Mass of the Electroweak Monopole

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We present two independent methods to estimate the mass of the electroweak monopole. Our result strongly implies the existence of a genuine electroweak monopole of mass around 4 to 7 TeV, which could be detected by MoEDAL at present LHC.

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The recent "discovery" of Higgs particle at LHC has reconfirmed that the electroweak theory of Weinberg describes the real world [1]. If so, one might ask what would be the next hot subject in the standard model. Certainly there could be different opinions, but one thing must be clear. We must look for the electroweak monopole because the theory provides the natural topology for the monopole [2, 3]. The existence of the topology strongly implies that the electroweak monopole must exist.

So it is timely that the latest MoEDAL detector ("The Magnificient Seventh") at LHC is actively searching for such monopole [4]. Under this circumstance we urgently need to have a theoretical estimate of the monopole mass. *The purpose of this Letter is to estimate the mass of the electroweak monopole to be around 4 to 7 TeV.*

It is well known that the Georgi-Glashow model has the 'tHooft-Polyakov monopole [5, 6]. But it has been asserted that the Weinberg-Salam model has no topological monopole of physical interest [7]. The basis for this "non-existence theorem" is that with the spontaneous symmetry breaking the quotient space $SU(2) \times U(1)/U(1)_{\rm em}$ allows no non-trivial second homotopy. This claim, however, is unfounded. In fact the electroweak monopole and dyon solutions in the standard model have been shown to exist [2, 3].

This is because the Weinberg-Salam model, with the hypercharge $U(1)$, could be viewed as a gauged CP^1 model in which the (normalized) Higgs doublet plays the role of the CP^1 field [2]. So the Weinberg-Salam model does have exactly the same nontrivial second homotopy as the Georgi-Glashow model which allows topological monopoles. Originally the electroweak monopole (known as the "Cho-Maison" monopole) was obtained by a numerical integration, but subsequently a mathematically rigorous existence proof has been established by Yang [3].

The importance of the Cho-Maison monopole is that it is the straightforward generalization of the Dirac monopole to the electroweak theory. This means that the monopole which should exist in the real world is not likely to be the Dirac monopole but this one.

Actually the Cho-Maison monopole may be viewed as a hybrid between the Dirac monopole and the 'tHooft-Polyakov monopole, because it has a $U(1)$ point singularity at the center even though the $SU(2)$ part is completely regular. Consequently it carries an infinite energy classically, which means that the mass of the monopole can be arbitrary. *A priori* there is nothing wrong with this, but nevertheless one may wonder whether one can estimate the mass of the electroweak monopole to help the MoEDAL experiment. In the following we show that this is possible.

Consider the standard Weinberg-Salam model,

$$\mathcal{L} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 - \frac{1}{4}G_{\mu\nu}^2 - |D_\mu\phi|^2 - \frac{\lambda}{2}\big(|\phi|^2 - \frac{\mu^2}{\lambda}\big)^2,$$
$$D_\mu\phi = \big(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{A}_\mu - i\frac{g'}{2}B_\mu\big)\phi, \qquad (1)$$

where ϕ is the Higgs doublet, $\vec{F}_{\mu\nu}$ and $G_{\mu\nu}$ are the gauge field strengths of $SU(2)$ and $U(1)$ with the potentials $\vec{A}_\mu$ and B_μ. Now choose the static spherically symmetric ansatz in the spherical coordinates (t, r, θ, φ)

$$\phi = \frac{1}{\sqrt{2}}\rho(r)\xi(\theta,\varphi), \quad \xi = i\begin{pmatrix} \sin(\theta/2)\ e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix},$$
$$\hat{\phi} = \xi^\dagger\vec{\tau}\xi = -\hat{r},$$
$$\vec{A}_\mu = \frac{1}{g}A(r)\partial_\mu t\ \hat{r} + \frac{1}{g}(f(r)-1)\ \hat{r}\times\partial_\mu\hat{r},$$
$$B_\mu = \frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi. \qquad (2)$$

Notice that the apparent string singularity along the negative z-axis in ξ and B_μ is a pure gauge artifact which can easily be removed making $U(1)$ non-trivial. So the above ansatz describes a most general spherically symmetric ansatz of the electroweak dyon.

