Measurement of $\gamma$ from $B^\pm \to Dh^\pm$ decays at LHCb—including the effect of $D^0-\bar{D}^0$ mixing

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I present a measurement of the CKM angle $\gamma$ from a combination of three LHCb measurements using the tree decays $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$. These measurements are based on a dataset corresponding to 1.0 fb$^{-1}$, collected in 2011. In contrast to what was presented at FPCP, these proceedings fully include the effect of $D^0-\bar{D}^0$ mixing on the combination. I also report on the inclusion of new, preliminary results on the Dalitz analysis of $B^\pm \to DK^\pm$, $D^0 \to K^0_S h^\pm h^\mp$, using a dataset corresponding to 3 fb$^{-1}$, collected in 2011–2012.

\textit{presented at}


1 Introduction

The CKM angle $\gamma$ is the least well measured angle of the unitarity triangle of the CKM matrix. The existing constraints come mainly from the $B$ factories BaBar and Belle, and mainly use analyses of tree-level $B^\pm \to DK^\pm$ decays. Both collaborations have independently combined all their relevant measurements in a frequentist way, and arrive at $\gamma = (69^{+17}_{-16})^\circ$ (BaBar \cite{1}) and $\gamma = (68^{+15}_{-14})^\circ$ (Belle \cite{2}).

The LHCb collaboration now has results on $\gamma$ available as well. As I presented at FPCP, and summarize in these proceedings, these too are combined in a frequentist way, and already with 1 fb$^{-1}$ of 2011 data LHCb reaches a precision comparable to that of the $B$ factories. For this not only the traditional $B^\pm \to DK^\pm$ decays are used, but also $B^\pm \to D\pi^\pm$ decays. Up to now these were mostly seen as pure normalization channels as their sensitivity to $\gamma$ is smaller than that of $B^\pm \to DK^\pm$ decays. But they do contribute to the overall precision.

Although $\gamma$ is still statistically limited, its uncertainty is steadily decreasing as new measurements appear. At some point systematic effects, that traditionally were neglected, need to be explicitly considered. Most prominently this is the effect of $D^0-\bar{D}^0$ mixing, which already now affects the analysis of $B^\pm \to D\pi^\pm$ decays. But also the possible effect of CP violation in $D^0$ decays plays a role. As a result of fruitful discussions held at FPCP, the combined $\gamma$ measurement by LHCb is now the first combination of measurements that explicitly considers both effects.

Anticipating the following sections, LHCb measures a value of $\gamma = (72.6^{+9.1}_{-15.9})^\circ$ \cite{3}. The agreement between the combinations by Belle, BaBar, and LHCb is impressive. It is very reassuring to see the theoretical framework under good control, and to see the efforts of over a decade of $e^+e^-$ physics and results obtained in LHC’s hadronic environment coming together so nicely. It is important, however, to keep in mind, that the three values are not entirely independent—they all share input measurements on the hadronic $D^0$ parameters. One prominent such parameter is the strong phase difference $\delta_{K\pi}$ between the decay amplitudes of $D^0 \to K^-\pi^+$ and $\bar{D}^0 \to K^-\pi^+$, which is, in the important ADS measurement of $B^\pm \to DK^\pm$, $D \to K\pi$, additive to $\gamma$ but also not very precisely known. For example, the latest HFAG CP-violation-allowed average is, in our phase convention, $\delta_{K\pi} = (199.5^{+8.6}_{-11.1})^\circ$ \cite{4}.

\textsuperscript{a}on behalf of the LHCb collaboration
2 Input measurements using 1.0 fb⁻¹

LHCb has published three measurements of \( B^\pm \rightarrow D K^\pm \) decays relevant to this \( \gamma \) measurement. The first two are a GLW/ADS measurement using two-body \( D^0 \) final states \([5]\), and an ADS measurement using the four-body \( D^0 \) final state \( K^{\pm} \pi^{\mp} \pi^{\pm} \pi^- \) \([6]\). Both of these measurements also use \( B^\pm \rightarrow D \pi^\pm \) decays, in addition to the established \( B^\pm \rightarrow D K^\pm \) decays. The third measurement is a Dalitz-model independent GGSZ analysis of \( D^0 \rightarrow R^0 h^\pm h^\mp \) decays \([7]\). All three measurements are based on 1 fb⁻¹ of data recorded in 2011, constituting about one third of the data recorded to date.

