CP phase from Higgs's boundary condition

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We propose a new mechanism to generate a CP phase originating from a non-trivial Higgs vacuum expectation value in an extra dimension. A twisted boundary condition for the Higgs doublet can produce an extra dimensional coordinate-dependent vacuum expectation value containing a CP phase degree of freedom. With this mechanism, we construct a phenomenological model on S^1 which can simultaneously and naturally explain the origins of the fermion generations, the quark mass hierarchy and the structure of the Cabibbo-Kobayashi-Maskawa matrix with the CP phase[1].

I. INTRODUCTION

A quest for the origins of the quark mass hierarchy, the structure of the flavor mixing, and the three generations of the fermions is one of the important tasks in particle physics. Lots of experiments have succeeded in measuring values of the quark masses and the elements of the Cabbibo-Kobayashi-Maskawa (CKM) matrix with good precision. A complex phase in the CKM matrix due to the three generations has been proposed to explain an origin of the CP violation [2], and the existence of the CP phase has been well established by B physics experiments. However, the Standard Model (SM) does not initiate us into the origins of quark mass hierarchy, the structure of the flavor mixing, and three generations of the fermions even though the SM contains these structures. Thus we consider that there is a more fundamental theory beyond the SM.

In the context of higher dimensional field theories, which are one of the candidates of beyond the SM, we propose a new mechanism to produce a CP phase and construct a five-dimensional (5D) phenomenological model with S^1 compactification which can naturally explain all the flavor structure of the SM, i.e. the origins of the fermion generations, the quark mass hierarchy and the structure of the CKM matrix with the CP phase. We put point interactions on S^1, which are additional boundary points on S^1, to realized the three generations from a single 5D fermion. It should be emphasized that 5D Yukawa couplings cannot be the origin of the CP phase in our model because the model contains only a single generation fermions for each 5D quark. A twisted boundary condition (BC) for the Higgs doublet leads to a CP phase degree of freedom. The Higgs VEV with an extra dimensional coordinate dependent phase is a key and will be derived in the next section. We also introduce an extra dimensional coordinate dependent vacuum expectation value (VEV) of a gauge singlet scalar field to realize the quark mass hierarchy. The Robin BC for the gauge singlet scalar field can lead to suitable form of the VEV. The structure of the flavor mixing is determined by the geometry of the extra dimension in our model.

II. HIGGS VEV WITH TWISTED BOUNDARY CONDITION

First, we discuss the property of the VEV of a $SU(2)_W$ Higgs doublet H with a twisted boundary condition on S^1. The action we consider is

$$S_H = \int d^4x \int_0^L dy \Big[-|\partial_M H|^2 + M^2 |H|^2 - \frac{\lambda}{2}|H|^4 \Big]. \tag{1}$$

We impose the twisted boundary condition on H as

$$H(y+L) = e^{i\theta} H(y). \tag{2}$$

Here, we take the range of θ as $-\pi < \theta \leq \pi$. We will obtain the VEV of $\langle H(y) \rangle$ minimizing the functional

$$\mathcal{E}[H] = \int_0^L dy \Big[\ |\partial_y H|^2 - M^2 |H|^2 + \frac{\lambda}{2}|H|^4 \ \Big]. \tag{3}$$

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[*] This talk is given by Y.Fujimoto in the conference HPNP2013.
[1] This talk is based on Ref.[1]

To find the minimization condition of the functional $\mathcal{E}$, we introduce $\mathcal{H}(y)$ as $H(y) = e^{i\frac{\theta}{L}y}\mathcal{H}(y)$, $\mathcal{H}(y+L) = \mathcal{H}(y)$. The VEV of $\langle \mathcal{H}(y) \rangle$ which minimize the functional $\mathcal{E}$ will lead us to the VEV of $\langle H(y) \rangle$. See Ref.[3] in details. The VEV $\langle H(y) \rangle$ is given, without any loss of generality, as follows:

- For $M^2 - \left(\frac{\theta}{L}\right)^2 > 0$ case

$$\langle H \rangle = \begin{cases} \frac{v}{\sqrt{2}} e^{i\frac{\theta}{L}y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \text{for} \quad -\pi < \theta < \pi \\ \frac{v}{\sqrt{2}} e^{i\frac{\pi}{L}y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{or} \quad \frac{v}{\sqrt{2}} e^{-i\frac{\pi}{L}y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \text{for} \quad \theta = \pi \end{cases}, \tag{4}$$