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In terms of the physical fields the Lagrangian (1) is written as

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\rho)^2 - \frac{\lambda}{8}\big(\rho^2 - \frac{2\mu^2}{\lambda}\big)^2 - \frac{1}{4}F_{\mu\nu}^{(\mathrm{em})2} - \frac{1}{4}Z_{\mu\nu}^2$$
$$-\frac{1}{2}|(D_\mu^{(\mathrm{em})}W_\nu - D_\nu^{(\mathrm{em})}W_\mu) + ie\frac{g}{g'}(Z_\mu W_\nu - Z_\nu W_\mu)|^2$$
$$-\frac{g^2+g'^2}{8}\rho^2 Z_\mu^2 - \frac{g^2}{4}\rho^2|W_\mu|^2 + ie\frac{g}{g'}Z_{\mu\nu}W_\mu^* W_\nu$$
$$+ieF_{\mu\nu}^{(\mathrm{em})}W_\mu^* W_\nu + \frac{g^2}{4}(W_\mu^* W_\nu - W_\nu^* W_\mu)^2, \qquad (3)$$

where ρ, W_μ, Z_μ are the Higgs, W, Z bosons, $D_\mu^{(\mathrm{em})} = \partial_\mu + ieA_\mu^{(\mathrm{em})}$, and $e = gg'/\sqrt{g^2+g'^2}$ is the electric charge. Moreover, the ansatz (2) becomes

$$\rho = \rho(r), \quad W_\mu = \frac{i}{g}\frac{f(r)}{\sqrt{2}}e^{i\varphi}(\partial_\mu\theta + i\sin\theta\partial_\mu\varphi),$$
$$A_\mu^{(\mathrm{em})} = e\big(\frac{A(r)}{g^2} + \frac{B(r)}{g'^2}\big)\partial_\mu t - \frac{1}{e}(1-\cos\theta)\partial_\mu\varphi,$$
$$Z_\mu = \frac{e}{gg'}\big(A(r) - B(r)\big)\partial_\mu t. \qquad (4)$$

With this we have the equations of motion

$$\ddot{f} - \frac{f^2-1}{r^2}f = \big(\frac{g^2}{4}\rho^2 - A^2\big)f,$$
$$\ddot{\rho} + \frac{2}{r}\dot{\rho} - \frac{f^2}{2r^2}\rho = -\frac{1}{4}(B-A)^2\rho + \lambda\big(\frac{\rho^2}{2} - \frac{\mu^2}{\lambda}\big)\rho,$$
$$\ddot{A} + \frac{2}{r}\dot{A} - \frac{2f^2}{r^2}A = \frac{g^2}{4}(A-B)\rho^2,$$
$$\ddot{B} + \frac{2}{r}\dot{B} = \frac{g'^2}{4}(B-A)\rho^2, \qquad (5)$$

which has a singular solution

$$f = 0, \quad \rho = \rho_0 = \sqrt{2\mu^2/\lambda},$$
$$A_\mu^{(\mathrm{em})} = -\frac{1}{e}(1-\cos\theta)\partial_\mu\varphi, \qquad Z_\mu = 0. \qquad (6)$$

This describes the point monopole in Weinberg-Salam model which has two remarkable features. First, this is not the Dirac monopole because it has the magnetic charge $4\pi/e$ (not $2\pi/e$). Second, this monopole with a non-trivial dressing of weak bosons becomes the Cho-Maison dyon which looks similar to the Julia-Zee dyon [6]. But, unlike the Julia-Zee dyon, the Cho-Maison dyon still carries the point singularity at the origin [2].

The point singularity of the Cho-Maison monopole makes it difficult to estimate the mass classically. On the other hand, if we treat the electroweak monopole as a finite energy soliton, we can do that. In this letter we present two ways to estimate mass of the electroweak monopole.

A. Scaling Argument

A simple way to make the energy of the monopole finite is to introduce a UV-cutoff which can remove the singularity. Assuming a UV-cutoff, we can use the Derrick's scaling argument to estimate the monopole mass.

If a finite energy monopole does exist, it should be stable under the rescaling of its field configuration. So consider a static monopole configuration and let

$$K_A = \frac{1}{4}\int \vec{F}_{ij}^2 d^3x, \quad K_B = \frac{1}{4}\int B_{ij}^2 d^3x$$
$$K_\phi = \int |\mathcal{D}_i\phi|^2 d^3x, \; V_\phi = \frac{\lambda}{2}\int \big(|\phi|^2 - \frac{\mu^2}{\lambda}\big)^2 d^3x. \quad (7)$$

With the ansatz (2) we have (with $A = B = 0$)

$$K_A = \frac{4\pi}{g^2}\int_0^\infty \big(\dot{f}^2 + \frac{(f^2-1)^2}{2r^2}\big)dr,$$
$$K_B = \frac{2\pi}{g'^2}\int_0^\infty \frac{1}{r^2}dr, \quad K_\phi = 2\pi\int_0^\infty (r\dot{\rho})^2 dr,$$
$$V_\phi = \frac{\pi}{2}\int_0^\infty \lambda r^2\big(\rho^2 - \rho_0^2\big)^2 dr. \qquad (8)$$

Notice that K_B makes the monopole energy infinite.

