Measurement of \( \gamma \) from
\[ B \to DK \] and related modes
at LHCb

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Outline

I. LHCb measurements
   - two-body GLW/ADS
   - four-body ADS
   - GGSZ  \{ 22 observables \}

II. Combination
   - B → DK
   - B → Dπ
   - full B → DK and B → Dπ

III. A new GGSZ result using additional 2fb⁻¹

→ see also talk by Matteo Rama!
• LHCb is a forward spectrometer operated in collider mode.
• Focus on precision measurements of b and c decays.
• CP violation, rare decays
CKM angle $\gamma$

$\gamma$ is the least well known angle of the unitarity triangle.

**“combined $\gamma$ measurements”**

\[
\begin{align*}
\gamma &= (66^{+12}_{-12})^\circ \\
\gamma &= (70.8 \pm 7.8)^\circ
\end{align*}
\]

**“$\gamma$ meas. not in triangle fit”**

CKMfitter ICHEP 2012

\[
\gamma = (68.0^{+4.1}_{-4.6})^\circ
\]

UTfit pre-Moriond 2013

\[
\gamma = (68.6 \pm 3.6)^\circ
\]
This was, and still is, the most important channel to measure $\gamma$.

We need to reconstruct the $D/\bar{D}$ meson in a final state accessible to both to achieve interference.

Choice of final state labels the “method”: GLW, ADS, GGSZ

Also possible: $B \rightarrow D\pi$! But little sensitivity.
\[ B \rightarrow DK \]

**“GLW”**

\[ \overline{D^0} K^+ \]

\[ B^+ \]

\[ (K^- K^+) \]

\[ D^0 K^+ \]

**“ADS”, “suppressed”**

\[ \overline{D^0} K^+ \]

\[ B^+ \]

\[ (K^- \pi^+) \]

\[ D^0 K^+ \]

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Gronau, London, Wyler

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Atwood, Dunietz, Soni

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**“GGSZ”, “Dalitz”**

- Use 3-body self-conjugate modes such as \( D \rightarrow K_s \pi^+ \pi^- \)
- hadronic D parameters vary across Dalitz plot
B → Dh: GLW/ADS observables

- Define observables as **yield ratios** (many systematics cancel).

- Charge **asymmetries**:

  \[ A_h^f = \frac{\Gamma(B^- \to [f]_D h^-) - \Gamma(B^+ \to [f]_D h^+)}{\Gamma(B^- \to [f]_D h^-) + \Gamma(B^+ \to [f]_D h^+)} \]

- **Kaon/pion** ratio:

  \[ R_{K/\pi}^f = \frac{\Gamma(B^\pm \to [f]_D K^\pm)}{\Gamma(B^\pm \to [f]_D \pi^\pm)} \]

  Form a system of equations. Need more observables than parameters! → many different decays

- **Suppressed/favored** decay ratio (2-body example):

  \[ R_{h}^{\pm} = \frac{\Gamma(B^\pm \to [\pi^\pm K^\mp]_D h^\pm)}{\Gamma(B^\pm \to [K^\pm \pi^\mp]_D h^\pm)} = r_B^2 + r_D^2 + 2r_B r_D \cos(\pm \gamma + \delta_B + \delta_D) \]

  strong phase difference: different for each decay mode!
$B \to D(\pi K)h$: suppressed ADS mode

$\mathcal{B}(B^\pm \to D_{ADS}K^\pm) \approx 2 \cdot 10^{-7}$

$A_{ADS} = -0.520 \pm 0.150 \pm 0.021$

13 observables in $B \to Dh, D \to hh$

B → D(πKππ)h: suppressed ADS mode

\[ A_{ADS} = -0.42 \pm 0.22 \]

5 observables in B → Dh, D → K3π

model independent GGSZ

- In the GGSZ method, one considers self-conjugate 3-body final states of the D meson: $D \to K_S^0 \pi^+ \pi^- \quad D \to K_S^0 K^+ K^-$

- A range of resonances introduces strong phase variations – no need for system of equations.

- Phase variation measured by CLEO. Used as input in binned analysis of the D Dalitz plot.

- Only $B^\pm \to DK^\pm$

- Control efficiency variation using $B^\pm \to D \pi^\pm$
model independent GGSZ

\[ D^0 \rightarrow K_S^0 \pi^- \pi^+ \]

\[ D^0 \rightarrow K_S^0 K^- K^+ \]

At the B-factories, this method is the best way to measure \( \gamma \)!