- For $M^2 - \left(\frac{\theta}{L}\right)^2 < 0$ case

$$\langle H(y) \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{5}$$

where v is given by

$$\left(\frac{v}{\sqrt{2}}\right)^2 := |\langle H(y) \rangle|^2 = \frac{1}{\lambda}\left(M^2 - \left(\frac{\theta}{L}\right)^2\right). \tag{6}$$

In the following, we will assume the case of $M^2 - \left(\frac{\theta}{L}\right)^2 > 0$. Now we discuss some properties of the derived VEV in Eq. (4). Differently from the SM, the VEV possesses y-position-dependence and its broken phase is realized only in the case of $M^2 - \left(\frac{\theta}{L}\right)^2 > 0$. But like the SM, the squared VEV (6) is still constant even though $\langle H(y) \rangle$ depends on y. This means that after $v/\sqrt{L}$ is set as 246 GeV, where the mass dimension of v is 3/2, the same situation as the SM occurs in the electroweak symmetry breaking (EWSB) sector. On the other hand, the y-dependence of the Higgs VEV in Eq. (4) is an important consequence for the Yukawa sector. Since the VEV of the Higgs doublet appears linearly in each Yukawa term, the overlap integrals which lead to effective 4D Yukawa couplings will produce non-trivial CP phase in the CKM matrix.

We also comment on the Higgs-quarks couplings in our model. The profiles of the VEV and the Higgs physical zero mode in our model are the same as $e^{i\frac{\theta}{L}y}$ up to the coefficients. This means that the strengths of the couplings are equivalent to those of the SM even though the mode function gets to be y-position dependent. As a result, the decay branching ratios of the Higgs boson are the same as those of the SM. Possible deviations in the partial widths of the one-loop induced processes could be small when we take the Kaluza-Klein (KK) scale around a few TeV.

III. THE MODEL WITH POINT INTERACTIONS ON S^1

Field localization in extra dimensions is known as an effective way of explaining the quark mass hierarchy and pattern of flavor mixing. For this purpose, we follow the strategy in [3], where point interactions are introduced in the bulk space to split and localize fermion profiles and also to produce a y-position-dependent VEV with an (almost) exponential shape, which generates the large fermion mass hierarchy. But in this letter, we set the extra dimension to be a circle S^1 not an interval as [3]. Under the situation, the twisted boundary condition (2) is compatible with the geometry. In the following part, we briefly explain how to construct our model. The 5D action for fermions is given by

$$S = \int d^4x \int_0^L dy \left[\bar{Q}(i\Gamma^M\partial_M + M_Q)Q + \bar{\mathcal{U}}(i\Gamma^M\partial_M + M_{\mathcal{U}})\mathcal{U} + \bar{\mathcal{D}}(i\Gamma^M\partial_M + M_{\mathcal{D}})\mathcal{D}\right], \tag{7}$$

where we introduce an $SU(2)_W$ doublet Q, an up-quark singlet $\mathcal{U}$, and a down-type singlet $\mathcal{D}$. We note that our model contains only one generation for 5D quarks but each 5D quark produces three generations of the 4D quarks, as we will see below.

We adopt the following BCs for $Q, \mathcal{U}, \mathcal{D}$ with an infinitesimal positive constant ε [3]:

$$Q_R = 0 \qquad \text{at} \quad y = L_0^{(q)} + \varepsilon,\ L_1^{(q)} \pm \varepsilon,\ L_2^{(q)} \pm \varepsilon,\ L_3^{(q)} - \varepsilon, \tag{8}$$

$$\mathcal{U}_L = 0 \qquad \text{at} \quad y = L_0^{(u)} + \varepsilon,\ L_1^{(u)} \pm \varepsilon,\ L_2^{(u)} \pm \varepsilon,\ L_3^{(u)} - \varepsilon, \tag{9}$$

$$\mathcal{D}_L = 0 \qquad \text{at} \quad y = L_0^{(d)} + \varepsilon,\ L_1^{(d)} \pm \varepsilon,\ L_2^{(d)} \pm \varepsilon,\ L_3^{(d)} - \varepsilon, \tag{10}$$

where Ψ_R and Ψ_L denote the eigenstates of γ^5, i.e. $\Psi_R \equiv \frac{1+\gamma^5}{2}\Psi$ and $\Psi_L \equiv \frac{1-\gamma^5}{2}\Psi$. A crucial consequence of the above BCs is that there appear three-fold degenerated left- (right-)handed zero modes in the mode expansions of Q ($\mathcal{U}, \mathcal{D}$) and that they form the three generations of the quarks. The details have been given in Ref. [3]. We will not repeat the discussions here.