The above three main inputs are combined with information on the hadronic \( D^0 \) parameters, on \( CP \) violation in \( D^0 \rightarrow h^+ h^- \) decays, and on the \( D^0 - \bar{D}^0 \) mixing parameters, taken from CLEO \([8]\), HFAG \([4]\), and LHCb \([9]\), respectively.

3 Effect of \( D^0 - \bar{D}^0 \) mixing

It was known since many years that \( D^0 - \bar{D}^0 \) mixing complicates the measurement of \( \gamma \) from \( B^\pm \rightarrow D K^\pm \) decays (and \( B^\pm \rightarrow D \pi^\pm \) decays) \([10-13]\). Traditionally, the effect was neglected, as it was estimated to be small compared to the achievable uncertainty on \( \gamma \). But it was clear that, in order to reach a degree-precision on \( \gamma \), the effect of \( D^0 - \bar{D}^0 \) mixing cannot be neglected anymore. It is straightforward to account for mixing in the relevant equations, fully retaining the outstanding theoretical cleanliness of the \( \gamma \) measurement.

Recently, the \( D^0 - \bar{D}^0 \) mixing effects on \( \gamma \) measurements were nicely summarized in Ref. \([14]\). In general, the effect is larger for \( B^\pm \rightarrow D \pi^\pm \) decays than for \( B^\pm \rightarrow D K^\pm \) decays, because the amplitude ratio \( r_B^K \) is about an order of magnitude smaller than \( r_B^\pi \). At leading order in the mixing parameters \( x \) and \( y \), and neglecting \( CP \) violation in mixing, (i) the GLW measurements are unaffected for both \( D K^\pm \) and \( D \pi^\pm \), (ii) the \( D K^\pm \) model-independent GGSZ measurement is unaffected, (iii) the \( D K^\pm \) ADS analysis is affected at the degree level, and (iv) the \( D \pi^\pm \) ADS analysis is largely affected, as the terms describing \( D^0 - \bar{D}^0 \) mixing are of the same order of magnitude as the interference terms giving the sensitivity to \( \gamma \).

The \( D^0 - \bar{D}^0 \) mixing effects also depend on the selection criteria that were used to select \( B^\pm \rightarrow Dh^\pm \) decays \([14]\). Since the \( D^0 \) decay time is a powerful discriminating variable, the \( D^0 \) decay time acceptance will not be flat, which needs to be considered in the measurement of \( \gamma \).

The ADS observables which receive mixing corrections are the charge-averaged ratios of \( B^\pm \rightarrow D K^\pm \) and \( B^\pm \rightarrow D \pi^\pm \) decays

\[
R_{K/\pi}^f = \frac{\Gamma(B^- \rightarrow D[\rightarrow f]K^-) + \Gamma(B^+ \rightarrow D[\rightarrow \bar{f}]K^+)}{\Gamma(B^- \rightarrow D[\rightarrow f]\pi^-) + \Gamma(B^+ \rightarrow D[\rightarrow \bar{f}]\pi^+)} , \tag{1}
\]

where \( f \) is the relevant final state. The ratios \( R_{K/\pi}^f \) are related to \( \gamma \) and the hadronic parameters through

\[
R_{K/\pi}^f = \frac{1 + (r_B^K r_f^K)^2 + 2r_B^K r_f^K \cos(\delta_B^K - \delta_f) \cos \gamma + M_{-}^K + M_{+}^K}{1 + (r_B^\pi r_f^\pi)^2 + 2r_B^\pi r_f^\pi \cos(\delta_B^\pi - \delta_f) \cos \gamma + M_{-}^\pi + M_{+}^\pi} , \tag{2}
\]

where \( \kappa \) denotes the coherence factor. The \( D \) mixing correction terms \( M_{\pm}^h \) are \([10]\)

\[
M_{\pm}^h = \left( \kappa r_f ((r_f^h)^2 - 1) \sin \delta_f + r_B^h (1 - r_f^2) \sin(\delta_B^h \pm \gamma) \right) a_D x_D \\
- \left( \kappa r_f ((r_f^h)^2 + 1) \cos \delta_f + r_B^h (1 + r_f^2) \cos(\delta_B^h \pm \gamma) \right) a_D y_D . \tag{3}
\]

