Measurements of the photon-meson transition form factors at BABAR

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(based on the V.Druzhinin talk at GPD2010)

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What is the $\gamma^*\gamma \to P$ form factor?

The amplitude of the $\gamma^*\gamma \to P$ transition

$$A = e^2 \varepsilon_{\mu \nu \alpha \beta} e_1^\mu e_2^\nu q_1^\alpha q_2^\beta F(q_1^2, q_2^2),$$

where $P$ is a pseudoscalar meson, contains one unknown function, depending on the photon virtualities.

The form factor is usually measured as a function of $Q^2 = |q_1|^2$. The second photon is real or almost real ($q_2^2 \approx 0$).

The form factor is known only for the two extreme cases. For $\pi^0$

$$\lim_{Q^2 \to 0} F(Q^2) = \sqrt{2}/(4\pi^2 f_\pi),$$

from the axial anomaly in the chiral limit, prediction for $\Gamma(\pi^0 \to \gamma\gamma)$

$$\lim_{Q^2 \to \infty} Q^2 F(Q^2) = \sqrt{2} f_\pi.$$

from perturbative QCD

$f_\pi \approx 0.131$ GeV is the pion decay constant
Why is the form factor interesting?

\[ F(Q^2) = \int T(x, Q^2) \varphi(x, Q^2) \, dx \]

Hard scattering amplitude for \( \gamma^* \gamma \rightarrow q\bar{q} \) transition which is calculable in pQCD

Nonperturbative meson distribution amplitude (DA) describing transition \( P \rightarrow q\bar{q} \)

\( x \) is the fraction of the meson momentum carried by one of the quarks

✓ The meson DA \( \varphi(x, Q^2) \) plays an important role in theoretical descriptions of many QCD processes \( (\gamma^* \rightarrow \pi^+\pi^-, \gamma\gamma \rightarrow \pi\pi, \chi_{c,0,1} \rightarrow \pi^+\pi^-, B \rightarrow \pi l\nu, B \rightarrow \pi\pi \ldots) \)

✓ Its shape (\( x \) dependence) is unknown, but its evolution with \( Q^2 \) is predicted by pQCD

✓ The models for DA shape can be tested using data on the form factor \( Q^2 \) dependence
Calculation of the $\gamma^* \gamma \rightarrow \pi^0$ form factor

The leading contribution:

$$Q^2 F(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 \frac{dx}{x} \varphi_\pi(x,Q^2) + O(\alpha_s) + O(\Lambda_{QCD}^2/Q^2)$$

$$\varphi_{ASY} = 6x(1-x)$$  G.P.Lepage and S.J.Brodsky, Phys.Lett. B87, 359 (1979)


- NLO and power corrections are large: 30, 20, 10 % at 4, 10, 50 GeV$^2$.
- Power corrections are 7% at 10 GeV$^2$ (twist-4 + due to hadronic component of a quasi-real photon).
- What is the model uncertainty of the power corrections?

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Calculation of the $\gamma^* \gamma \rightarrow \pi^0$ form factor


$$\varphi(x, Q^2) = \varphi_{ASY} \left[ 1 + \sum_{n \geq 1} a_{2n}(Q^2) C_{2n}^{3/2} (2x - 1) \right], \quad C_{2n}^{3/2} \text{ are Gegenbauer polynomials}$$

The QCD evolution of the DA is very slow. The $Q^2$ needed to decrease the $a_2$ coefficient found at 1 GeV$^2$ by a factor of 3 is about 70000 GeV$^2$
How can the form factor be measured?