The fields $Q, \mathcal{U}, \mathcal{D}$ with the BCs in Eqs (8)–(10) are KK-decomposed as follows:

$$Q(x,y) = \begin{pmatrix} U(x,y) \\ D(x,y) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{3} u_{iL}^{(0)}(x) f_{q_{iL}^{(0)}}(y) \\ \sum_{i=1}^{3} d_{iL}^{(0)}(x) f_{q_{iL}^{(0)}}(y) \end{pmatrix} + (\text{KK modes}), \tag{11}$$

$$\mathcal{U}(x,y) = \sum_{i=1}^{3} u_{iR}^{(0)}(x) f_{u_{iR}^{(0)}}(y) + (\text{KK modes}), \tag{12}$$

$$\mathcal{D}(x,y) = \sum_{i=1}^{3} d_{iR}^{(0)}(x) f_{d_{iR}^{(0)}}(y) + (\text{KK modes}). \tag{13}$$

Here the zero mode functions are obtained in the following forms:

$$f_{q_{iL}^{(0)}}(y) = \mathcal{N}_i^{(q)} e^{M_Q(y-L_{i-1}^{(q)})}\Big[\theta(y-L_{i-1}^{(q)})\theta(L_i^{(q)}-y)\Big] \qquad \text{in } [L_0^{(q)}, L_3^{(q)}], \tag{14}$$

$$f_{u_{iR}^{(0)}}(y) = \mathcal{N}_i^{(u)} e^{-M_{\mathcal{U}}(y-L_{i-1}^{(u)})}\Big[\theta(y-L_{i-1}^{(u)})\theta(L_i^{(u)}-y)\Big] \qquad \text{in } [L_0^{(u)}, L_3^{(u)}], \tag{15}$$

$$f_{d_{iR}^{(0)}}(y) = \mathcal{N}_i^{(d)} e^{-M_{\mathcal{D}}(y-L_{i-1}^{(d)})}\Big[\theta(y-L_{i-1}^{(d)})\theta(L_i^{(d)}-y)\Big] \qquad \text{in } [L_0^{(d)}, L_3^{(d)}], \tag{16}$$

where

$$\Delta L_i^{(l)} = L_i^{(l)} - L_{i-1}^{(l)} \qquad (\text{for } i=1,2,3;\ l=q,u,d), \tag{17}$$

$$\mathcal{N}_i^{(q)} = \sqrt{\frac{2M_Q}{e^{2M_Q \Delta L_i^{(q)}}-1}}, \quad \mathcal{N}_i^{(u)} = \sqrt{\frac{2M_{\mathcal{U}}}{1-e^{-2M_{\mathcal{U}}\Delta L_i^{(u)}}}}, \quad \mathcal{N}_i^{(d)} = \sqrt{\frac{2M_{\mathcal{D}}}{1-e^{-2M_{\mathcal{D}}\Delta L_i^{(d)}}}}. \tag{18}$$

$\mathcal{N}_i^{(q)}, \mathcal{N}_i^{(u)}, \mathcal{N}_i^{(d)}$ are the wavefunction normalization factors for $f_{q_{iL}^{(0)}}, f_{u_{iL}^{(0)}}, f_{d_{iL}^{(0)}}$, respectively.

Since the length of the total system is universal, $L_3^{(l)} - L_0^{(l)}$ ($l = q, u, d$) should be equal to the circumference of S^1, i.e.

$$L := L_3^{(q)} - L_0^{(q)} = L_3^{(u)} - L_0^{(u)} = L_3^{(d)} - L_0^{(d)}. \tag{19}$$

Note that all the mode functions in Eqs. (14)–(16) (and a form of a singlet VEV in Eq. (23)) are periodic with the common period L, whereas we do not indicate that thing explicitly in Eqs. (14)–(16).