Now, consider the scale transformation

$$\vec{x} \to \lambda\vec{x}, \qquad (9)$$

which changes $\vec{A}_k(\vec{x}) \to \lambda\vec{A}_k(\lambda\vec{x})$, $B_k(\vec{x}) \to \lambda B_k(\lambda\vec{x})$, and $\phi(\vec{x}) \to \phi(\lambda\vec{x})$. Under this we have

$$K_A \to \lambda K_A, \quad K_B \to \lambda K_B,$$
$$K_\phi \to \lambda^{-1}K_\phi, \quad V_\phi \to \lambda^{-3}V_\phi. \qquad (10)$$

So we have the following condition for the stable monopole configuration

$$K_A + K_B = K_\phi + 3V_\phi, \qquad (11)$$

from which we can infer the value of K_B.

For the Cho-Maison monopole we have (with $M_W \simeq 80.4$ GeV, $M_H \simeq 125$ GeV, and $\sin^2\theta_{\mathrm{w}} = 0.2312$)

$$K_A \simeq 0.1852 \times \frac{4\pi}{e^2}M_W, \quad K_\phi \simeq 0.1577 \times \frac{4\pi}{e^2}M_W,$$
$$V_\phi \simeq 0.0011 \times \frac{4\pi}{e^2}M_W. \qquad (12)$$

This, with (11), tells that

$$K_B \simeq 0.0058 \times \frac{4\pi}{e^2}M_W. \qquad (13)$$

From this we can estimate the energy of the monopole

$$E \simeq 0.3498 \times \frac{4\pi}{e^2}M_W \simeq 3.85 \text{ TeV}. \qquad (14)$$

This strongly implies that the electroweak monopole of mass around a few TeV could be possible.

One might wonder if there is any independent backup argument which can support this estimate of the

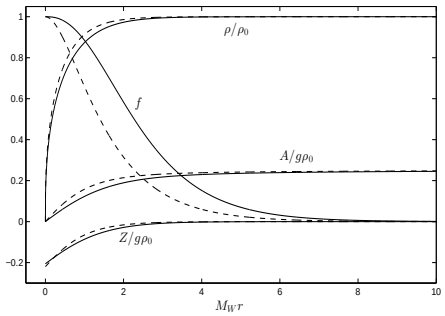

FIG. 1: The electroweak dyon solution. The solid and dotted lines represent the finite energy dyon and the Cho-Maison dyon, where $Z = A - B$ and we have put $A(\infty) = M_W/2$.

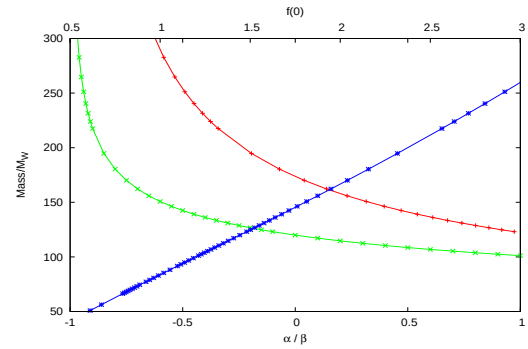

FIG. 2: The energy dependence of the electroweak monopole on α, β, or $f(0)$. The red and green curves represents the α and β dependence, and the blue curve represents the $f(0)$ dependence.

monopole mass. Now we show that we can actually regularize the Cho-Maison monopole, and that the regularized monopole has the energy predicted by the above argument.