- **Two-photon production of the meson**
  - \(-S+M^2 < q_1^2 < 0, \ q_2^2 \approx 0, \ Q^2 \equiv -q_1^2\)
  - \(d\sigma/dQ^2\) falls as \(1/Q^6\)
  - At \(\sqrt{s}=10.6\ \text{GeV}\) for \(e^+e^- \rightarrow e^+e^- \pi^0\)
    \(d\sigma/dQ^2(10\ \text{GeV}^2) \approx 10\ \text{fb}/\text{GeV}^2\)

- **Annihilation process** \(e^+e^- \rightarrow P\gamma\)
  - \(Q^2 = S > M^2\)
  - \(\sigma \propto 1/S^2\)
  - \(\sigma(e^+e^- \rightarrow \eta\gamma) \approx 5\ \text{fb}\) at \(\sqrt{s}=10.6\ \text{GeV}\)

- **Dalitz decay** \(P \rightarrow \gamma\ e^+e^-\)
  - \(0 < Q^2 < M^2\)
  - \(M^2d\Gamma/dQ^2 \approx (2\alpha/\pi)\Gamma(P \rightarrow \gamma\gamma)\) at \(Q^2/M^2 \approx 1/4\)
Available statistics

- The cross section studied is $< 10 \text{ fb} \ (10^{-38} \text{ cm}^2)$
- B-factory at SLAC and BABAR detector
  - peak luminosity is about $10^{34} \text{ cm}^{-2}\text{sec}^{-1}$
  - integrated luminosity collected during 8-year data taking period is about $450 \text{ fb}^{-1}$
- Expected number of events for the $\gamma^*\gamma \rightarrow \pi^0$ form factor measurement is $L \times \sigma \times \varepsilon = 450 \times 10 \times 0.15 \approx 700/\text{GeV}^2$ at $Q^2 = 10 \text{ GeV}^2$
- $dN/dQ^2$ falls with $Q^2$ increase as $Q^{-6}$
- Previous CLEO measurement of the $\gamma^*\gamma \rightarrow \pi^0, \eta, \eta'$ transition form factors (J.Gronberg et al., Phys.Rev. D57, 33 (1998)) was based on 3 fb$^{-1}$
Two-photon reaction $e^+ e^- \rightarrow e^+ e^- P$

- Electrons are scattered predominantly at small angles.
- **Single-tag mode:**
  - one of electrons is detected
  - $Q^2 = -q_1^2 = 2EE/(1-\cos \theta)$,
  - $q_2^2 \approx 0$
  - $F(Q^2,0)$

- ✓ electron is detected and identified
- ✓ meson P are detected and fully reconstructed
- ✓ electron + meson system has low $p_\perp$
- ✓ missing mass in an event is close to zero
Specific features of $e^+ e^- \rightarrow e^+ e^- \pi^0$

- Low final particle multiplicity and only one charged particle (electron).
  - Such events are usually removed at the trigger and filter stages
  - Special trigger line should be designed to select $e^+ e^- \rightarrow e^+ e^- \pi^0$ events
- Large QED background
  - $e^+ e^- \rightarrow e^+ e^- \gamma \gamma$ in which one of the photons is emitted along the beam axis, and one of the electrons is soft
  - Virtual Compton scattering (VCS): $e^+ e^- \rightarrow e^+ e^- \gamma$ with one of the final electrons going along the collision axis
  - The photon from QED process together with a soft photon, for example, from beam background, may give the invariant mass close to the $\pi^0$ mass.
Trigger selection for $e^+e^- \rightarrow e^+e^-\pi^0$

- The $e^+e^- \rightarrow e^+e^-\pi^0$ events do not pass the standard BABAR trigger and background filters.
- Fortunately, a special trigger line was designed to select VCS events (electron+photon with zero recoil mass) for detector calibration.
- Two photons from the $\pi^0$ decay are close and usually form single cluster (with two bumps) in the detector calorimeter.

The VCS trigger treats this cluster as a photon.

The $e^+e^- \rightarrow e^+e^-\pi^0$ events are efficiently selected by the VCS trigger.
Two-photon mass spectrum

The data were divided into 17 $Q^2$ intervals. The size of the interval is increased with $Q^2$ growth.
$e^+e^- \rightarrow e^+e^-\pi^0$, cross section


Systematic uncertainty independent on $Q^2$ is 3%.
$e^+e^- \rightarrow e^+e^-\pi^0$, form factor


In $Q^2$ range $4-9$ GeV$^2$ our results are in a reasonable agreement with CLEO data but have significantly better accuracy.