4 observables: “cartesian coordinates”

\[ x_\pm = r_B \cos(\delta_B \pm \gamma) \]
\[ y_\pm = r_B \sin(\delta_B \pm \gamma) \]
**K_s** reconstruction

- At LHCb, about 70% of the reconstructible KS decays are “down-down”.
- Decays behind first tracker are unusable!
Combination

- We now have measured **22 γ-related observables**. What does it mean for γ?

- **Combine the inputs!**
  - frequentist procedure
  - assume (mostly) Gaussian observables
  - assume Gaussian systematics
  - correct for undercoverage and some neglected systematic correlations

- **Strategy:**
  - for the first time include the B → Dπ system
  - consider CP violation in charm decays
  - partially consider charm mixing

\[
\mathcal{L}(\vec{y}) = \frac{1}{N} \exp \left( -\frac{1}{2} (\vec{y} - \vec{y}_t)^T V_{\text{cov}}^{-1} (\vec{y} - \vec{y}_t) \right)
\]

\[
\chi^2(\vec{y}) = -2 \ln \mathcal{L}(\vec{y})
\]
Combination

- Three LHCb input measurements:
  - $B \rightarrow D_h, D \rightarrow hh$ (two-body GLW/ADS)
  - $B \rightarrow D_h, D \rightarrow K\pi\pi\pi$ (four-body ADS)
  - $B \rightarrow DK, D \rightarrow Kshh$ (GGSZ)

- Other inputs:
  - CLEO measurement of $D \rightarrow hh, K\pi\pi\pi$ systems
  - Heavy Fl. Avg. Group averages for CPV in charm
  - (as crosscheck:) LHCb charm mixing result (arXiv:1211.1230 / PRL)

- Results are presented for three combinations:
  - “DK only” (in-line with previous experiments)
  - “$D\pi$ only”
  - “DK & $D\pi$”
statistical treatment

- The combined likelihood has a very rich structure:
  - many **nuisance parameters**
  - many trigonometrical functions, thus **many local minima**
  - **varying dimensionality** of the likelihood, depending on the value of the nuisance parameters

- Use a Feldman-Cousins based frequentist method.
- Compute the actual distribution of the test statistic ($\Delta\chi^2$) using toy Monte Carlo.
- Nuisances assume their profiled best-fit values.

\[ x_{\pm} = r_B \cos(\delta_B \pm \gamma) \]
CP violation in $D^0$ decays / $D^0$ mixing

- Any **CP violation** in the decays $D \rightarrow KK$ or $D \rightarrow \pi\pi$ will affect the GLW method.

  $A_{CP}^{dir}(KK) = (-0.31 \pm 0.24) \times 10^{-2}$

  $A_{CP}^{dir}(\pi\pi) = (+0.36 \pm 0.25) \times 10^{-2}$

  Measurements combined by the Heavy Fl. Avg. Group

- We take this into account by modifying the GLW asymmetries, but leaving the ratios unchanged:

  $$A_{\pi}^{KK} = \frac{2r_B^\pi \sin \delta_B^\pi \sin \gamma}{1 + (r_B^\pi)^2 + 2r_B^\pi \cos \delta_B^\pi \cos \gamma} + A_{CP}^{dir}(KK)$$

- This is valid up to a small weak phase in the D decay (London et al., arXiv:1301.5631).

- **$D^0$ mixing**: considered in description of D decay (constrained through CLEO measurement), but ignored in B decay: possible $\gamma$ shift of $O(x_D, y_D)$ → will have to be fixed!
**B → DK**

**LHCb:** $\gamma = (70.5^{+14.9}_{-15.6})^\circ$ (corrected)

[Graph showing exclusion regions for LHCb]

**Belle:** $\gamma = (68^{+15}_{-14})^\circ$

[Graph showing exclusion regions for Belle]

**BaBar:** $\gamma = (69^{+17}_{-16})^\circ$

[Graph showing exclusion regions for BaBar]
Comparing:
1 fb⁻¹ GLW/ADS and
1 fb⁻¹ GGSZ
Agreement of inputs

Make a test:

- predict the traditional ADS observables, $R_{AD}$, $A_{AD}$, in $B \to DK$, $D \to K\pi$, using all other LHCb $1fb^{-1}$ inputs
- (the combination uses $R_+$, $R_-$ instead)
- the agreement is impressive
For the first time, we include $B \rightarrow D\pi$ into a $\gamma$ measurement.

