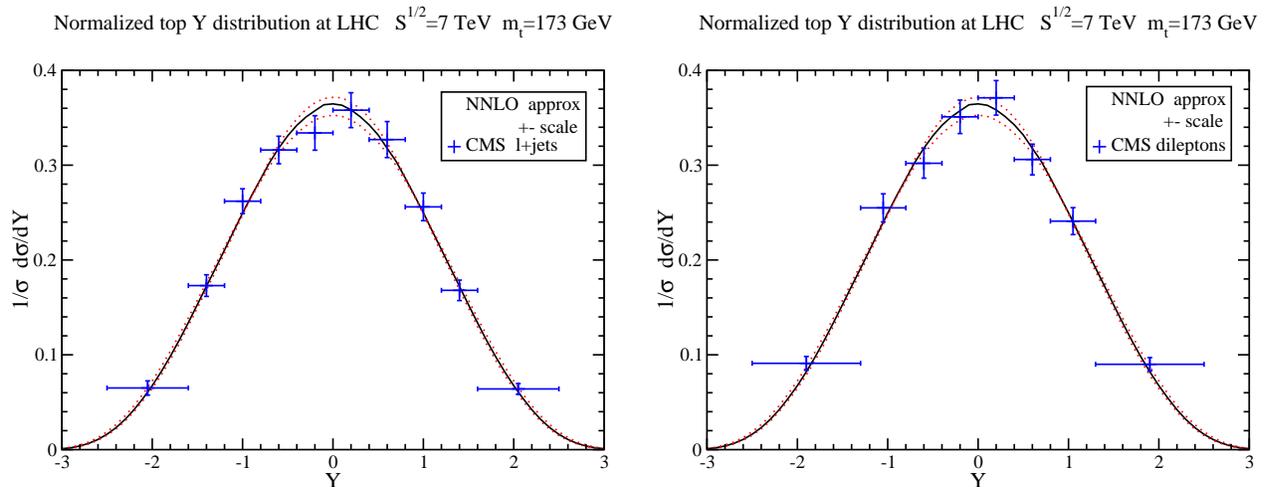


Figure 4: Normalized approximate NNLO top-quark rapidity distributions [15] at the LHC and comparison with CMS data [17] in the  $\ell$ +jets (left) and dilepton (right) channels.

NLO and NNLO total cross sections. The fact that the approximation also works extremely well for differential distributions at NLO provides confidence that the approximate NNLO distributions should be nearly indistinguishable from any future exact results. The theoretical top-quark transverse momentum [3] and rapidity [15] distributions have predicted very well the recent LHC data; the agreement is excellent. Future approximate  $N^3$ LO calculations will likely provide small additional enhancements for both total and differential cross sections and are currently under study.

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