At $Q^2 > 10$ GeV$^2$ the measured form factor exceeds the asymptotic limit $\sqrt{2}f_\pi = 0.185$ GeV. Most models for the pion distribution amplitude give form factors approaching the limit from below.

Our data in the range $4-40$ GeV$^2$ are well described by the formula

$$Q^2|F(Q^2)| = A \left( \frac{Q^2}{10 \text{ GeV}^2} \right)^{\beta}$$

with $A = 0.182 \pm 0.002$ GeV and $\beta = 0.25 \pm 0.02$, i.e. $F \sim 1/Q^{3/2}$.

Systematic uncertainty independent on $Q^2$ is $2.3\%$. 

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\[ e^+e^- \rightarrow e^+e^-\pi^0, \text{ after publication} \]

S.V. Mikhailov, N.G. Stefanis, Nucl. Phys. B821, 291(2009); arXiv:0909.5128; arXiv: 0910.3498. The NNLO pQCD corrections was partly taken into account. They was estimated to be about 5% at \( Q^2 \sim 10 \text{ GeV}^2 \).

The BABAR data contradict the QCD factorization for any pion DA with the end points (x=0,1) behavior \( \sim x(1-x) \).
\( e^+e^- \to e^+e^-\pi^0 \), after publication


A flat pion distribution amplitude \( \varphi_\pi(x) \approx 1 \) is used to reproduce \( Q^2 \) dependence of BABAR data.

To avoid divergence the infrared regulator \( m^2 \) can be introduced

\[
Q^2 F_{\pi\gamma}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx \frac{\varphi_\pi(x, Q)}{x + m^2/Q^2}
\]

The result has a logarithmic rise with the \( Q^2 \) increase

\[
Q^2 F(Q^2) = \frac{\sqrt{2} f_\pi}{3} \ln \left( 1 + \frac{Q^2}{m^2} \right)
\]

with \( m^2 \approx 0.6 \text{ GeV}^2 \).
$e^+e^- \rightarrow e^+e^-\pi^0$, *after publication*

V.L. Chernyak, arXiv:0912.0623

The twist-4 power correction, $\Delta F/F(Q^2) \sim -(0.6\, \text{GeV}^2)/Q^2$, is only part of the total power correction.

Taking, for example, $\Delta F/F(Q^2) = -1.5/Q^2-(1.2/Q^2)^2$ for CZ DA leads to good data description.
$e^+e^- \rightarrow e^+e^- \eta(\gamma)$, event selection

arXiv:1101.1142v1, submitted to PRD.

$\eta \rightarrow \pi^+\pi^-\pi^0$, $\pi^0 \rightarrow \gamma\gamma$

$N_s = 3060 \pm 70$

$\eta' \rightarrow \pi^+\pi^-\eta$, $\eta \rightarrow \gamma\gamma$

$N_s = 5010 \pm 90$
Mass spectra for $\eta$ and $\eta'$ events

The fit is performed in 11 $Q^2$ intervals.
The systematic uncertainties independent of $Q^2$ are 2.9% for the $\eta$ form factor and 3.5% for the $\eta'$ form factor.
• CLEO (Phys. Rev. D79, 111101, 2009) and BABAR (Phys. Rev. D74, 012002, 2006) data on the time-like transition form factors are added.
• They are extracted from the $e^+e^- \rightarrow \eta^{(/')}\gamma$ cross section measurements at $Q^2=14.2$ GeV$^2$ (CLEO) and 112 GeV$^2$ (BABAR).
• At large $Q^2$ the time- and space-like values are expected to be close.
• This is confirmed by the CLEO result.
• The BABAR time-like data allow to extend the $Q^2$ region up to 112 GeV$^2$
Discussion: $\eta$ and $\eta'$ form factors

The BABAR data are fit with $Q^2F(Q^2)=b+a \ln Q^2$ (GeV$^2$) with $\chi^2/n=6.7/10$ for $\eta$ and 14.6/10 for $\eta'$. The fitted rise ($a \approx 0.2$ GeV$^2$) is about 3 times weaker than that for $\pi^0$.