The coefficient \( a_D \) parameterizes the effect of a finite \( D^0 \) decay resolution and non-flat acceptance. It takes the value of \( a_D = 1 \) in case of an ideal, flat acceptance and negligible time resolution.
For a realistic acceptance and resolution model present in the two- and four-body GLW/ADS analyses of Refs. [5,6], it is estimated to be $a_D = 1.20 \pm 0.04$. The charge asymmetries are

$$A^f_h = \frac{\Gamma(B^- \to D[\to f]h^-) - \Gamma(B^+ \to D[\to f]h^+)}{\Gamma(B^- \to D[\to f]h^-) + \Gamma(B^+ \to D[\to f]h^+)},$$

which are related to $\gamma$ and the hadronic parameters through

$$A^f_h = \frac{2r^h_B r_f \kappa \sin(\delta_B^h - \delta_f) \sin \gamma + M_+^h - M_-^h}{1 + (r^h_B r_f)^2 + 2r^h_B r_f \kappa \cos(\delta_B^h - \delta_f) \cos \gamma + M_+^h + M_-^h},$$

where $r_B^h$ denotes $r_B^K$ and $r_B^\pi$. Finally, the non charge-averaged ratios of suppressed and favored $D$ final states are

$$R^\pm_h = \frac{\Gamma(B^\pm \to D[\to f_{\text{sup}}]h^\pm)}{\Gamma(B^\pm \to D[\to f]h^\pm)} = \frac{r_f^2 + (r_B^h)^2 + 2r_B^h r_f \kappa \cos(\delta_B^h + \delta_f \pm \gamma) - [M_{\pm}^h]_{\text{sup}}}{1 + (r_B^h r_f)^2 + 2r_B^h r_f \kappa \cos(\delta_B^h - \delta_f \pm \gamma) + M_{\pm}^h}.$$

where $f_{\text{sup}}$ is the suppressed final state, and $f$ the allowed one. The suppressed $D$ mixing correction terms are given, at leading order in $x_D$ and $y_D$, by

$$[M_{\pm}^h]_{\text{sup}} = \left(\kappa r_f ((r_B^h)^2 - 1) \sin \delta_f + r_B^h (1 - r_f^2) \sin(\delta_B^h \pm \gamma)\right) a_D x_D + \left(\kappa r_f ((r_B^h)^2 + 1) \cos \delta_f + r_B^h (1 + r_f^2) \cos(\delta_B^h \pm \gamma)\right) a_D y_D.$$

4 Results on $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$, using 1.0 fb$^{-1}$

The combination follows a frequentist procedure (plug-in method) and is fully described in Ref. [3]. The frequentist coverage is computed at the best-fit point, and the final confidence intervals are enlarged to correct for a small undercoverage. The results for $\gamma$ are shown in Table 1 for three cases: only combining $B^\pm \to DK^\pm$ inputs, only combining $B^\pm \to D\pi^\pm$ inputs, and the full $DK^\pm$ and $D\pi^\pm$ combination. The $B^\pm \to D\pi^\pm$ decays do indeed add sensitivity to the full combination. This can be explained by the observed high value of $r_B^\pi$, that drives the sensitivity, $r_B^\pi = 0.015_{-0.009}^{+0.012}$ at 68% CL (obtained from the full combination), which is much larger than (but consistent with) the expectation of $r_B^\pi = 0.006$, as estimated from Ref. [15]. Figures 1 and 2 illustrate these results.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$DK^\pm$ combination</th>
<th>$D\pi^\pm$ combination</th>
<th>$DK^\pm$ and $D\pi^\pm$ combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>72.0°</td>
<td>18.9°</td>
<td>72.6°</td>
</tr>
<tr>
<td>68% CL</td>
<td>[56.4, 86.7]°</td>
<td>[7.4, 99.2]° $\cup$ [167.9, 176.4]°</td>
<td>[55.4, 82.3]°</td>
</tr>
<tr>
<td>95% CL</td>
<td>[42.6, 99.6]°</td>
<td>no constraint</td>
<td>[40.2, 92.7]°</td>
</tr>
</tbody>
</table>

5 General agreement of input measurements

The agreement of the input measurements is excellent, leading to fit probabilities well above 50%. These are estimated using both a simple $\chi^2$ test, and a more accurate approach with pseudoexperiments, and given in Table 2.