Data are compatible with rather high values of $r_{B}^{\pi}$.

Sensitivity scales roughly like $1/r_{B}^{\pi}$.

\[ r_{B}^{\pi} \approx \left| \frac{V_{ub}^{*}V_{cd}}{V_{cb}^{*}V_{ud}} \right| \times \left| C \right|/\left| T + C \right| \approx 0.006 \]

color suppression
For the first time, we include $B \rightarrow D\pi$ into a $\gamma$ measurement.

Data are compatible with rather high values of $r_B^{\pi}$

Sensitivity scales roughly like $1/r_B^{\pi}$

$\color{suppression}{r_B^{\pi} \approx \left| (V_{ub}^* V_{cd})/(V_{cb}^* V_{ud}) \right| \times |C|/|T + C| \approx 0.006}$

For the first time, we include $B \rightarrow D\pi$ into a $\gamma$ measurement.
$B \to D K$ and $B \to D\pi$

![Graphs showing distributions of $1-CL$ vs. $r_B^K$ and $r_B^\eta$ with LHCb data.](image)
B → DK and B → Dπ

high $\gamma$ corresponds to large $r_B\pi$

intrinsic angular symmetry

 naïve statistical treatment

Validation

- Goodness-of-fit probability:

<table>
<thead>
<tr>
<th>Combination</th>
<th>$n_{\text{obs}}$</th>
<th>$n_{\text{fit}}$</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$P[%]$ (pseudoexperiments)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DK^\pm$</td>
<td>26</td>
<td>15</td>
<td>7.47</td>
<td>83.2 ± 0.9</td>
</tr>
<tr>
<td>$D\pi^\pm$</td>
<td>19</td>
<td>14</td>
<td>3.00</td>
<td>86.3 ± 0.8</td>
</tr>
<tr>
<td>full</td>
<td>35</td>
<td>17</td>
<td>12.24</td>
<td>88.2 ± 0.7</td>
</tr>
</tbody>
</table>

- Coverage test. Intervals for $\gamma$ are **corrected for undercoverage**.

<table>
<thead>
<tr>
<th>Combination</th>
<th>$\eta$</th>
<th>$\alpha$ (plug-in)</th>
<th>$\alpha$ (profile likelihood)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DK^\pm$</td>
<td>0.6827 (1σ)</td>
<td>0.6646 ± 0.0067</td>
<td><strong>0.6299 ± 0.0069</strong></td>
</tr>
<tr>
<td></td>
<td>0.9545 (2σ)</td>
<td>0.9453 ± 0.0032</td>
<td>0.9318 ± 0.0036</td>
</tr>
<tr>
<td>$D\pi^\pm$</td>
<td>0.6827 (1σ)</td>
<td>0.6532 ± 0.0048</td>
<td>0.6019 ± 0.0050</td>
</tr>
<tr>
<td></td>
<td>0.9545 (2σ)</td>
<td>0.9492 ± 0.0022</td>
<td>0.9389 ± 0.0024</td>
</tr>
<tr>
<td>$DK^\pm$ and $D\pi^\pm$</td>
<td>0.6827 (1σ)</td>
<td>0.6616 ± 0.0067</td>
<td>0.6156 ± 0.0069</td>
</tr>
<tr>
<td></td>
<td>0.9545 (2σ)</td>
<td>0.9586 ± 0.0028</td>
<td>0.9352 ± 0.0035</td>
</tr>
</tbody>
</table>

- Berger-Boos-like method: confirms intervals.
- Bayesian approach: confirms intervals.
- Assign systematic error due to some neglected syst. correlations.
corrected results

The results, corrected for undercoverage and neglected systematic correlations, are:

\[ B^{\pm} \rightarrow DK^{\pm} \]

\[ \gamma = (70.5^{+14.9}_{-15.6})^\circ \text{ at } 68\% \text{ CL} \]

---

\[ B^{\pm} \rightarrow DK^{\pm} \text{ and } B^{\pm} \rightarrow D\pi^{\pm} \]

\[ \gamma = 86.0^\circ \]

\[ \gamma \in [49.0, 72.5]^\circ \cup [79.1, 94.2]^\circ \text{ at } 68\% \text{ CL} \]
A new GGSZ result

$LHCb$ preliminary $\int L \, dt = 2.0 \, fb^{-1}$

- $B^\pm \rightarrow DK^\pm$,
- $D \rightarrow K_S^0 \pi^+ \pi^-$

plots show “down-down” $K_S$ reconstruction only

$D \rightarrow K^0_S K^+ K^-$
A new GGSZ result

$B^\pm \rightarrow D K^{\pm}$,

$D \rightarrow K_S^0 \pi^+ \pi^-$

new!