The fit by a constant for $Q^2>15$ GeV$^2$ also gives reasonable quality: $\chi^2/n=5.6/5$ for $\eta$ and 2.6/5 for $\eta'$. 
$\eta - \eta'$ mixing in the quark flavor basis

$|n\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle)$, $|s\rangle = |\bar{s}s\rangle$, $\phi \approx 41^\circ$

$|\eta\rangle = \cos \phi |n\rangle - \sin \phi |s\rangle$, $|\eta'\rangle = \sin \phi |n\rangle + \cos \phi |s\rangle$.

The form factors for the $|n\rangle$ and $|s\rangle$ states are introduced

$F_\eta = \cos \phi F_n - \sin \phi F_s$, $F_{\eta'} = \sin \phi F_n + \cos \phi F_s$,

with asymptotic limits $Q^2 F_s(Q^2) = \frac{2}{3} f_s$, $Q^2 F_n(Q^2) = \frac{5\sqrt{2}}{3} f_n$,

where decay constants is expected to be $f_n = f_\pi$, $f_s = 1.34 f_\pi$

One can expect that the DA for the $|n\rangle$ state is close to the $\pi^0$ DA. Under this assumption the only difference between the $|n\rangle$ and $\pi^0$ DAs is a factor of 3/5 coming from the quark charges.
Form factor for $|n\rangle$ and $|s\rangle$ state

- The $Q^2$ dependencies of the measured $|n\rangle$ and $\pi^0$ form factors are strongly different.
- The data on the $|n\rangle$ form factor are described well by the model with BMS DA.

- For $|s\rangle$ all data points lie well below the pQCD prediction for the asymptotic DA.
- Is DA for $|s\rangle$ narrower than the asymptotic DA?
- The result for $|s\rangle$ strongly depends on mixing parameters, for example, on a possible two-gluon contents in $\eta'$. 

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Concluding remarks

- After the CLEO publication on the photon-meson transition form factors in 1998 it was generally accepted that the pion DA is close to asymptotic form in near-end-point regions. Many theoretical works (predictions) using such near-asymptotic DAs were published.
- The BABAR measurement indicates that the pion DA is significantly wider than the asymptotic form. If the experiment is correct, many theoretical predictions should be revised.
- The next measurement of the pion-photon transition form factor confirming or refuting BABAR result will be performed at Super-B factories in 5-10 years. Trigger!
- Therefore, study of other reactions sensitive to the DA shape and careful theoretical analysis of already measured reactions should be performed.
Concluding remarks

- The processes with pseudoscalars, which have already been measured and which theoretical description should be updated:
  - The $\gamma^*\gamma \rightarrow \eta(\ell)$ transition form factors. There are new BABAR data.
  - The pion and kaon electromagnetic form factors. There are recent CLEO time-like measurements at $Q^2=14$ GeV$^2$
  - Belle measurements of the $\gamma\gamma \rightarrow \pi\pi, KK, \eta\pi$ cross sections for $W_{\gamma\gamma}$ up to 4.1 GeV
  - $\chi_{c,0,2} \rightarrow \pi\pi, KK, \eta\eta, \ldots$ (BELLE, CLEO, BES)
  - ...
Concluding remarks

The processes sensitive to the pseudoscalar DA shape which can be measured using B-factory data