We also made an internal consistency check of the $B^\pm \to DK^\pm$ combination, inspired by K. Trabelsi, by excluding the ADS observables $R_{\text{ADS}}$ and $A_{\text{ADS}}$ and instead predicting them.
Figure 1: Graphs showing $1 - \text{CL}$ for $\gamma$, for (a) the $DK^\pm$ combination, for (b) the $D\pi^\pm$ combination, and (c) for the full $DK^\pm$ and $D\pi^\pm$ combination. The reported numbers correspond to the best-fit values and the uncertainties are computed using the respective 68.3% CL confidence intervals (not corrected for undercoverage and neglected systematic correlations).

Figure 2: Profile likelihood contours of (a) $\gamma$ vs. $r^K_B$, and (b) $\gamma$ vs. $r^\pi_B$, for the full $DK^\pm$ and $D\pi^\pm$ combination. The contours are the $n\sigma$ profile likelihood contours, where $\Delta \chi^2 = n^2$ with $n = 1, 2$. The markers denote the best-fit values.

using all other LHCb inputs. The prediction agrees beautifully with the measurements, as shown in Figure 3. In fact, Belle sees a very similar picture (also in Fig. 3).

Table 2: Numbers of observables $n_{\text{obs}}$, numbers of free parameters in the fit $n_{\text{fit}}$, the minimum $\chi^2$ at the best-fit point, and fit probabilities of the best-fit point for the three combinations. The quoted uncertainties are due to the limited number of pseudoexperiments.

<table>
<thead>
<tr>
<th>Combination</th>
<th>$n_{\text{obs}}$</th>
<th>$n_{\text{fit}}$</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$P[%]$ ($\chi^2$ distribution)</th>
<th>$P[%]$ (pseudoexperiments)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DK^\pm$</td>
<td>29</td>
<td>15</td>
<td>10.48</td>
<td>72.6</td>
<td>73.9 ± 0.2</td>
</tr>
<tr>
<td>$D\pi^\pm$</td>
<td>22</td>
<td>14</td>
<td>6.28</td>
<td>61.6</td>
<td>61.2 ± 0.3</td>
</tr>
<tr>
<td>full</td>
<td>38</td>
<td>17</td>
<td>13.06</td>
<td>90.6</td>
<td>90.9 ± 0.1</td>
</tr>
</tbody>
</table>

$^b$Note that the nominal combination does not use $R_{ADS}$ and $A_{ADS}$ directly, but they were re-introduced for the purpose of this test.
6 Preliminary results on $B^\pm \to DK^\pm$, $D^0 \to K^0_S h^\pm h^\mp$, using 3.0 fb$^{-1}$

Going beyond the published analyses using 1 fb$^{-1}$, LHCb has also a new preliminary result of the GGSZ analysis of $B^\pm \to DK^\pm$, $D^0 \to K^0_S h^\pm h^\mp$, using 3.0 fb$^{-1}$ [16]. When these are used in the $DK^\pm$ only combination, the best single-experiment precision for $\gamma$ is achieved: $\gamma = (67 \pm 12)^\circ$. Figure 4 shows the corresponding, quite Gaussian 1 − CL graph.

7 Conclusion

The CKM angle $\gamma$ is now being constrained by three collaborations, Babar, Belle, and LHCb. It wasn’t until recently that all have published their individual averages, so that now all three can be seen side by side: They all reach similar precision, and they all agree. Despite this being a big success, it is clear that one wants to go beyond the current precision, aiming at a degree uncertainty on $\gamma$. Along this way it is mandatory to correctly consider small systematic effects, that have been neglected so far—most notably the effect of $D^0$–$\bar{D}^0$ mixing. The LHCb combination is the first to do so.
References


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