$D \rightarrow K_S^0 K^+ K^-$
A new GGSZ result

2012 result

dominant internal systematics:
assumption of no CPV in $B \rightarrow D\pi$
second leading: fit shape

\[
\begin{align*}
x_+ &= (-8.7 \pm 3.1 \pm 1.6 \pm 0.6) \times 10^{-2} \\
x_- &= (5.3 \pm 3.2 \pm 0.9 \pm 0.9) \times 10^{-2} \\
y_+ &= (0.1 \pm 3.6 \pm 1.4 \pm 1.9) \times 10^{-2} \\
y_- &= (9.9 \pm 3.6 \pm 2.2 \pm 1.6) \times 10^{-2}
\end{align*}
\]
combined 1 fb$^{-1}$+2 fb$^{-1}$ GGSZ result

\[ \begin{aligned}
\langle x_+ \rangle &= (-8.9 \pm 3.1) \times 10^{-2} \\
\langle y_+ \rangle &= (-0.1 \pm 3.7) \times 10^{-2} \\
\langle x_- \rangle &= (3.5 \pm 2.9) \times 10^{-2} \\
\langle y_- \rangle &= (7.9 \pm 3.8) \times 10^{-2}
\end{aligned} \]

combined taking into account systematic correlations (CLEO phase information)

3-dimensional Feldman-Cousins, projecting the 20% CL shape

\[ \begin{aligned}
\gamma &= (57 \pm 16)\degree \\
\delta_B^K &= (124^{+15}_{-17})\degree \\
\rho_B^K &= (8.8^{+2.3}_{-2.4}) \times 10^{-2}
\end{aligned} \]
impact on LHCb $\gamma$ ($B \rightarrow DK$)

Comparing:
1 fb$^{-1}$ GLW/ADS and 1 fb$^{-1}$ GGSZ

naïve statistical treatment
impact on LHCb $\gamma$ ($B \to DK$)

Comparing:
1 fb$^{-1}$ GLW/ADS and 3 fb$^{-1}$ GGSZ

naïve statistical treatment
impact on LHCb $\gamma (B \rightarrow DK)$

Comparing:
- $1 \text{fb}^{-1} B \rightarrow DK$
- $3 \text{fb}^{-1} B \rightarrow DK$

full statistical treatment

$\gamma = (67 \pm 12)^{\circ}$ at 68% CL preliminary
Conclusion

- LHCb has a complete set of 1 fb$^{-1}$ results: GLW, ADS, GGSZ
- New results using 3 fb$^{-1}$ start to appear.
- The “factory approach” by LHCb starts going beyond the traditional methods.
- $B \rightarrow D\pi$ modes used to measure $\gamma$.
- As the precision increases, we will soon have to be more accurate with D mixing.
- The overall consistency is impressive: goodness-of-fit, predictions of observables, agreement with BaBar and Belle, ...

\[ \gamma = (70.5^{+14.9}_{-15.6})^\circ \text{ at } 68\% \text{ CL} \]

1 fb$^{-1}$ LHCb measurements

\[ LHCb-CONF-2013-006 \]

We understand what we're doing!
Backup
Outlook

- model dependent GGSZ
- model independent GGSZ: $B \to D\pi$
- $B \to DK, D \to K_SK\pi$ (ADS)
- time dependent $B_s \to D_s K$
- time dependent $B^0 \to D\pi$
- Bayesian combination
- ...
LHCb

- $b\bar{b}$ pair production angles strongly correlated
- covers $1.9 < \eta < 4.9$
- 100'000 $bb$ pairs produced per second ($10^4 \times B$ factories)

$\sigma(b\bar{b}) = 284 \pm 53\mu b$  \hspace{1cm} [PLB 694 (2010) 209]

$\sigma(c\bar{c}) \approx 20 \times \sigma(b\bar{b})$  \hspace{1cm} [LHCb-CONF-2010-013]
flavor tagging
Luminosity