In this model, the large mass hierarchy is naturally explained with the following Yukawa sector

$$S_{\mathcal{Y}} = \int d^4x \int_0^L dy \Big\{ \Phi\Big[-\mathcal{Y}^{(u)}\overline{Q}(i\sigma_2 H^*)\mathcal{U} - \mathcal{Y}^{(d)}\overline{Q}H\mathcal{D}\Big] + \text{h.c.}\Big\}, \tag{20}$$

where $\mathcal{Y}^{(u)}/\mathcal{Y}^{(d)}$ is the Yukawa coupling for up/down type quark; H and Φ are an $SU(2)_W$ scalar doublet and a singlet. It should be noted that although the Yukawa couplings $\mathcal{Y}^{(u)}$ and $\mathcal{Y}^{(d)}$ can be complex, they cannot be an origin of the CP phase of the CKM matrix because our model contains only a single quark generation, so that the number of the 5D Yukawa couplings is not enough to produce a CP phase in the CKM matrix. An schematic figure of our system is depicted in Fig. 1. Note that the five terms of $\overline{Q}(i\sigma_2 H^*)\mathcal{U}, \overline{Q}H\mathcal{D}, \Phi\overline{Q}Q, \Phi\overline{\mathcal{U}}\mathcal{U}, \Phi\overline{\mathcal{D}}\mathcal{D}$ with the Pauli matrix σ_2 are excluded by introducing a discrete symmetry $H \to -H, \Phi \to -\Phi$. Φ is a gauge singlet and there is no problem with gauge universality violation.

The 5D action and the BCs for Φ are assumed to be of the form [3, 4]

$$S_\Phi = \int d^4x \int_0^L dy \Big\{ \Phi^\dagger\Big(\partial_M\partial^M - M_\Phi^2\Big)\Phi - \frac{\lambda_\Phi}{2}\Big(\Phi^\dagger\Phi\Big)^2\Big\}, \tag{21}$$

$$\begin{aligned} \Phi + L_+\partial_y\Phi = 0 \qquad & \text{at} \quad y = L_0^{(\Phi)} + \varepsilon, \\ \Phi - L_-\partial_y\Phi = 0 \qquad & \text{at} \quad y = L_3^{(\Phi)} - \varepsilon, \end{aligned} \tag{22}$$

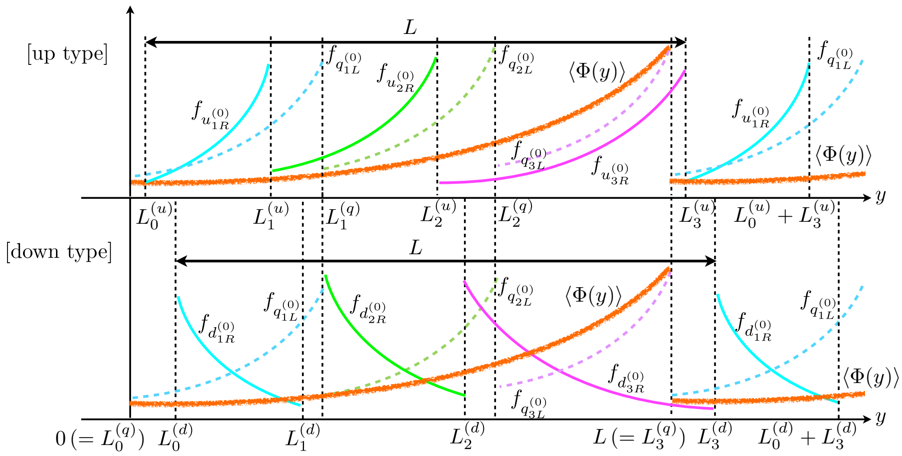

FIG. 1: The wavefunction profiles of the quarks and the VEV of $\Phi(y)$ are schematically depicted. Here we take $L_0^{(q)} = L_0^{(\Phi)} = 0$. Note that all the profiles have the periodicity along y with the same period L. Differently from the model on an interval in Ref. [3], we can find the $(1,3)$ elements of the mass matrices due to the periodicity along y-direction.