B. Electromagnetic Regularization

Notice that (3) describes the "bare" theory which should change to an "effective" theory after the quantum correction. Moreover, the "real" dyon we are looking for must be the solution of the effective theory. Now, suppose the quantum correction changes the coupling strengths of the magnetic moment and quartic self interactions of W boson by $\delta\mathcal{L}$,

$$\delta\mathcal{L} = i\alpha e F^{(em)}_{\mu\nu} W^*_\mu W_\nu + \frac{\beta}{4} g^2 (W^*_\mu W_\nu - W^*_\nu W_\mu)^2, \quad (15)$$

where α and β are the constants to be fixed later. In this case the energy of the dyon is given by

$$\begin{aligned} E &= E_0 + E_1, \\ E_0 &= \frac{2\pi}{g^2}\int_0^\infty \frac{dr}{r^2}\Big\{1 + \frac{g^2}{g'^2} - 2(1+\alpha)f^2 + (1+\beta)f^4\Big\}, \\ E_1 &= \frac{4\pi}{g^2}\int_0^\infty dr\Big\{\frac{g^2}{2}(r\dot\rho)^2 + \dot f^2 + \frac{1}{2}(r\dot A)^2 + \frac{g^2}{2g'^2}(r\dot B)^2 \\ &\quad + \frac{g^2}{4} f^2\rho^2 + \frac{\lambda g^2 r^2}{4}\big(\rho^2 - \rho_0^2\big)^2 \\ &\quad + f^2 A^2 + \frac{g^2 r^2}{8}(B-A)^2\rho^2\Big\}. \end{aligned} \quad (16)$$

Clearly E_1 could be made finite with a proper boundary condition, but notice that when $\alpha = \beta = 0$, E_0 becomes infinite.

To make E_0 finite we must require

$$\begin{aligned} 1 + \frac{g^2}{g'^2} - 2(1+\alpha)f^2(0) + (1+\beta)f^4(0) = 0, \\ (1+\alpha)f(0) - (1+\beta)f^3(0) = 0, \end{aligned} \quad (17)$$

or

$$f(0) = \frac{g}{e\sqrt{1+\alpha}}, \quad 1 + \beta = (1+\alpha)^2\frac{e^2}{g^2}. \quad (18)$$

With this the equations of motion (5) remains exactly the same, except that the first equation is replaced by

$$\ddot f - \frac{1+\alpha}{r^2}\Big(\frac{f^2}{f(0)^2} - 1\Big)f = \Big(\frac{g^2}{4}\rho^2 - A^2\Big)f. \quad (19)$$

This can be integrated with the boundary conditions

$$\begin{aligned} f(0) = 1, \quad A(0) = 0, \quad B(0) = b_0, \quad \rho(0) = 0, \\ f(\infty) = 0, \quad A(\infty) = B(\infty), \quad \rho(\infty) = \rho_0. \end{aligned} \quad (20)$$

The finite energy dyon solution, together with the Cho-Maison dyon, is shown in Fig. 1. *It is really remarkable that the finite energy solution looks almost identical to the Cho-Maison solution, even though it no longer has the singularity at the origin.*

The monopole (with $A = B = 0$) has the energy

$$E \simeq 0.6104 \times \frac{4\pi}{e^2} M_W \simeq 6.72 \text{ TeV}. \quad (21)$$

This strongly supports the mass estimate based on the scaling argument. In general (18) tells that the monopole mass depends on one of the parameters $f(0)$, α, and β. This energy dependence is shown in Fig. 2.

What is more, changing the W boson mass from $g\rho_0/2$ to $e\rho_0$ we can obtain the explicitly analytic monopole solution. To see this let $\alpha = 0$ ($f(0) = g/e$) and add the mass correction term $\delta\mathcal{L}'$ in the Lagrangian,

$$\delta\mathcal{L}' = -\Big(e^2 - \frac{g^2}{4}\Big)\rho^2|W_\mu|^2. \quad (22)$$

In this case we can show that the energy of the monopole in the limit $\lambda = 0$ is bounded from below by

$$E \geq \frac{4\pi}{e}\rho_0 = \frac{8\pi}{e^2}\sin\theta_{\rm w} M_W \simeq 5.08 \text{ TeV}. \quad (23)$$

Moreover, we can saturate this bound with the following Bogomol'nyi type equation

$$\dot f \pm e\rho f = 0, \quad \dot\rho \pm \frac{1}{er^2}\Big(\frac{e^2}{g^2}f^2 - 1\Big) = 0, \quad (24)$$

which has the analytic solution which looks very much like the Prasad-Sommerfield solution [6]

$$f = \frac{g\rho_0 r}{\sinh(e\rho_0 r)}, \quad \rho = \rho_0 \coth(e\rho_0 r) - \frac{1}{er}. \tag{25}$$

It has generally been believed that finite energy monopole solutions can exist only at the grand unification scale [8]. But our analysis tells that a genuine finite energy electroweak monopole at much lower scale is possible.