LHCb Integrated Luminosity pp collisions 2010-2012

\[(1.11 + 2.08) \text{ fb}^{-1}\]
LHCb – Kaon/pion separation

- Ring Imaging Cherenkov Detectors
- 3 radiators covering wide momentum range

\[ \cos \theta = \frac{1}{\beta n} \]
B → D(hh)K: Results

\[
\begin{align*}
R_{K/\pi}^{K\pi} &= 0.0774 \pm 0.0012 \pm 0.0018 \\
R_{K/\pi}^{KK} &= 0.0773 \pm 0.0030 \pm 0.0018 \\
R_{K/\pi}^{\pi\pi} &= 0.0803 \pm 0.0056 \pm 0.0017 \\
A_{\pi}^{K\pi} &= -0.0001 \pm 0.0036 \pm 0.0095 \\
A_{K}^{K\pi} &= 0.0044 \pm 0.0144 \pm 0.0174 \\
A_{K}^{KK} &= 0.1480 \pm 0.0369 \pm 0.0097 \\
A_{\pi}^{\pi\pi} &= 0.1351 \pm 0.0661 \pm 0.0095 \\
A_{\pi}^{KK} &= -0.0199 \pm 0.0091 \pm 0.0116 \\
A_{\pi}^{\pi\pi} &= -0.0009 \pm 0.0165 \pm 0.0099 \\
R_{K}^{-} &= 0.0073 \pm 0.0023 \pm 0.0004 \\
R_{K}^{+} &= 0.0232 \pm 0.0034 \pm 0.0007 \\
R_{\pi}^{-} &= 0.00469 \pm 0.00038 \pm 0.00008 \\
R_{\pi}^{+} &= 0.00352 \pm 0.00033 \pm 0.00007
\end{align*}
\]
multi-body D decays

- Interference can only occur at same points in phase space, i.e. the requirement “same final state” is not enough.
- The magnitudes of the D decay amplitudes and the strong phase difference become **functions of the phase space**.
- Introduce effective quantities averaged over phase space!

\[
r_{K3\pi}^2 = \frac{\int \bar{A}_D(\bar{m})^2 d\bar{m}}{\int A_D(\bar{m})^2 d\bar{m}}
\]

\[
\kappa_{K3\pi} e^{i\delta_{K3\pi}} = \frac{\int A_D(\bar{m}) \bar{A}_D(\bar{m}) e^{i\delta(\bar{m})} d\bar{m}}{\sqrt{\int \bar{A}_D(\bar{m})^2 d\bar{m} \times \int A_D(\bar{m})^2 d\bar{m}}}
\]

\[
R_\pm = r_B^2 + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\pm \gamma + \delta_B + \delta_{K3\pi})
\]

the “coherence factor”, external input

a new (eff.) strong phase diff.
four-body ADS

“LHCb-style” observables:

\[
\begin{align*}
R_{K^3/\pi}^K &= R_{cab} \frac{1 + r_B^2 r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B T_{K^3/\pi} \cos(\delta_B - \delta_{K^3/\pi}) \cos \gamma}{1 + r_B^{\pi^2} r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \cos(\delta_B^{\pi^2} - \delta_{K^3/\pi}) \cos \gamma}, \\
A_{K^3/\pi} &= \frac{2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \sin(\delta_B - \delta_{K^3/\pi}) \sin(\gamma)}{1 + r_B^{\pi^2} r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \cos(\delta_B^{\pi^2} - \delta_{K^3/\pi}) \cos \gamma}, \\
A_{K^3/\pi} &= \frac{2 \kappa_{K^3/\pi} r_B r_{K^3/\pi} \sin(\delta_B - \delta_{K^3/\pi}) \sin(\gamma)}{1 + r_B^2 r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B r_{K^3/\pi} \cos(\delta_B - \delta_{K^3/\pi}) \cos \gamma}, \\
R_{\pi^-}^{K3} &= \frac{r_B^{\pi^2} + r_{K^3/\pi}^{\pi^2} + 2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \cos(\delta_B^{\pi^2} + \delta_{K^3/\pi} - \gamma)}{1 + r_B^{\pi^2} r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \cos(\delta_B^{\pi^2} - \delta_{K^3/\pi} - \gamma)}, \\
R_{\pi^+}^{K3} &= \frac{r_B^{\pi^2} + r_{K^3/\pi}^{\pi^2} + 2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \cos(\delta_B^{\pi^2} + \delta_{K^3/\pi} + \gamma)}{1 + r_B^{\pi^2} r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B^{\pi^2} r_{K^3/\pi} \cos(\delta_B^{\pi^2} - \delta_{K^3/\pi} + \gamma)}, \\
R_{K^-}^{K3} &= \frac{r_B^2 + r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B T_{K^3/\pi} \cos(\delta_B + \delta_{K^3/\pi} - \gamma)}{1 + r_B^2 r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B T_{K^3/\pi} \cos(\delta_B - \delta_{K^3/\pi} - \gamma)}, \\
R_{K^+}^{K3} &= \frac{r_B^2 + r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B T_{K^3/\pi} \cos(\delta_B + \delta_{K^3/\pi} + \gamma)}{1 + r_B^2 r_{K^3/\pi}^2 + 2 \kappa_{K^3/\pi} r_B T_{K^3/\pi} \cos(\delta_B - \delta_{K^3/\pi} + \gamma)}.
\end{align*}
\]