where $L_\pm$ can take values in the range of $-\infty \le L_\pm \le \infty$ and $L_0^{(\Phi)}$ and $L_3^{(\Phi)}$ indicate the locations of the two "end points" of the singlet. The VEV of Φ with the BCs, named Robin BCs, in Eq. (22) is expressed in terms of Jacobi's elliptic functions in general and its phase structure has been discussed in Ref [4]. We adopt a specific form in the region $[L_0^{(\Phi)} + \varepsilon, L_3^{(\Phi)} - \varepsilon]$[3]:

$$\langle\Phi(y)\rangle = \left[\frac{M_\Phi}{\sqrt{\lambda_\Phi}}\left\{\sqrt{1+X}-1\right\}^{\frac{1}{2}}\right] \times \frac{1}{\mathrm{cn}\left(M_\Phi\{1+X\}^{1/4}(y-y_0), \sqrt{\frac{1}{2}(1+\frac{1}{\sqrt{1+X}})}\right)}, \quad \left(X := \frac{4\lambda_\Phi|Q|}{M_\Phi^4}\right). \quad (23)$$

Here y_0 and Q are parameters which appear after integration on y and we focus on the choice of $Q < 0$. We note that the values of y_0 and Q are automatically determined after choosing those of $L_\pm$. As shown in Ref. [3], we get the form of $\langle\Phi(y)\rangle$ to be an (almost) exponential function of y by choosing suitable parameter configurations. Although there is a discontinuity in the wavefunction profile of $\langle\Phi\rangle$ between $y = L_0^{(\Phi)} + \varepsilon$ and $y = L_3^{(\Phi)} - \varepsilon$ in Eqs. (22), this type of BCs is derived from the variational principle on S^1 and leads to no inconsistency [4]. The BCs for the 5D $SU(3)_C, SU(2)_W, U(1)_Y$ gauge bosons G_M, W_M, B_M are selected as

$$G_M|_{y=0} = G_M|_{y=L}, \qquad \partial_y G_M|_{y=0} = \partial_y G_M|_{y=L}, \quad (24)$$

where we only show the G_M's case. In this configuration, we obtain the SM gauge bosons in zero modes. Based on the discussion in Section II, we conclude that the W and Z bosons become massive and their masses are suitably created through "our" Higgs mechanism as $m_W \simeq 81\,\mathrm{GeV}, m_Z \simeq 90\,\mathrm{GeV}$. We mention that, on S^1 geometry, $G_y^{(0)}$, $W_y^{(0)}$, and $B_y^{(0)}$ would exist as massless 4D scalars at the tree level, but they will become massive via quantum corrections and are expected to be uplifted to near KK states. We will discuss those modes in another paper. We should note that in our model on S^1 with point interactions, the 5D gauge symmetries are intact under the BCs (2),(8)-(10),(22),(24).

IV. RESULTS

In this section, we would like to find a set of parameter configurations in which the quark mass hierarchy and the structure of the CKM matrix with the CP phase are derived naturally. In the following analysis, we rescale all the dimensional valuables by the S^1 circumference L to make them dimensionless and the rescaled valuables are indicated with the tilde $\tilde{}$.

We set the parameters concerning the scalar singlet Φ as

$$\tilde{M}_\Phi = 8.67, \quad \tilde{y}_0 = -0.1, \quad \tilde{\lambda}_\Phi = 0.001, \quad |\tilde{Q}| = 0.001, \quad (25)$$

where the VEV profile becomes an (almost) exponential function of y, which is suitable for generating the large mass hierarchy. In this case, the values of $L_\pm$ in Eq. (22) correspond to

$$\frac{1}{\tilde{L}_+} = -6.07, \quad \frac{1}{\tilde{L}_-} = 8.69, \tag{26}$$

where the broken phase is realized [3].