Of course (15) does not describe the true quantum correction, so that the finite energy solution can only be viewed as an approximate solution of the standard model. But this is not our point. The point here is to show that a small correction like (15), with no new interaction, can regularize the Cho-Maison dyon. In this respect the finite energy solution has important meaning. As Fig. 1 demonstrates, it describes an excellent approximation of the Cho-Maison dyon from which we can estimate the mass of the electroweak monopole.

Independent of the details there is an intuitive argument which strongly supports the above analysis. Roughly speaking, the mass of the electroweak monopole should come from the same mechanism which generates the mass to the weak bosons, except that the coupling is given by the monopole charge. This means that the monopole mass should be of the order of M_W/α_{em}, where α_{em} is the electromagnetic fine structure constant. This solidly supports the above analysis.

The fact that the standard model has the monopole topology tells that the experimental verification of the electroweak monopole could be the final test of the stansard model. Certainly our mass estimate should be very useful to detect such monopole experimentally. Another crucial information is that this monopole obeys the Schwinger quantization rule $eg = 4\pi n$. This is because the $U(1)$ subgroup of $SU(2)$ has the period 4π.

Certainly the existence of the electroweak monopole of the mass around 4 to 7 TeV should have important physical implications. *But what is remarkable is that the monopole of this mass is within the energy range of the present LHC, so that the MoEDAL should be able to detect it* [4].

We close with the following remarks:
1. We emphasize that, unlike the Dirac's monopole in electrodynamics which is optional, the Cho-Maison monopole must exist in the standard model. In electrodynamics the $U(1)$ gauge group need not be non-trivial, so that the Maxwell's theory does not have to have the monopole. But in the standard model $U(1)_{em}$ comes from the $U(1)$ subgroup of $SU(2)$ (and the hypercharge $U(1)$), which is well known to be non-trivial. This means that in the standard model $U(1)_{em}$ must be non-trivial. So, if the standard model is correct, we must have the monopole. This is why the experimental detection of the monopole should be the final test of the standard model.
2. The monopole of mass around 4 to 7 TeV can only be produced at LHC, and one might wonder what is the monopole-antimonopole pair production rate. Intuitively the production rate must be similar to the WW production rate, except that here the coupling constant is fixed by the monopole charge. This implies that the monopole-antimonopole production rate at LHC (above the threshold) must be about $(1/\alpha_{em})^2$ times bigger than the WW production rate.
3. The electroweak monopole invites exciting new questions. What is the spin of the monopole, and how can we predict it? How can we construct the quantum field theory of the monopole? What are the new physical processes induced by the monopole? What are the impacts of the monopole in cosmology, in particular in the early universe? Clearly the electroweak monopole opens up new physics.

A detailed discussion of our work will be published in a separate paper [9].

Acknowledgments

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[1] G. Aad et al. (ATLAS Collaboration), Phys. Lett. **B716**, 1 (2012); S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. **B716**, 30 (2012); T. Aaltonen et al. (CDF and D0 Collaborations), Phys. Rev. Lett. **109**, 071804 (2012).
[2] Y. M. Cho and D. Maison, Phys. Lett. **B391**, 360 (1997); W. S. Bae and Y. M. Cho, JKPS **46**, 791 (2005).
[3] Yisong Yang, Proc. Roy. Soc. **A454**, 155 (1998). See also Yisong Yang, *Solitons in Field Theory and Nonlinear Analysis* (Springer Monographs in Mathematics), p. 322 (Springer-Verlag) 2001.
[4] J. Pinfold, Rad. Meas. **44**, 834 (2009); *Progress in High-Energy Physics and Nuclear Safety* (edited by V. Begun et al.), p. 217 (Springer Science and Business Media) 2009. See also CERN Courier **52**, No. 7, p. 10 (2012).
[5] G. 'tHooft, Nucl. Phys. **B79**, 276 (1974); A. Polyakov, JETP Lett. **20**, 194 (1974).
[6] B. Julia and A. Zee, Phys. Rev. **D11**, 2227 (1975); M. Prasad and C. Sommerfield, Phys. Rev. Lett. **35**, 760 (1975).
[7] T. Vachaspati and M. Barriola, Phys. Rev. Lett. **69**, 1867 (1992); M. Barriola, T. Vachaspati, and M. Bucher, Phys. Rev. **D50**, 2819 (1994).
[8] C. Dokos and T. Tomaras, Phys. Rev. **D21**, 2940 (1980); Y. M. Cho, Phys. Rev. Lett. **44**, 1115 (1980).
[9] Y. M. Cho, Kyoungtae Kimm, and J. H. Yoon, to be published.