\(D^0 \rightarrow K \pi \pi \pi\)
four-body ADS

\[ R^{K3\pi}_{K/\pi} \equiv \frac{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^+]_{D}K^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_{D}K^+)}{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^+]_{D}\pi^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_{D}\pi^+)} , \]

\[ A^{K3\pi}_{h} \equiv \frac{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^-]_{D}h^-) - \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_{D}h^+)}{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^-]_{D}h^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_{D}h^+)} , \]

\[ R^{K3\pi,\pm}_{h} \equiv \frac{\Gamma(B^\pm \to [\pi^\pm K^+\pi^-\pi^-]_{D}h^\pm)}{\Gamma(B^\pm \to [K^\pm\pi^+\pi^-\pi^-]_{D}h^\pm)} . \]

\[ R^{K3\pi}_{K/\pi} = 0.0771 \pm 0.0017 \pm 0.0026 \]
\[ A^{K3\pi}_{K} = -0.029 \pm 0.020 \pm 0.018 \]
\[ A^{K3\pi}_{\pi} = -0.006 \pm 0.005 \pm 0.010 \]
\[ R^{K3\pi,-}_{K} = 0.0072 \pm 0.0036 \pm 0.0008 \]
\[ R^{K3\pi,+}_{K} = 0.0175 \pm 0.0043 \pm 0.0010 \]
\[ R^{K3\pi,-}_{\pi} = 0.00417 \pm 0.00054 \pm 0.00011 \]
\[ R^{K3\pi,+}_{\pi} = 0.00321 \pm 0.00048 \pm 0.00011 \]
Express GLW observables in terms of cart. coordinates:

\[ x_\pm = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{4} \]

\[ r^2 = x_\pm^2 + y_\pm^2 = \frac{R_{CP+} + R_{CP-} - 2}{2} \]
Plugin method

Scan for one specific physics parameter, $x$:

1. Find global minimum $\chi^2_{\text{min}}$ and the most probable values for $x$.
2. Fix $x$ to $x_0$ and minimize with respect to the non-fixed parameters, i.e. obtain $\bar{x}'$, and $(\chi^2_{\text{min}})'$. Calculate $\Delta \chi^2 = \chi^2_{\text{min}} - (\chi^2_{\text{min}})'$.
3. Generate a Toy MC result for $\tilde{y}$, $\tilde{y}_{\text{toy}}$, by interpreting the likelihood as a PDF of $\tilde{y}$.
4. Repeat the first two steps on the toy result, i.e. calculate $\Delta \chi^2_{\text{toy}}$.
5. Calculate $(1 - \text{CL})$ as the fraction

$$1 - \text{CL} = \frac{N(\Delta \chi^2_{\text{toy}} > \Delta \chi^2)}{N_{\text{toy}}}.$$  \hfill (5)

Doesn't guarantee coverage (but tends to be close).

Use the best fit-values values for the parameters.
Agreement of inputs

• Make a test:
  • predict the traditional ADS observables, $R_{ADS}$, $A_{ADS}$, in $B \to DK$, $D \to K\pi$, using all other LHCb $1fb^{-1}$ inputs
  • the agreement is impressive
impact on LHCb $\gamma$ (B $\rightarrow$ DK)

Comparing:
1 fb$^{-1}$ GLW/ADS and
1 fb$^{-1}$ GGSZ

full statistical treatment
impact on LHCb $\gamma$ ($B \to DK$)

Comparing:
1 fb$^{-1}$ GLW/ADS and 3 fb$^{-1}$ GGSZ

full statistical treatment