As in the previous analysis [3], the signs of the fermion bulk masses are assigned as $M_Q > 0, M_{\mathcal{U}} < 0, M_{\mathcal{D}} > 0$ to make much larger overlapping in up quark sector than in down ones for top mass. Here we assume the positions of the two "end points" of both the quark doublet and the scalar singlet are the same

$$L_0^{(q)} = L_0^{(\Phi)} = 0, \quad L_3^{(q)} = L_3^{(\Phi)} = L, \tag{27}$$

where we set $L_0^{(q)}$ and $L_0^{(\Phi)}$ as zero. In addition, we also assume that the orders of the positions of point interactions are settled as

$$\begin{aligned} &0 < L_0^{(u)} < L_1^{(u)} < L_1^{(q)} < L_2^{(u)} < L_2^{(q)} < L < L_3^{(u)}, \\ &0 < L_0^{(d)} < L_1^{(d)} < L_1^{(q)} < L_2^{(d)} < L_2^{(q)} < L < L_3^{(d)}. \end{aligned} \tag{28}$$

Here our up quark mass matrix $\mathcal{M}^{(u)}$ and that of down ones $\mathcal{M}^{(d)}$ take the forms

$$\mathcal{M}^{(u)} = \begin{bmatrix} m_{11}^{(u)} & m_{12}^{(u)} & m_{13}^{(u)} \\ 0 & m_{22}^{(u)} & m_{21}^{(u)} \\ 0 & 0 & m_{33}^{(u)} \end{bmatrix}, \quad \mathcal{M}^{(d)} = \begin{bmatrix} m_{11}^{(d)} & m_{12}^{(d)} & m_{13}^{(d)} \\ 0 & m_{22}^{(d)} & m_{21}^{(d)} \\ 0 & 0 & m_{33}^{(d)} \end{bmatrix}, \tag{29}$$

where the row (column) index of the mass matrices shows the generations of the left- (right-)handed fermions, respectively. Differently from the model on an interval in Ref. [3], the $(1,3)$ elements of the mass matrices are allowed geometrically due to the periodicity along y-direction.

The parameters which we use for calculation are

$$\begin{aligned} &\tilde{L}_0^{(q)} = 0, && \tilde{L}_1^{(q)} = 0.298, && \tilde{L}_2^{(q)} = 0.659, && \tilde{L}_3^{(q)} = 1, \\ &\tilde{L}_0^{(u)} = 0.0245, && \tilde{L}_1^{(u)} = 0.0260, && \tilde{L}_2^{(u)} = 0.520, && \tilde{L}_3^{(u)} = 1.03, \\ &\tilde{L}_0^{(d)} = 0.0703, && \tilde{L}_1^{(d)} = 0.178, && \tilde{L}_2^{(d)} = 0.646, && \tilde{L}_3^{(d)} = 1.07, \\ &\tilde{M}_Q = 0.654, && \tilde{M}_{\mathcal{U}} = -0.690, && \tilde{M}_{\mathcal{D}} = 0.595, && \theta = 3.0, \end{aligned} \tag{30}$$

where the twist angle θ is a dimensionless value and should be within the range $-\pi < \theta \leq \pi$. We should remind that in our system, the EWSB is only realized on the condition of $M^2 - \left(\frac{\theta}{L}\right)^2 > 0$ as in Eq. (4). Recently, the ATLAS and CMS experiments have announced that the physical Higgs mass is around 126 GeV with 5σ confidence level [5, 6]. $\tilde{\lambda}$ is 0.262 irrespective of the value of L, while $\tilde{M}$ is slightly dependent on the value of L as 3.01303 (3.00052) in the case of $M_{\mathrm{KK}} = 2\,\mathrm{TeV}$ ($M_{\mathrm{KK}} = 10\,\mathrm{TeV}$), where M_{KK} is a typical scale of the KK mode and defined as $2\pi/L$. Here some tuning is required to obtain the suitable values realizing the EWSB.

After the diagonalization of the two mass matrices, the quark masses are evaluated as

$$\begin{aligned} &m_{\mathrm{up}} = 2.06\,\mathrm{MeV}, && m_{\mathrm{charm}} = 1.25\,\mathrm{GeV}, && m_{\mathrm{top}} = 174\,\mathrm{GeV}, \\ &m_{\mathrm{down}} = 4.91\,\mathrm{MeV}, && m_{\mathrm{strange}} = 102\,\mathrm{MeV}, && m_{\mathrm{bottom}} = 4.18\,\mathrm{GeV}, \\ &\frac{m_{\mathrm{up}}}{m_{\mathrm{up}}|_{\mathrm{exp.}}} = 0.897, && \frac{m_{\mathrm{charm}}}{m_{\mathrm{charm}}|_{\mathrm{exp.}}} = 0.978, && \frac{m_{\mathrm{top}}}{m_{\mathrm{top}}|_{\mathrm{exp.}}} = 1.00, \\ &\frac{m_{\mathrm{down}}}{m_{\mathrm{down}}|_{\mathrm{exp.}}} = 1.02, && \frac{m_{\mathrm{strange}}}{m_{\mathrm{strange}}|_{\mathrm{exp.}}} = 1.07, && \frac{m_{\mathrm{bottom}}}{m_{\mathrm{bottom}}|_{\mathrm{exp.}}} = 1.00, \end{aligned} \tag{31}$$