- $\gamma\gamma \rightarrow \eta\eta, \eta'/\eta', \eta\eta'$
- single tag studies of $\gamma\gamma$ reactions: $\pi^+\pi^-$, $\eta\pi$, …
- update of the $e^+e^-\rightarrow\eta(\gamma)$ cross section measurements
- kaon electromagnetic form factor at 112 GeV$^2$
- $e^+e^-\rightarrow VP$
- …
\textbf{e}^+\text{e}^- \rightarrow \text{VP cross sections}

\begin{align*}
F(\gamma^* \rightarrow \text{VP}) & \sim \int_{\delta}^{1} \frac{\varphi_p(x)}{x^2} \, dx \quad \text{with} \quad \delta = \frac{m_0^2}{s}, \\
\text{for } \varphi_p(x) \sim x(1-x): \quad & \sigma(e^+e^- \rightarrow \text{VP}) \sim \frac{\ln^2 \left( \frac{s}{s_0} \right)}{s^4}, \\
\text{for flat } \varphi_p(x): \quad & \sigma(e^+e^- \rightarrow \text{VP}) \sim \frac{1}{s^2}.
\end{align*}

V.L. Chernyak, 
\texttt{arXiv:0912.0623}

The $\gamma^* \rightarrow \text{VP}$ form factors are highly sensitive to the end-point behavior of the pseudoscalar DA.

- The $e^+e^- \rightarrow \text{VP}$ cross sections have been measured by CLEO for $V=\rho, \omega, \phi$, and $P=\pi, \eta, \eta'$ at $s=14 \text{ GeV}^2$.
- The BABAR and Belle have performed measurements for $\phi\eta^{(*)}$, $\rho\eta^{(*)}$ at $112 \text{ GeV}^2$. The cross section's dependencies reasonably agree with the QCD predictions for conventional DA's.
- The cross sections for all other VP combinations definitely can be measured at BABAR and Belle.
- The expected cross section for the $\omega\pi$ final state at $112 \text{ GeV}^2$ is about 4 fb for a conventional DA and 200 fb for flat DA.
Summary

- The $\gamma^*\gamma \rightarrow \pi^0$, $\eta$, $\eta'$ transition form factors have been measured for $Q^2$ range from 4 to 40 GeV$^2$.
- The unexpected $Q^2$ dependence of the $\gamma^*\gamma \rightarrow \pi^0$ form factor is observed.
- The measured $Q^2$ dependencies for the $\gamma\gamma^* \rightarrow \eta$ and $\gamma\gamma^* \rightarrow \eta'$ transition form factors strongly differ from that for $\gamma\gamma^* \rightarrow \pi^0$.
- The $\eta'$ data are in good agreement with the result of QCD calculation with a conventional DA.
- For $\eta$ the agreement is worse. A mild logarithmic rise of $Q^2F(Q^2)$ is not excluded.
- There are many processes sensitive to the DA shape measured and not measured yet. The theoretical input is required to stimulate experimentalists.
$e^+e^- \rightarrow e^+e^- \eta_c$, form factor

- The form factor is normalized to $F(0)$ obtained from no-tag data
- The form factor data are fit with the monopole function

$$F(Q^2) = F(0)/(1 + Q^2 / \Lambda)$$

- The result $\Lambda = 8.5 \pm 0.6 \pm 0.7$ GeV$^2$ does not contradict to the vector dominance model with $\Lambda = m_{J/\psi}^2 = 9.6$ GeV$^2$.
- pQCD: Due to relatively large $c$-quark mass, the $\eta_c$ form factor is rather insensitive to the shape of the $\eta_c$ distribution amplitude. $\Lambda$ is expected to be about 10 GeV$^2$ (T. Feldmann, P.Kroll, Phys. Lett. B 413, 410 (1997)).
- Lattice QCD: $\Lambda = 8.4 \pm 0.4$ GeV$^2$ (J.J.Dudek, R.G.Edwards, Phys. Rev. Lett. 97, 172001 (2006)).

Systematic uncertainty independent of $Q^2$ is 4.3%.