and the absolute values of the CKM matrix elements are given as

$$|V_{\mathrm{CKM}}| = \begin{bmatrix} 0.971 & 0.238 & 0.00318 \\ 0.238 & 0.970 & 0.0372 \\ 0.00829 & 0.0364 & 0.999 \end{bmatrix}, \quad \left|\frac{V_{\mathrm{CKM}}}{V_{\mathrm{CKM}}|_{\mathrm{exp.}}}\right| = \begin{bmatrix} 0.997 & 1.06 & 0.906 \\ 1.06 & 0.997 & 0.902 \\ 0.957 & 0.900 & 1.00 \end{bmatrix}. \tag{32}$$

The Jarlskog parameter J is

$$J = 2.56 \times 10^{-5}, \quad \frac{J}{J|_{\text{exp.}}} = 0.865, \tag{33}$$

where we also provide the differences from the latest experimental values in Ref. [7]. All the deviations from the latest experimental values are within about 15% and we can conclude that the situation of the SM is suitably generated.

V. SUMMARY AND DISCUSSION

In this letter, we proposed a new mechanism for generating CP phase via the Higgs vacuum expectation value originating from geometry of an extra dimension. A twisted boundary condition for the Higgs doublet has been found to lead to an extra dimensional coordinate-dependent VEV with a non-trivial CP phase degree of freedom. This mechanism is useful for realizing CP violation in an extra-dimensional model.

As an application of this idea, we have constructed a phenomenological model with an extra dimension which can simultaneously and naturally explain the origin of the fermion generations, the quark mass hierarchy, and the CKM structure with the CP phase based on [3]. The point interactions realize the three fermion generations and the situation where all the quark profiles are split and localized. With the help of the almost exponential function of the scalar VEV, which appears in the Yukawa sector, we can generate the phenomenologically-desirable circumstances where all the flavor structures are realized with good precision and almost all dimensionless scaled parameters take values of natural $\mathcal{O}(10)$ magnitudes.

One of the most important remaining tasks is to construct a model which can explain both of the quark and lepton flavor structures simultaneously. Then, it is necessary to explain why the neutrino masses are so light and the flavor mixings in the lepton sector are large. The result will be reported elsewhere.

Another important topics is the stability of the system. Our system is possibly threatened with instability. Some mechanisms will be required to stabilize the moduli representing the positions of point interactions (branes). In a multiply-connected space of S^1, there is another origin of gauge symmetry breaking i.e. the Hosotani mechanism. Since further gauge symmetry breaking causes a problem in our model, we need to insure that the Hosotani mechanism does not occur. To this end, we might introduce additional 5D matter to prevent zero modes of y-components of gauge fields from acquiring non-vanishing VEVs. We will leave those issues in future work.

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[1] Y. Fujimoto, K. Nishiwaki, M. Sakamoto, arXiv:1301.7253 [hep-ph]
[2] M. Kobayashi, T. Maskawa, *Prog. Theor. Phys.* **49** (1973), 652.
[3] Y. Fujimoto, T. Nagasawa, K. Nishiwaki, M. Sakamoto, *Prog. Theor. Exp. Phys.* (2013) **023B07**.
[4] Y. Fujimoto, T. Nagasawa, S. Ohya, M. Sakamoto, *Prog. Theor. Phys.* **126** (2011), 841.
[5] ATLAS Collaboration, G. Aad *et. al.*, *Phys. Lett.* **B716** (2012), 1.
[6] CMS Collaboration, S. Chatrchyan *et. al.*, *Phys. Lett.* **B716** (2012), 30.
[7] Particle Data Group Collaboration, J. Beringer *et al.*, *Phys. Rev.* **D86** (2012), 010001.